ANALYSIS AND NUMERICAL AVERAGE RUN LENGTH OF MEWMA, EEWMA, AND NEEWMA CONTROL CHARTS USING THE MARKOV CHAIN APPROACH

MR. APITAD KRAICHOK

A DISSERTATION SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN
APPLIED STATISTICS
DEPARTMENT OF APPLIED STATISTICS
GRADUATE COLLEGE
KING MONGKUT'S UNIVERSITY OF TECHNOLOGY NORTH BANGKOK
ACADEMIC YEAR 2026

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ABSTRACT

This research presents a method for computing the average run length (ARL) of the Modified Exponentially Weighted Moving Average (MEWMA), Extended Exponentially Weighted Moving Average (EEWMA), and New Extended Exponentially Weighted Moving Average (NEEWMA) control charts, which extend the Exponentially Weighted Moving Average (EWMA) control chart and are efficient in detecting small shifts in a process. The ARL is computed by the Markov Chain Approach (MCA), which relies on the cumulative distribution function and on the transitions between in-control and out-of-control states. The monitoring statistics of these charts are formed from more than one observation, so that the transition probability is governed by a joint rather than a marginal distribution and the classical construction cannot be applied directly. Two approaches are therefore proposed: the Joint Distribution Approach (JDA), which evaluates the joint distribution exactly and is developed for the MEWMA chart, and the Point Estimator Approach (PEA), which replaces the lagged observations by an estimator of the process mean and extends to a statistic of any lag depth. Closed-form expressions for the ARL and the standard deviation of the run length are derived, and the process is assumed to follow continuous symmetric, continuous asymmetric, and discrete distributions. Accuracy is assessed by five measures of error together with the computation time required. The proposed method agrees closely with Monte Carlo simulation, the two never differing by as much as one run-length unit under a standard normal process, and its computation time is fixed by the number of partitions of the state space alone, whereas the time required by simulation rises with the length of the runs generated. The method is further used to explain chart behaviour under each form of process distribution, where the effect of the second and third weighted parameters is found to reverse between distributional families, and the charts are applied to four real-world datasets.

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Keywords: average run length, Markov chain approach, exponentially weighted moving average, control chart, statistical process control

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CHAPTER 1

Introduction

This chapter introduces the background and motivation of the present study, which is concerned with the estimation of the Average Run Length (ARL) of EWMA-type control charts by means of the Markov chain approach (MCA) method. The discussion begins with the role of statistical process control (SPC) in contemporary industrial practice and the development of control charts for detecting shifts in the process mean, leading to a consideration of the existing methods for computing the ARL and the limitations associated with each of them, from which the research problem is identified. The objectives of the study, the scope within which the investigation is confined, and the anticipated benefits of the research are then stated in turn. The chapter is organized into the following sections:

1.1 Background and Statement of the Problem
1.2 Purpose of the Study
1.3 Scope of the Study
1.4 Benefits of the Study

The details pertaining to each section are presented in turn below.

1.1 Background and Statement of the Problem

Currently, the primary factor in industrial operations is output of goods, which are crucial for generating revenue for companies or industries. Therefore, producing goods quickly and with minimal error is essential. Today, various industries use Statistical Process Control (SPC) as a key tool for monitoring, controlling, and improving production processes to ensure they meet specified standards. One of the most widely used methods in SPC is the control chart, which is divided into two types: quantitative control charts and qualitative control charts. These two types of charts differ according to the nature of the data being monitored:

1) Quantitative control charts are used for continuous data or data that track continuous distribution, such as water weight, rainfall amount, or stock prices.

2) Qualitative control charts are used for categorical or discrete data, such as the number of defective products or the number of surface defects of a product.

SPC tools, or control charts, are widely accepted and considered a fundamental technique. Control charts display data over time with control limits that specify the acceptable range for a process under normal conditions. If the measured values are within this range, the process is under control; otherwise, the process is out of control.

The first control chart was introduced by Shewhart in 1924, designed to detect changes in process mean values. And it is extremely effective in identifying large changes (Shewhart, 1924). In 1959, Roberts introduced the Exponentially Weighted Moving Average (EWMA) control chart, an effective alternative to the traditional

Shewhart chart for detecting small changes in the process mean. Currently, EWMA control charts have been developed in various forms as follows:

1) Modified Exponentially Weighted Moving Average (MEWMA) (Patel & Divecha, 2011)
2) Extended Exponentially Weighted Moving Average (EEWMA) (Naveed et al., 2018)
3) New Extended Exponentially Weighted Moving Average (NEEWMA) (Javed et al., 2024)

The selection of an appropriate control chart for quality control considers the Average Run Length (ARL), which is the expected value of the Run Length (RL), which is the number of samples taken before the process exceeds the control limit.

Generally, ARL is estimated using the Monte Carlo simulation approach (MC) to model process data and calculate ARL, or by applying mathematical methods such as the Markov chain approach (MCA) and integral equations, or the Martingale method, each with its own limitations.

Among these methods, the Markov chain approach is attractive because it yields a closed-form expression that can be evaluated once rather than re-simulated for every parameter setting. Its classical formulation, however, rests on an assumption that the EWMA-type charts considered here are not satisfy. In the formulation of Brook and Evans (1972), the monitoring statistic at any point in time is a function of the current observation and of the preceding value of the statistic alone, so that the probability of moving from one state to another reduces to a difference of two univariate distribution functions. The MEWMA statistics, by contrast, incorporate the difference between the current observation and the one immediately preceding it, while the EEWMA and NEEWMA statistics assign separate weights to two and three consecutive observations respectively. In each case the observations entering the statistics are more than one, and the transition probability is therefore governed by a joint rather than a marginal distribution. The classical construction cannot be applied to these charts without modification, and for this reason no closed-form expression for their ARL has previously been available.

The consequence is that practitioners have been obliged to rely on Monte Carlo simulation, whose cost is proportional to the length of the runs being generated and therefore rises sharply at the in-control levels commonly prescribed in practice. The problem addressed by this study is thus the construction of a transition probability matrix that accommodates monitoring statistics depending on several consecutive observations, so that the ARL of the MEWMA, EEWMA, and NEEWMA control charts may be obtained in closed form.

This research proposes estimating the average run length (ARL) of the MEWMA, EEWMA, and NEEWMA control charts using the MCA method, by means of two constructions of the transition probability matrix that accommodate the lagged

observations entering each statistic and compares the resulting ARL values and computation times with those obtained by the MC method.

1.2 Purpose of the Study

1.2.1 To construct closed-form formulas for the ARL of the MEWMA, EEWMA, and NEEWMA control charts using the Markov chain approach (MCA).

1.2.2 To compare the efficiency of the closed-form ARL formulas obtained from the MCA with the Monte Carlo simulation approach (MC) in terms of closeness of estimation and computational time.

1.2.3 To analyses the performance of the EEWMA and NEEWMA control charts using the MCA method under symmetric, asymmetric, and discrete process distributions.

1.2.4 To apply the MEWMA, EEWMA, and NEEWMA control charts to real-world data in various fields.

1.3 Scope of the Study

1.3.1. Quality control charts, as follows:

1.3.1.1. Exponentially Weighted Moving Average (EWMA) Control Chart

1.3.1.2. Modified Exponentially Weighted Moving Average (MEWMA) Control Chart

1.3.1.3. Extended Exponentially Weighted Moving Average (EEWMA) Control Chart

1.3.1.4. New Extended Exponentially Weighted Moving Average (NEEWMA) Control Chart

1.3.2. Distribution of Data Processes as follows:

1.3.2.1. Symmetric distribution as follows:

1) Normal distribution
2) Logistic distribution
3) Laplace distribution

1.3.2.2. Asymmetric distribution as follows:

1) Gamma distribution
2) Weibull distribution
3) Gumbel distribution
4) Two-parameter Rayleigh distribution

1.3.2.3. Discrete distribution as follows:

1) Poisson distribution
2) Binomial distribution
3) Negative Binomial distribution

1.3.3. Methods for calculating the Average (ARL) and Standard Deviation (SDRL) Run Length as follows:

1) Monte Carlo simulation approach (MC)

2) Markov chain approach (MCA)

1.4 Benefits of the Study

1.4.1 Closed-form formulas for the ARL of the MEWMA, EEWMA, and NEEWMA control charts are obtained, providing an analytical alternative to simulation-based computation.

1.4.2 The MCA method computes the ARL with an accuracy comparable to that of the Monte Carlo simulation approach while requiring considerably less computational time.

1.4.3 The performance of the EEWMA and NEEWMA control charts under symmetric, asymmetric, and discrete process distributions is established, providing guidance on the selection of chart parameters appropriate to each process condition.

1.4.4 The proposed control charts can be applied to real-world data in various fields, and the approach may serve as a guideline for developing ARL computation methods for other types of control charts.

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CHAPTER 2

Literature Review

This chapter provides the theoretical foundation underlying the estimation of the Average Run Length (ARL) by means of the Markov chain approach (MCA) method. The discussion encompasses the theory of Markov chains, the properties of EWMA-type control charts, and the statistical distributions employed to characterize the underlying process, all of which constitute the theoretical basis upon which the proposed methodology in the subsequent chapters is developed. The chapter is organized into the following sections:

2.1 Statistical Distributions
2.2 Control charts
2.3 Run Length (RL)
2.4 Monte Carlo simulation approach (MC)
2.5 Markov chain approach (MCA)

The theoretical details pertaining to each section are presented in turn below.

2.1 Statistical Distributions

In this section, we present the statistical distributions used in this study to describe the process being monitored. The distribution matters because the run-length behavior of a control chart depends on where the monitoring statistic comes from. In practice, many industrial and service processes are skewed or heavy-tailed, and quality characteristics such as the number of defective items are discrete by nature. The distributions considered here are therefore divided into three groups: symmetric, asymmetric, and discrete. This allows the proposed method to be tested across a wide range of process conditions rather than under a single distributional assumption. For each distribution, the probability density or probability mass function is given, along with its mean, variance, and other properties relevant to this study.

2.1.1 Symmetric distribution

Symmetric distributions are a class of probability distributions in which the left and right sides of the distribution are mirror images of each other around a central point, typically the mean μ. Formally, a distribution is symmetric if its probability density function (PDF) satisfies:

$$f(x-\mu)=f(x+\mu)$$

This property implies that the mean, median, and mode coincide at the center of the distribution. Symmetric distributions are commonly used in statistical modeling due to their mathematical simplicity, ease of analysis, and clear interpretability. However, it is important to acknowledge that many real-world industrial and service processes

often display skewness, heavy tails, or discrete characteristics because of operational variability and rare events. Thus, assuming symmetry should be considered mainly a simplifying assumption that aids theoretical development rather than an accurate reflection of all practical data scenarios.

In this study, we focus on three important symmetric distributions: the normal, logistic, and Laplace distributions.

2.1.1.1 Normal distribution

Normal distribution, commonly referred to as the Gaussian distribution, is one of the most widely used symmetric distributions in statistics and process monitoring. Its bell-shaped curve distinguishes it and is widely adopted due to its robust theoretical properties and analytical tractability. PDF is defined as follows:

$$f(x)=\frac{1}{\sqrt{2\pi\beta^2}}\exp\left\{-\frac{(x-\mu)^2}{2\beta^2}\right\};\ x\in\mathbb{R}$$

where μ is the location parameter and $\beta>0$ is the scale parameter.

The mean and variance of the distribution are $E(X)=\mu$ and $V(X)=\beta^2$. The moment generating function (MGF) is

$$M_X(t)=\exp\left\{\mu t+\frac{\beta^2t^2}{2}\right\}.$$

Theorem 1: Let $X\sim N(\mu,\beta^2)$, then $Y=aX+b\sim N(a\mu+b,a^2\beta^2)$

Proof

$$M_Y(t)=e^{bt}M_X(at)=\exp\left\{(a\mu+b)t+\frac{(a\beta)^2t^2}{2}\right\}.$$

So that $M_Y(t)=\exp\left\{(a\mu+b)t+\frac{(a\beta)^2t^2}{2}\right\}$, hence $Y=aX+b\sim N(a\mu+b,a^2\beta^2)$.

Remark 1: If $a=\frac{1}{\beta}$ and $b=-\frac{\mu}{\beta}$ then $Y=\frac{X-\mu}{\beta}\sim N(0,1)$.

2.1.1.2 Logistic distribution

The logistic distribution is another symmetric distribution that closely resembles the normal distribution in shape but features heavier tails, thereby providing increased robustness to moderate deviations and extreme observations. Owing to its analytical simplicity and heavy-tailed characteristics, it is frequently employed in probabilistic modeling and robust statistical analysis. PDF is expressed as follows:

$$f(x)=\left(1+\exp\left\{-\left(\frac{x-\mu}{\beta}\right)\right\}\right)^{-2}\exp\left\{-\left(\frac{x-\mu}{\beta}\right)\right\};\ x\in\mathbb{R}.$$

where μ is the location parameter and $\beta>0$ is the scale parameter.

The mean and variance of the distribution are $E(X)=\mu$ and $V(X)=\frac{\pi^2\beta^2}{3}$. The moment generating function (MGF) is

$$M_X(t)=\frac{1}{1-\beta^2t^2}\exp\{\mu t\};\ |t|<\frac{1}{\beta}.$$

Theorem 2: Let $X\sim LG(\mu,\beta)$, then $Y=aX+b\sim LG(a\mu+b,a\beta)$.

Proof

$$M_Y(t)=e^{bt}M_X(at)=\frac{1}{1-(a\beta)^2t^2}\exp\{(a\mu+b)t\}.$$

So that $M_Y(t)=\frac{1}{1-(a\beta)^2t^2}\exp\{(a\mu+b)t\}$, hence $Y=aX+b\sim LG(a\mu+b,a\beta)$.

Remark 2: If $a=\frac{1}{\beta}$ and $b=-\frac{\mu}{\beta}$ then $Y=\frac{X-\mu}{\beta}\sim LG(0,1)$.

2.1.1.3 Laplace distribution

The Laplace distribution, also referred to as the double exponential distribution, is characterized by a sharper peak at the center and heavier tails compared to the normal distribution. These features render it particularly suitable for modeling data exhibiting abrupt changes or greater variability around the central value. PDF is expressed as follows:

$$f(x)=\frac{1}{2\beta}\exp\left\{-\left|\frac{x-\mu}{\beta}\right|\right\};\ x\in\mathbb{R}.$$

where μ is the location parameter and $\beta>0$ is the scale parameter.

The mean and variance of the distribution are $E(X)=\mu$ and $V(X)=2\beta^2$. The moment generating function (MGF) is

$$M_X(t)=\exp\{\mu t\}\mathrm{B}(1-\beta t,1+\beta t);\ t\in\left(-\frac{1}{\beta},\frac{1}{\beta}\right).$$

where $\mathrm{B}(a,b)$ is the beta function with $\mathrm{B}(a,b)=\int_0^1 t^{a-1}(1-t)^{b-1}dt$.

Theorem 3: Let $X\sim LP(\mu,\beta)$, then $Y=aX+b\sim LP(a\mu+b,a\beta)$.

Proof

$$M_Y(t)=e^{bt}M_X(at)=\exp\{(a\mu+b)t\}\mathrm{B}(1-a\beta t,1+a\beta t).$$

So that $M_Y(t)=\exp\{(a\mu+b)t\}\mathrm{B}(1-a\beta t,1+a\beta t)$, hence

$$Y=aX+b\sim LP(a\mu+b,a\beta).$$

Remark 3: If $a=\frac{1}{\beta}$ and $b=-\frac{\mu}{\beta}$ then $Y=\frac{X-\mu}{\beta}\sim LP(0,1)$.

2.1.2 Asymmetric distribution

In statistical modeling, asymmetric (or skewed) distributions play a crucial role in describing phenomena where observations are not symmetrically distributed around the mean. Unlike symmetric distributions such as the normal distribution, asymmetric distributions provide greater flexibility for modeling real-world data characterized by skewness, heavy tails, or extreme values. These distributions are commonly classified into location–scale and shape–scale families according to the role of their parameters in controlling location, dispersion, and skewness.

This section focuses on four widely used asymmetric distributions, which are classified into two families. The shape–scale family includes the Gamma and Weibull distributions, and the location–scale family includes the Gumbel and two-parameter Rayleigh distributions. The main characteristics and applications of these distributions are discussed in the following subsections.

2.1.2.1 Shape-scale Family distribution

The shape-scale family comprises probability distributions characterized by a scale parameter and shape parameters. Let Y denote a standardized random variable whose distribution depends only on the shape parameter. A random variable X is said to belong to a shape-scale family if it can be represented as

$$X=\beta Y,$$

Where $\beta>0$ is the scale parameter, and Y is a standardized random variable governed by one or more shape parameters. Consequently, shape-scale distributions provide considerable flexibility for modeling positive and asymmetric data frequently encountered in reliability analysis, survival studies, environmental sciences, and risk assessment.

2.1.1.1.1 Gamma distribution

The gamma distribution is a continuous probability distribution commonly used to model waiting times and positively skewed data. It is defined by two parameters: the shape parameter $\alpha>0$ and a scale parameter $\beta>0$. Its probability density function (PDF) is given by:

$$f(x)=\frac{x^{\alpha-1}}{\Gamma(\alpha)\beta^{\alpha}}\exp\left\{-\frac{x}{\beta}\right\};\ x>0.$$

and the mean and variance of the distribution are $E(X)=\alpha\beta$ and $V(X)=\alpha\beta^2$.

2.1.1.1.2 Weibull distribution

The Weibull distribution is extensively used in reliability engineering and survival analysis. It is characterized by a shape parameter $\alpha>0$ and a scale parameter $\beta>0$. Its PDF is given by:

$$f(x)=\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\exp\left\{-\left(\frac{x}{\beta}\right)^{\alpha}\right\};\ x>0.$$

and the mean is and variance of the distribution are

$$E(X)=\beta\cdot\Gamma\left(1+\frac{1}{\alpha}\right)\text{ and }V(X)=\beta^2\left(\Gamma\left(1+\frac{2}{\alpha}\right)-\left(\Gamma\left(1+\frac{1}{\alpha}\right)\right)^2\right).$$

2.1.2.2 Location-scale Family distribution

The location-scale family consists of probability distributions that can be generated through a linear transformation of a standard random variable. These distributions are characterized by two parameters: a location parameter and a scale parameter, which control the position and dispersion of the distribution, respectively. A random variable X belongs to a location-scale family if it can be expressed as

$$X=\theta+\beta Z,$$

where $\theta\in\mathbb{R}$ is the location parameter, $\beta>0$ is the scale parameter, and Z is a standardized random variable with a fixed reference distribution. Owing to their flexibility and interpretability, location-scale distributions are widely used in statistical modeling and data analysis.

2.1.1.2.1 Gumbel distribution

The Gumbel distribution, also known as the extreme value type I distribution, is used to model the distribution of extreme values such as maxima or minima. It is defined by a location parameter $\theta\in\mathbb{R}$ and a scale parameter $\beta>0$. The PDF is given by:

$$f(x)=\frac{1}{\beta}\exp\left\{-\left(\left(\frac{x-\theta}{\beta}\right)+\exp\left\{-\left(\frac{x-\theta}{\beta}\right)\right\}\right)\right\};\ x\in\mathbb{R}.$$

and the mean and variance of the distribution are $E(x)=\theta+\gamma\beta$ and $V(x)=\frac{\pi^2}{6}\beta^2$

where $\gamma=0.577216$ denotes the Euler-Mascheroni constant.

2.1.1.2.2 Two-parameter Rayleigh distribution

The two-parameter Rayleigh distribution is an extension of the standard Rayleigh distribution that includes both location and scale

parameters. It is defined by a location parameter θ and a scale parameter $\beta > 0$. Its PDF is given by:

$$f(x) = \left(\frac{x-\theta}{\beta^2}\right)\exp\left\{-\frac{1}{2}\left(\frac{x-\theta}{\beta}\right)^2\right\};\ x > \theta.$$

and the mean and variance of the distribution are $E(x) = \theta + \sqrt{\frac{\pi}{2}}\beta$ and $V(x) = \frac{4-\pi}{2}\beta^2$.

2.1.3 Discrete distribution

Discrete distributions are fundamental tools in statistical modeling for describing the probability characteristics of countable outcomes. A key discrete distribution is based on the concept of a Bernoulli trial, which is a random experiment with exactly two possible outcomes, "success" and "failure", occurring with probabilities p and $1-p$, respectively.

Bernoulli trials provide the foundation for several important discrete distributions, including the binomial, negative binomial, and Poisson distributions. These distributions differ not only in their probability structures but also in their dispersion properties. Dispersion refers to the relationship between the variance and the means of a distribution. In count data analysis, three common cases are often considered: **equidispersion**, where the variance equals the mean; **overdispersion**, where the variance exceeds the mean; and **underdispersion**, where the variance is smaller than the mean. Understanding these properties is important for selecting an appropriate probabilistic model.

2.1.3.1 Binomial distribution

The binomial distribution arises as the sum of independent Bernoulli trials and models the number of successes in a fixed number of trials. It is defined by the number of trials $n \in I^+$ and the probability of success $p \in (0,1)$. The probability mass function (PMF) is given by

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x};\ x = 0,1,...,n,\ n \in I^+.$$

The mean and variance of distribution are $E(X) = np$ and $V(X) = np(1-p)$. Since $V(X) = (1-p)E(X) < E(X)$, it exhibits **underdispersion**, meaning that its variance is smaller than its mean.

2.1.3.2 Negative Binomial distribution

The negative binomial distribution extends the Bernoulli trial framework by modeling the number of failures required to achieve a fixed number of successes. It is defined by the number of successes $r \in I^+$ and the probability of success $p \in (0,1)$. The PMF is

$$p(x) = \binom{x+r-1}{x} p^r (1-p)^x;\ x = 1,2,...,\ r \in I^+.$$

The mean and variance of the distribution are $E(X) = \frac{r(1-p)}{p}$ and $V(X) = \frac{r(1-p)}{p^2}$. Since $V(X) = \frac{E(X)}{p} > E(X)$, it exhibits **overdispersion**, making it particularly useful for modeling count data with variability exceeding that predicted by the Poisson distribution.

2.1.3.3 Poisson distribution

The Poisson distribution models the number of events occurring within a fixed interval under the assumptions of independence and a constant event rate. It is defined by the average rate of occurrence $\lambda > 0$. The PMF is

$$p(x) = \frac{e^{-\lambda}\lambda^x}{x!};\ x = 1,2,...$$

The mean and variance are $E(X) = V(X) = \lambda$. Since the mean and variance are equal, the Poisson distribution is characterized by **equidispersion**.

2.2 Control charts

In this section, we present the control charts employed in this study, including the EWMA, MEWMA, EEWMA, and NEEWMA control charts. Control charts are statistical monitoring tools used to assess whether a process operates under stable conditions by comparing calculated monitoring statistics against predefined control limits. The process observations are assumed to be sequentially collected over time and are represented as follows:

Let X_{it} is a random sample from a statistical distribution with mean μ_0 and variance σ_0^2 with subsample $i = 1,2,\ldots,n$ and time $t = 1,2,\ldots$

2.2.1 Exponentially Weighted Moving Average (EWMA) Control Chart

The EWMA control chart was originally proposed by Roberts. It is designed to enhance the detection of small to moderate shifts in the process by assigning greater weight to recent observations, while still incorporating historical

data through a recursive structure. Due to its computational simplicity, smoothing ability, and effectiveness in continuous monitoring, the EWMA chart is widely adopted in statistical quality control. However, its performance relies on assumptions such as independent observations and appropriate parameter selection, which may affect effectiveness in practical scenarios involving autocorrelation or non-standard process behaviors. The EWMA statistics are expressed as follows:

For the process without subsample:

$$Z_t = \lambda X_t + (1-\lambda) Z_{t-1}.$$

For the process with subsample:

$$Z_t = \lambda \bar{X}_t + (1-\lambda) Z_{t-1}.$$

Where Z_t is the EWMA statistic at time t, and λ_1 is the weighted parameter; $0 < \lambda_1 \leq 1$.

The mean and variance of the EWMA statistic are $E(Z_t) = \mu_0$ and $V(Z_t) = Q^Z \sigma^2$, where: $Q^Z = \dfrac{\lambda_1}{2-\lambda_1}$.

The upper and lower control limits of EWMA chart are as follows:

$$UCL^Z / LCL^Z = \mu_0 \pm L_Z \sigma_0 \sqrt{Q^Z}.$$

where L_Z is the limit parameter of EWMA, which represents the width of the upper control limit and lower control limit of the EWMA control chart. The EWMA stopping time (τ_Z) can then be written as follows:

$$\tau_Z = \inf\{t > 0 : Z_t > UCL_Z \text{ or } Z_t < LCL_Z\}.$$

2.2.2 Modified Exponentially Weighted Moving Average (MEWMA) Control Chart

In 2011, Patel and Divecha proposed and developed an improved EWMA control chart that is highly effective in detecting small shifts in the process mean. This control chart addresses issues related to both independent and autocorrelated observations. Later, Khan et al. redesigned the structure of the modified EWMA control statistics. Their new control chart statistics were developed to enhance the effectiveness of the modified EWMA control charts through the adjustment of control chart parameters. Specifically, two parameters with different weighting schemes were introduced, as described below. The MEWMA statistics are formulated as follows

For the process without subsample:

$$M_t = \lambda_1 X_t + (1-\lambda_1) M_{t-1} + k(X_t - X_{t-1}).$$

For the process with subsample:

$$M_t = \lambda_1 \bar{X}_t + (1-\lambda_1) M_{t-1} + k\left(\bar{X}_t - \bar{X}_{t-1}\right).$$

It can be rewritten as follows:

For the process without subsample:

$$M_t = (\lambda_1 + k) X_t - kX_{t-1} + (1-\lambda_1) M_{t-1}.$$

For the process with subsample:

$$M_t = (\lambda_1 + k) \bar{X}_t - k\bar{X}_{t-1} + (1-\lambda_1) M_{t-1}.$$

where M_t is MEWMA statistic at time t, where the no change statistic X_0 is independent random variable the same distribution as a random sample.

λ_1 is a weighted parameter; $0 < \lambda_1 \leq 1$.

k is a different parameter; $k \in \mathbb{R}$

The mean and variance of MEWMA statistic is $E(M_t) = \mu_0$ and $V(M_t) = Q_t \cdot \sigma_0^2$, respectively. The upper and lower control limits of MEWMA chart are

$$UCL^M / LCL^M = \mu_0 + L_M \sigma_0^2 \sqrt{Q^M}$$

where $Q^M = \dfrac{\lambda_1 + 2\lambda_1 k + 2k^2}{2 - \lambda_1}$

The corresponding stopping time for the MEWMA procedure is given by:

$$\tau_M = \inf\{t > 0 : M_t > UCL_M \text{ or } M_t < LCL_M\}.$$

2.2.3 Extended Exponentially Weighted Moving Average (EEWMA) Control Chart

The EEWMA control chart was introduced by Naveed et al. as an extension of the traditional EWMA. By incorporating an additional weighting mechanism, the EEWMA chart offers greater flexibility and improved sensitivity for detecting small to moderate shifts while maintaining robustness against random variability. This additional smoothing enhances adaptability to various monitoring environments; however, it also increases the chart's dependence on parameter configuration and model specification. Careful parameter calibration is therefore necessary to balance shift detection sensitivity with false alarm control. The EEWMA statistics are defined as follows:

For the process without subsample:

$$E_t = \lambda_1 X_t - \lambda_2 X_{t-1} + (1 - \lambda_1 + \lambda_2) E_{t-1}.$$

For the process with subsample:

$$E_t = \lambda_1 \bar{X}_t - \lambda_2 \bar{X}_{t-1} + (1 - \lambda_1 + \lambda_2) E_{t-1}.$$

where E_t is the EEWMA statistic at time t, and λ_1 is the weighted parameter; $0<\lambda_1\leq 1$, λ_2 is the lag-1 weighted parameter; $0\leq\lambda_2<\lambda_1$.

The mean and variance of the EEWMA statistic are $E(E_t)=\mu_0$ and $V(E_t)=Q^E\sigma^2$, where:

$$Q^E=\frac{\lambda_1^2+\lambda_2^2-2\lambda_1\lambda_2(1-\lambda_1+\lambda_2)}{2(\lambda_1-\lambda_2)-(\lambda_1-\lambda_2)^2},$$

It can be expanded to a simplified formula, that is,

$$Q^E=\frac{\lambda_1-\lambda_2+2\lambda_1\lambda_2}{2-\lambda_1+\lambda_2}.$$

The upper and lower control limits of EEWMA chart are as follows:

$$UCL^E/LCL^E=\mu_0\pm L_E\sigma_0\sqrt{Q^E},$$

where L_E is the limit parameter of EEWMA, which represents the width of the upper control limit and lower control limit of the EEWMA control chart. The EEWMA stopping time (τ_E) can then be written as follows:

$$\tau_E=\inf\{t>0: E_t>UCL_E \text{ or } E_t<LCL_E\}.$$

Furthermore, the EEWMA control chart becomes the EWMA control chart when $\lambda_2=0$.

2.2.4 New Extended Exponentially Weighted Moving Average (NEEWMA) Control Chart

To further enhance monitoring performance, Javed et al (2024) proposed the NEEWMA control chart by adding another exponential weighting parameter to the EEWMA framework. This additional smoothing parameter allows the NEEWMA chart to better balance responsiveness to subtle process shifts with noise reduction, improving detection performance for small to moderate changes. Nevertheless, the presence of multiple smoothing parameters increases model complexity and sensitivity to parameter selection, potentially complicating practical implementation and interpretation. Improper parameter choices may negatively impact monitoring effectiveness or robustness under varying process conditions. The NEEWMA statistics are expressed as follows:

For the process without subsample:

$$N_t=\lambda_1X_t-\lambda_2X_{t-1}-\lambda_3X_{t-2}+(1-\lambda_1+\lambda_2+\lambda_3)N_{t-1}$$

For the process with subsample:

$$N_t=\lambda_1\bar{X}_t-\lambda_2\bar{X}_{t-1}-\lambda_3\bar{X}_{t-2}+(1-\lambda_1+\lambda_2+\lambda_3)N_{t-1}$$

where N_t is the NEEWMA statistic at time t, and λ_1 is the weighted parameter; $\lambda_2+\lambda_3<\lambda_1\le 1$, λ_2 is the lag-1 weighted parameter; $\lambda_3<\lambda_2<\lambda_1$, λ_3 is the lag-2 weighted parameter; $0\le\lambda_3<\lambda_2$.

The mean and variance of the NEEWMA statistic are $E(N_t)=\mu_0$ and $V(N_t)=Q^N\sigma^2$, where:

$$Q^N=\frac{\left(\lambda_1^2+\lambda_2^2+\lambda_3^2\right)-2\lambda_2\left(\lambda_1-\lambda_3\right)(1-b)-2\lambda_1\lambda_2(1-b)^2}{1-(1-b)^2},$$

where $b=\lambda_1-\lambda_2-\lambda_3$, and it can be expanded to the simplified formula, that is,

$$Q^N=\frac{\lambda_1-\lambda_2-\lambda_3+2\lambda_2\left(\lambda_1-\lambda_3\right)}{2-\lambda_1+\lambda_2+\lambda_3}+2\lambda_1\lambda_3.$$

The upper and lower control limits of NEEWMA chart are as follows:

$$UCL^N/LCL^N=\mu_0\pm L_N\sigma_0\sqrt{Q^N},$$

where L_N is the limit parameter of NEEWMA, which represents the width of the upper control limit and lower control limit of the NEEWMA control chart. The NEEWMA stopping time (τ_N) can then be written as follows:

$$\tau_N=\inf\{t>0: N_t>UCL_N \text{ or } N_t<LCL_N\}.$$

Furthermore, the NEEWMA control chart becomes the EEWMA control chart when $\lambda_3=0$, and the EWMA control chart when both $\lambda_2=0$ and $\lambda_3=0$

2.3 Run Length (RL)

Run Length is the number of samples obtained before a change in process is detected. RL is related to the stopping time given by $RL=\tau-1$. The distribution of RL is a geometric distribution with PMF is

$$p_{RL}(k)=P(RL=k)=p(1-p)^{k-1};\ k=1,2,\ldots$$

where p is the probability of an out-of-control process

Average Run Length (ARL) is the expected value of RL; it is given by:

$$ARL=E(RL)=\sum_{k=1}^{\infty}k\cdot p_{RL}(k),$$

The ARLs have two values, as follows. First, the ARL before an out-of-control is detected when the process is in control, defined as ARL_0. Second, ARL before an out-of-control is detected after the process mean changed, defined as ARL_1.

The standard deviations of RL are given by:

$$SDRL=E\left(RL^2\right)-\left(E(RL)\right)^2.$$

Where $E\left(RL^2\right)$ is the second moment of RL given by:

$$E\left(RL^2\right)=\sum_{k=1}^{\infty}k^2P\left(RL=k\right).$$

However, the RL distribution described above follows a geometric distribution only when the control chart statistic exhibits no autocorrelation, as in the Shewhart control chart. In practice, however, many recently developed control charts exhibit autocorrelation — for example, the EWMA, MEWMA, EEWMA, and NEEWMA control charts — because their statistics are computed using both current and historical observations to detect process changes, which inherently induces autocorrelation in the chart statistics. Consequently, the RL distribution of control charts whose statistics are autocorrelated does not follow a geometric distribution, and calculating these probabilities becomes challenging, since the exact distribution of the control chart statistic is often unknown, making analytical evaluation difficult.

As a result, the control chart statistic is commonly assumed to follow a Markov chain process, and ARL and SDRL are typically estimated using numerical or simulation-based methods. Among these, Monte Carlo simulation (MC) is widely used due to its flexibility and ease of implementation across various distributional settings. Nevertheless, reliable MC estimation often requires a large number of simulations runs, particularly for high in-control ARL values, which increases computational cost, extends execution time, and introduces sampling variability in the results. To address these limitations, analytical approaches such as the Markov chain approach (MCA) have been developed. MCA provides a more stable and computationally efficient framework for estimating ARL and SDRL by modeling the transition behavior of the monitoring statistics through transition probability matrices. Therefore, this study employs both the MCA and MC to evaluate and compare control chart performance.

2.4 Monte Carlo simulation approach (MC)

MC simulation is a traditional method used to estimate ARL and SDRL of control charts when these measures cannot be obtained analytically. Furthermore, MC results can be used to validate the accuracy of alternative methods, as the approach relies on statistical data generation and applies control charts to detect shifts in simulated data, thereby evaluating the run length performance under specified conditions. The approximations of ARL and SDRL are given by

$$ARL_{MC}=\frac{1}{M}\sum_{t=1}^{M}RL_t,$$

and

$$SDRL_{MC}=\sqrt{\frac{1}{M-1}\sum_{t=1}^{M}\left(RL_t-ARL_{MC}\right)^2}.$$

where M is the number of simulation loops to approximate ARL and SDRL in this research, assuming $M=100,000$.

2.5 Markov chain approach (MCA)

MCA is one of the most effective methods to study the behavior of a control chart. This approach has been discussed by many authors (e.g., Champ and Rigdon (1991); Lucas and Saccucci (1990)). They introduced the MCA to approximate an ARL state in an in-control process,

Assuming the observation is an in-control state and an out-of-control state. The transition probability P_{ij} is the probability of moving from state to state in one step and is given by

$$P_{ij} = \Pr\left(X_{t+1} = x_j \mid X_t = x_i\right).$$

the transition probability matrix $\mathbf{P}$ and the elements of matrix P_{ij} can be rewritten as

$$\mathbf{P} = \left[P_{ij}\right]_{(n+1)\times(n+1)} = \begin{bmatrix} \mathbf{R} & (\mathbf{I}-\mathbf{R})\mathbf{1} \\ \mathbf{0} & 1 \end{bmatrix}.$$

where: $\mathbf{P}$ is the one-step transition probability matrix.

$\mathbf{R}$ is the matrix of probabilities of statistics changing states from in to in, as follows:

$$\mathbf{R} = \left[P_{ij}\right]_{n\times n};\ i, j = 1,\ldots,n,$$

$(\mathbf{I}-\mathbf{R})\mathbf{1}$ is the vector of probabilities of statistics changing states from in to out, as follows:

$$\mathbf{I} = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix}_{n\times n},\ \mathbf{1} = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}^T_{1\times n},\ \mathbf{0} = \begin{bmatrix} 0 & \cdots & 0 \end{bmatrix}_{1\times n},$$

$\mathbf{0}$ is the vector of probabilities of statistics changing states from out to in, as follows:

$$\mathbf{0} = \begin{bmatrix} 0 & \cdots & 0 \end{bmatrix}_{1\times n}$$

The k-stage transition probability matrix is useful for evaluating ARL because it contains the probability

that the chain goes from one state to another stage in k steps. This matrix is

$$\mathbf{P}_k = \begin{bmatrix} \mathbf{R}^k & \left(\mathbf{I}-\mathbf{R}^k\right)\mathbf{1} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$$

The vector $\left(\mathbf{I}-\mathbf{R}^k\right)\mathbf{1}$ is the vector of transition probabilities from state $i \le n$ to state $n+1$ in k steps.

Hence,

$$\Pr\left(\tau_r = k \mid X_0 = x_i\right) = p_i^{(0)}\left(\mathbf{I}-\mathbf{R}^k\right)\mathbf{1}$$

where $p_i^{(0)}$ is the no change probability vector with 1 at position i^{th} and 0 otherwise, which represents the probability that the stopping time is given by

$$\Pr(\tau = k \mid X_0 = x_i) = \Pr(\tau \le k \mid X_0 = x_i) - \Pr(\tau \le k-1 \mid X_0 = x_i) = p_i^{(0)}\left(\mathbf{R}^{k-1} - \mathbf{R}^{k}\right)\mathbf{1}$$

So, the transition probability of RL is given by

$$\Pr(RL = k) = \Pr(\tau_r = k+1 \mid X_0 = x_i) = \mathbf{p}_i^{(0)}\mathbf{R}^{k}\left(\mathbf{I} - \mathbf{R}\right)\mathbf{1}$$

where $k = 0,1,2,\ldots$



CHAPTER 3

Research Framework

This chapter describes the research framework and the methodological procedures adopted in this study to develop the Markov chain approach (MCA) method for estimating the Average Run Length (ARL) of the MEWMA, EEWMA, and NEEWMA control charts. Building upon the theoretical foundation established in Chapter 2, the chapter outlines the sequence of research stages undertaken, derives the analytical expressions employed to evaluate the performance measures of the control charts, and introduces the two proposed approaches for constructing the transition probability matrix, namely the Joint Distribution Approach (JDA) and the Point Estimator Approach (PEA). The criteria adopted for comparing the proposed method with the conventional Monte Carlo (MC) simulation approach, together with the parameter settings specified for numerical experiments, are subsequently presented. The chapter is organized into the following sections:

3.1 Research Framework
3.2 Performance Measures of Control charts
3.3 Proposed Method in Transition Probability
3.4 Performance Evaluation of Approaches
3.5 Experimental Design

The methodological details pertaining to each section are presented in turn below.

3.1 Research Framework

The research framework of this study was developed to construct and evaluate an MCA for estimating the ARL of the MEWMA, EEWMA, and NEEWMA control charts. The research process consisted of six stages, as follows.

3.1.1 Problem Identification and Literature Review

Reviewing the relevant literature on Statistical Process Control (SPC), control charts, Average Run Length (ARL), and existing ARL estimation methods, including Monte Carlo simulation, the Markov chain approach, and Markov chain approach techniques. The review was conducted to identify the strengths, limitations, and research gaps associated with current ARL computation methods.

3.1.2 Constructs closed-form formula of ARL using MCA

The mathematical formulations of the MEWMA, EEWMA, and NEEWMA control charts were examined, and the statistical properties of their monitoring statistics were investigated to establish a theoretical basis for ARL estimation. On this basis, closed-form expressions for the ARL and the SDRL were derived from the first and second moments of the run-length distribution, together with two approaches for constructing the transition probability matrix, namely the Joint Distribution Approach (JDA) and the Point Estimator Approach (PEA).

3.1.3 Efficiency Evaluation of the Proposed ARL Estimation Approaches

The performance of the proposed MCA method was evaluated under the normal distribution. The ARL and SDRL values obtained from the MCA were compared with those generated by the conventional Monte Carlo (MC) simulation approach to assess the closeness of estimation and the computational time efficiency of the two methods, thereby validating the proposed approaches prior to their extension to other distributions.

3.1.4 Performance Analysis of EEWMA and NEEWMA using MCA with PEA_0

The MCA method with PEA_0 was applied to analyses the performance of the EWMA, EEWMA, and NEEWMA control charts under symmetric, asymmetric, and discrete process distributions. For each case, the accuracy and the computational time efficiency of the MCA relative to the MC method were examined, and the effects of the chart parameters and of the distributional parameters upon the detection performance of the charts were investigated.

3.1.5 Real Data Application

The proposed method was applied to real-world datasets to demonstrate its practical applicability. The results obtained from these applications were analyses and compared with the findings of the numerical study.

3.1.6 Discussion, Conclusion, and Recommendations

The findings of the study were summarized, the theoretical and practical implications of the results were discussed, and recommendations for future research were provided.

3.2 Performance Measures of Control charts

This section discusses the equations for calculating ARL and SDRL using the MCA method, which employs transition probability. The details are as follows:

Theorem 4: Let $\mathbf{R}$ is a matrix in $p \times p$ dimension then

$$\sum_{k=0}^{\infty}\mathbf{R}^k=(\mathbf{I}-\mathbf{R})^{-1},\ \sum_{k=0}^{\infty}k\mathbf{R}^k=\mathbf{R}(\mathbf{I}-\mathbf{R})^{-2},\ \sum_{k=0}^{\infty}k^2\mathbf{R}^k=\mathbf{R}(\mathbf{I}+\mathbf{R})(\mathbf{I}-\mathbf{R})^{-3}$$

Proof

Equation 1: $\sum_{k=0}^{\infty}\mathbf{R}^k=(\mathbf{I}-\mathbf{R})^{-1}$.

Define, $\mathbf{S}=\sum_{k=0}^{\infty}\mathbf{R}^k=\mathbf{I}+\mathbf{R}+\mathbf{R}^2+\ldots=\mathbf{I}+\mathbf{RS}$.

So, $\sum_{k=0}^{\infty}\mathbf{R}^k=(\mathbf{I}-\mathbf{R})^{-1}$.

Equation 2: $\sum_{k=0}^{\infty} k\mathbf{R}^k = \mathbf{R}(\mathbf{I}-\mathbf{R})^{-2}$

Define, $T = \sum_{k=0}^{\infty} k\mathbf{R}^k = \mathbf{R} + 2\mathbf{R}^2 + 3\mathbf{R}^3 + \ldots$ and $\mathbf{RT} = \mathbf{R}^2 + 2\mathbf{R}^3 + 3\mathbf{R}^4 + \ldots$

Then $\mathbf{T} - \mathbf{RT} = \mathbf{R} + \mathbf{R}^2 + \mathbf{R}^3 + \ldots = \mathbf{S} - \mathbf{I}$ and $(\mathbf{I}-\mathbf{R})\mathbf{T} = \mathbf{R}(\mathbf{I}-\mathbf{R})^{-1}$

So, $\sum_{k=0}^{\infty} k\mathbf{R}^k = \mathbf{R}(\mathbf{I}-\mathbf{R})^{-2}$.

Equation 3: $\sum_{k=0}^{\infty} k^2\mathbf{R}^k = \mathbf{R}(\mathbf{I}+\mathbf{R})(\mathbf{I}-\mathbf{R})^{-3}$

Define, $\mathbf{U} = \sum_{k=0}^{\infty} k^2\mathbf{R}^k = \mathbf{R} + 4\mathbf{R}^2 + 9\mathbf{R}^3 + \ldots$ and $\mathbf{RU} = \mathbf{R}^2 + 4\mathbf{R}^3 + 9\mathbf{R}^4 + \ldots$,

then $\mathbf{U} - \mathbf{RU} = \mathbf{R} + 3\mathbf{R}^2 + 5\mathbf{R}^3 + \ldots = \sum_{k=1}^{\infty}(2k-1)\mathbf{R}^k = 2\sum_{k=1}^{\infty} k\mathbf{R}^k - \sum_{k=1}^{\infty}\mathbf{R}^k$

$$(\mathbf{I}-\mathbf{R})\mathbf{U} = 2\mathbf{R}(\mathbf{I}-\mathbf{R})^{-2} - \mathbf{R}(\mathbf{I}-\mathbf{R})^{-1}$$

$$(\mathbf{I}-\mathbf{R})\mathbf{U} = 2\mathbf{R}(\mathbf{I}+\mathbf{R})(\mathbf{I}-\mathbf{R})^{-2}.$$

So, $\sum_{k=0}^{\infty} k^2\mathbf{R}^k = \mathbf{R}(\mathbf{I}+\mathbf{R})(\mathbf{I}-\mathbf{R})^{-3}$.

Thus, the first moment of RL (ARL) and the second moment of RL can approximately be written as

$$E(RL) = \sum_{k=0}^{\infty} k \Pr(RL = k) = \sum_{k=0}^{\infty} k\left[\mathbf{p}_i^{(0)}\mathbf{R}^k(\mathbf{I}-\mathbf{R})\mathbf{1}\right] = \mathbf{p}_i^{(0)}\left(\sum_{k=0}^{\infty} k\mathbf{R}^k\right)(\mathbf{I}-\mathbf{R})\mathbf{1}$$
$$= \mathbf{p}_i^{(0)}\mathbf{R}(\mathbf{I}-\mathbf{R})^{-1}\mathbf{1}$$

and

$$E(RL^2) = \sum_{k=0}^{\infty} k^2 \Pr(RL = k) = \sum_{k=0}^{\infty} k^2\left[\mathbf{p}_i^{(0)}\mathbf{R}^k(\mathbf{I}-\mathbf{R})\mathbf{1}\right] = \mathbf{p}_i^{(0)}\left(\sum_{k=0}^{\infty} k^2\mathbf{R}^k\right)(\mathbf{I}-\mathbf{R})\mathbf{1}$$
$$= \mathbf{p}_i^{(0)}\mathbf{R}(\mathbf{I}+\mathbf{R})(\mathbf{I}-\mathbf{R})^{-2}\mathbf{1}$$

so that the ARL and SDRL can approximately follow:

$$ARL_{MCA} = E(RL) = \mathbf{p}_i^{(0)}\mathbf{R}(\mathbf{I}-\mathbf{R})^{-1}\mathbf{1},$$

and

$$SDRL_{MCA} = \sqrt{E\left(RL^2\right) - \left(E\left(RL\right)\right)^2} = \sqrt{\mathbf{p}_i^{(0)}\mathbf{R}\left(\mathbf{I}+\mathbf{R}\right)\left(\mathbf{I}-\mathbf{R}\right)^{-2}\mathbf{1} - ARL_{MCA}^2}\ .$$

where $p_i^{(0)}$ is the no change probability vector with 1 at position i^{th} and 0 otherwise, $\mathbf{I}$ is the identity matrix, $\mathbf{1}$ is a vector of 1, $\mathbf{R}$ is the matrix of probabilities of statistics changing states from in-control stage to in-control stage, as follows:

A key component in approximating ARL and SDRL is the transition probability, which depends on the control chart structure. In the proposed MCA, n denotes the number of partitions in the state space between LCL and UCL, this study uses $n = 1{,}000$ to achieve stable and accurate ARL approximations.

3.3 Proposed Method in Transition Probability

This study proposes two approaches for constructing the transition probability matrix of the MEWMA, EEWMA, and NEEWMA control charts, namely the Joint Distribution Approach (JDA) and the Point Estimator Approach (PEA). The need for two approaches arises from the structure of the monitoring statistics themselves. In the classical formulation of Brook and Evans (1972), the statistic at any point in time depends on the current observation and on the preceding value of the statistic alone, so that the transition probability reduces to a difference of two univariate distribution functions. The three charts considered here do not satisfy this condition: the MEWMA statistic is formed from the difference between the current observation and the one immediately preceding it, while the EEWMA and NEEWMA statistics assign separate weights to two and to three consecutive observations respectively. In each case the transition probability is governed by the joint distribution of the observations entering the statistic, and the two approaches proposed below differ in how that joint distribution is handled.

Introducing the proposed approaches, the transition probability is first defined. The transition probability, denoted by P_{ij}, represents the probability of moving from state i to state j in one transition step and is defined as

$$P_{ij} = \Pr\left(X_{t+1} = x_j \mid X_t = x_i\right).$$

The transition probability matrix for the in-control states is then expressed as

$$\mathbf{R} = \left[P_{ij}\right]_{n\times n};\ i,j = 1,\ldots,n\ .$$

where each element corresponds to the probability of transitioning between two transient states while the process remains in the in-control region. The transition probability matrix is a fundamental component of the MCA, as it is used to estimate

performance measures such as the ARL and the SDRL. Since the transition probabilities depend on the monitoring statistics of each control chart, the mathematical formulation of P_{ij} differs for the EEWMA and NEEWMA control charts. The derivation of the transition probabilities for the in-control states is presented in the following sections.

It is a key component in approximate ARL and SDRL, where each control chart has its own probability equation format. The basis of the transition probability of the in-control state is as follows:

Let $\bar{X}_t$ be a sample mean of random variable with subsample n from distribution with a mean μ and variance σ^2/n with time $t=1,2,3,\ldots$, and let S_t denote the control chart statistic for $\bar{X}_t$. The control limits, namely the lower control limit LCL^S and upper control limit UCL^S, are defined as

$$UCL^S / LCL^S = \mu_0 \pm L_S \sigma_0 \sqrt{Q^S}.$$

Where L_S is a control limit coefficient and Q^S is a parameter depending on the type of control chart.

Then, the transition probability of the control chart is as follows:

$$P_{ij}^S = \Pr\left(L_j^S \le S_t \le U_j^S \mid S_{t-1} = z_i^S\right).$$

where S_t is statistics of the control chart at time t.

z_i^S is the middle of the control chart at time $t-1$ in i position as follows:

$$z_i^S = LCL^S + \frac{(2i-1)\left(UCL^S - LCL^S\right)}{2n} = \mu_0 - L_S \sigma_0 \sqrt{Q^S}\left(1 - \frac{2i-1}{n}\right).$$

L_j^S is the lower limit of the control chart in the partition j as follows:

$$L_j^S = LCL^S + \frac{(j-1)\left(UCL^S - LCL^S\right)}{n} = \mu_0 - L_S \sigma_0 \sqrt{Q^S}\left(1 - \frac{2(j-1)}{n}\right).$$

U_j^S is the upper limit of the control chart in the partition j as follows:

$$U_j^S = LCL^S + \frac{j\left(UCL^S - LCL^S\right)}{n} = \mu_0 - L_S \sigma_0 \sqrt{Q^S}\left(1 - \frac{2j}{n}\right).$$

For EWMA control charts, the transition probability equation P_{ij} can be rewritten in the EWMA control charts by Equations and as follows:

$$\begin{aligned}
P_{ij}^{Z} &= \Pr\left(L_j^Z \le Z_t \le U_j^Z \mid Z_{t-1} = z_i^Z\right) \\
&= \Pr\left(L_j^Z \le \lambda_1 \bar{X}_t + (1-\lambda_1) z_i^Z \le U_j^Z\right) \\
&= \Pr\left(\frac{L_j^Z - (1-\lambda_1) z_i^Z}{\lambda_1} \le \bar{X}_t \le \frac{U_j^Z - (1-\lambda_1) z_i^Z}{\lambda_1}\right) \\
&= \Pr\left(\bar{X}_t \le \frac{U_j^Z - (1-\lambda_1) z_i^Z}{\lambda_1}\right) - \Pr\left(\bar{X}_t \le \frac{L_j^Z - (1-\lambda_1) z_i^Z}{\lambda_1}\right).
\end{aligned}$$

Thus, the transition probability of EWMA is given by

$$P_{ij}^{Z} = F_{\bar{X}}\left(\frac{U_j^Z - (1-\lambda_1) z_i^Z}{\lambda_1}\right) - F_{\bar{X}}\left(\frac{L_j^Z - (1-\lambda_1) z_i^Z}{\lambda_1}\right),$$

and

$$P_{ij}^{Z} = F_{\bar{X}}\left(\mu_0 - A_{\sigma_0}^Z \cdot B_{ij}^Z\right) - F_{\bar{X}}\left(\mu_0 - A_{\sigma_0}^Z \cdot \left(B_{ij}^Z + \frac{2}{n}\right)\right)$$

where $A_{\sigma_0}^Z = \dfrac{L_Z \sigma_0 \sqrt{Q^Z}}{\lambda_1}$ and $B_{ij}^Z = \left(1 - \dfrac{2i}{n}\right) + (1-\lambda_1)\left(1 - \dfrac{2i-1}{n}\right)$.

Consequently, for the MEWMA, EEWMA, and NEEWMA control charts, the transition probability derived in Equation serves as the no change equation upon which the subsequent ARL calculations are formulated. By substituting the respective chart statistics into this general transition probability framework, the transition probabilities for the MEWMA, EEWMA, and NEEWMA charts, denoted by Equations P_{ij}^{M}, P_{ij}^{E}, and P_{ij}^{N}, respectively, can be rewritten as follows:

For MEWMA:

$$\begin{aligned}
P_{ij}^{M} &= \Pr\left(L_j^M \le M_t \le U_j^M \mid M_{t-1} = z_i^M\right) \\
&= \Pr\left(L_j^M \le \lambda_1 \bar{X}_t + (1-\lambda_1) z_i^M - k\left(\bar{X}_t - \bar{X}_{t-1}\right) \le U_j^M\right) \\
&= \Pr\left(L_j^M - (1-\lambda_1) z_i^M \le (\lambda_1 + k)\bar{X}_t - k\bar{X}_{t-1} \le U_j^M - (1-\lambda_1) z_i^M\right) \\
&= \Pr\left((\lambda_1 + k)\bar{X}_t - k\bar{X}_{t-1} \le U_j^M - (1-\lambda_1) z_i^M\right) \\
&\quad - \Pr\left((\lambda_1 + k)\bar{X}_t - k\bar{X}_{t-1} \le L_j^M - (1-\lambda_1) z_i^M\right).
\end{aligned}$$

Thus, the transition probability of MEWMA is given by

$$P_{ij}^{M} = \Pr\left(a_M \bar{X}_t - b_M \bar{X}_{t-1} \le U_j^M - q_M \cdot z_i^M\right) - \Pr\left(a_M \bar{X}_t - b_M \bar{X}_{t-1} \le L_j^M - q_M \cdot z_i^M\right).$$

where $a_M = \lambda_1 + k$, $b_M = k$ and $q_M = 1 - \lambda_1$.

For EEWMA:

$$\begin{aligned}
P_{ij}^{E} &= \Pr\left(L_j^E \le E_t \le U_j^E \mid E_{t-1} = z_i^E\right) \\
&= \Pr\left(L_j^E \le \lambda_1 \bar{X}_t - \lambda_2 \bar{X}_{t-1} + \left(1 - \lambda_1 + \lambda_2\right) z_i^E \le U_j^E\right) \\
&= \Pr\left(L_j^E - \left(1 - \lambda_1 + \lambda_2\right) z_i^E \le \lambda_1 \bar{X}_t - \lambda_2 \bar{X}_{t-1} \le U_j^E - \left(1 - \lambda_1 + \lambda_2\right) z_i^E\right) \\
&= \Pr\left(\lambda_1 \bar{X}_t - \lambda_2 \bar{X}_{t-1} \le U_j^E - \left(1 - \lambda_1 + \lambda_2\right) z_i^E\right) \\
&\quad - \Pr\left(\lambda_1 \bar{X}_t - \lambda_2 \bar{X}_{t-1} \le L_j^E - \left(1 - \lambda_1 + \lambda_2\right) z_i^E\right).
\end{aligned}$$

Thus, the transition probability of EEWMA is given by

$$P_{ij}^{E} = \Pr\left(a_E \bar{X}_t - b_E \bar{X}_{t-1} \le U_j^E - q_E \cdot z_i^E\right) - \Pr\left(a_E \bar{X}_t - b_E \bar{X}_{t-1} \le L_j^E - q_E \cdot z_i^E\right).$$

where $a_E = \lambda_1$, $b_E = \lambda_2$ and $q_E = 1 - \lambda_1 + \lambda_2$.

For NEEWMA:

$$\begin{aligned}
P_{ij}^{N} &= \Pr\left(L_j^N \le N_t \le U_j^N \mid N_{t-1} = z_i^N\right) \\
&= \Pr\left(L_j^N \le \lambda_1 \bar{X}_t - \lambda_2 \bar{X}_{t-1} - \lambda_3 \bar{X}_{t-2} + \left(1 - \lambda_1 + \lambda_2 + \lambda_3\right) z_i^N \le U_j^N\right) \\
&= \Pr\left(L_j^N - \left(1 - \lambda_1 + \lambda_2 + \lambda_3\right) z_i^N \le \lambda_1 \bar{X}_t - \lambda_2 \bar{X}_{t-1} - \lambda_3 \bar{X}_{t-2} \le U_j^N - \left(1 - \lambda_1 + \lambda_2 + \lambda_3\right) z_i^N\right) \\
&= \Pr\left(\lambda_1 \bar{X}_t - \lambda_2 \bar{X}_{t-1} - \lambda_3 \bar{X}_{t-2} \le U_j^N - \left(1 - \lambda_1 + \lambda_2 + \lambda_3\right) z_i^N\right) \\
&\quad - \Pr\left(\lambda_1 \bar{X}_t - \lambda_2 \bar{X}_{t-1} - \lambda_3 \bar{X}_{t-2} \le L_j^N - \left(1 - \lambda_1 + \lambda_2 + \lambda_3\right) z_i^N\right).
\end{aligned}$$

Thus, the transition probability of NEEWMA is given by

$$P_{ij}^{N} = \Pr\left(a_N \bar{X}_t - b_N \bar{X}_{t-1} - c_N \bar{X}_{t-2} \le U_j^N - q_N \cdot z_i^N\right) - \Pr\left(a_N \bar{X}_t - b_N \bar{X}_{t-1} - c_N \bar{X}_{t-2} \le L_j^N - q_N \cdot z_i^N\right).$$

where $a_N = \lambda_1$, $b_N = \lambda_2$, $c_N = \lambda_3$ and $q_N = 1 - \lambda_1 + \lambda_2 + \lambda_3$.

From the equation of the transition probability for the MEWMA, EEWMA, and NEEWMA control charts, they were found that equations are characterized by the joint distribution of the monitoring statistics $\bar{X}_t$, $\bar{X}_{t-1}$, and $\bar{X}_{t-2}$. Consequently, the accurate estimation of these joint probabilities is essential for constructing the transition probability matrix.

The Joint Distribution Approach (JDA) evaluates the transition probability directly from the joint distribution of the observations entering the monitoring statistic,

without replacing any of them, and therefore preserves the dependence structure of the statistic in full; the resulting expression is tractable only in the bivariate case, however, which is why the approach is developed for the MEWMA chart alone, the EEWMA and NEEWMA statistics involving two and three lagged observations and requiring a triradiate and a quadrivariate integral respectively. The Point Estimator Approach (PEA)instead replaces the lagged observations by an estimator of the process mean, which reduces the joint probability to a univariate one and extends to a statistic of any lag depth; since the uniformly minimum variance unbiased estimator of the mean is the sample mean, the substitution amounts to representing the historical observations by the mean of the process from which they were drawn. Which means is appropriate depends on how those observations enter the statistic, and the approach is accordingly divided into two variants. For the MEWMA chart, whose statistic contains the difference between the current observation and the immediately preceding one, the quantity being subtracted is a single recent observation reflecting the level of the process as it now stands, so the out-of-control mean μ_1 is used and the variant is designated PEA1. For the EEWMA and NEEWMA charts, whose statistics carry weighted contributions from observations further back in time, it cannot be assumed that a shift has already occurred at the point from which those observations are drawn, so the in-control mean μ_0 is used and the variant is designated PEA0. The details of each approach are presented in the following sections.

3.3.1. Joint Distribution Approach (JDA)

The Joint Distribution Approach is exactly calculating the transition probabilities based on the joint probability distribution of the random variables. By utilizing the joint distribution, the transition probability matrix for each control chart can be derived. The derivation of the transition probability for the MEWMA control charts is presented in the equation as follows:

$$P_{ij}^{M} = \Pr\left(a_M \bar{X}_t - b_M \bar{X}_{t-1} \le U_j^M - q_M \cdot z_i^M\right) - \Pr\left(a_M \bar{X}_t - b_M \bar{X}_{t-1} \le L_j^M - q_M \cdot z_i^M\right).$$

Assume $S = a_M \bar{X}_t - b_M \bar{X}_{t-1}$ is a random variable so that

$$P_{ij}^{M} = F_S\left(U_j^M - q_M \cdot z_i^M\right) - F_S\left(L_j^M - q_M \cdot z_i^M\right),$$

where $F_S(s)$ is the cumulative density function of $S = a_M \bar{X}_t - b_M \bar{X}_{t-1}$.

The proposed study assumes that $\bar{X}_t \sim NID\left(\mu, \frac{\sigma_0^2}{n}\right)$ with $t = 1,2,3,\ldots$

then the distribution of $S = a_M \bar{X}_t - b_M \bar{X}_{t-1}$ is

$$S \sim N\left(\left(a_M - b_M\right)\cdot \mu, \left(a_M^2 + b_M^2\right)\sigma^2 / n\right)$$

and

$$Y = \frac{S - (a_M - b_M)\cdot\mu}{\sigma\sqrt{(a_M^2 + b_M^2)/n}} \sim N(0,1),$$

so that the transition probability for the MEWMA control charts with JDA

$$P_{ij}^M = \Phi\left(\frac{U_j^M - q_M \cdot z_i^M - (a_M - b_M)\cdot\mu}{\sigma\sqrt{(a_M^2 + b_M^2)/n}}\right) - \Phi\left(\frac{L_j^M - q_M \cdot z_i^M - (a_M - b_M)\cdot\mu}{\sigma\sqrt{(a_M^2 + b_M^2)/n}}\right),$$

where $\Phi(y)$ is the cumulative density function of the standard normal distribution.

3.3.2. Point Estimator Approach (PEA)

The Point Estimator Approach (PEA) is a method for calculating transition probability based on the theory of inferential statistics, which states that the UMVUE of the mean μ is $\bar{X}$. Therefore, the variable is an estimator of the mean. In estimating probability, the value is represented by the mean μ of the distribution process. Under these processes, there are two types: in-control and out-of-control stages. The process state affects the meaning of the process. Therefore, the PEA estimation method is divided into two types: representing historical data by μ_0 (PEA with in-control stage) and μ_0 (PEA with out-of-control stage). Each type is used in a different control chart. The MEWMA control chart represents historical data by μ_1, while the EEWMA and NEEWMA control charts also represent historical data by μ_0. Each control chart has a transition probability equation as follows:

3.3.2.1. PEA with out-of-control stage for MEWMA (PEA_1)

This method calculates the transition probability of the MEWMA control chart with approximate historical data $\bar{X}_{t-1}$ by μ_1, resulting in the following equation:

$$\begin{aligned}
P_{ij}^M &= \Pr\left(a_M\bar{X}_t - b_M\bar{X}_{t-1} \le U_j^M - q_M\cdot z_i^M\right) - \Pr\left(a_M\bar{X}_t - b_M\bar{X}_{t-1} \le L_j^M - q_M\cdot z_i^M\right) \\
&= \Pr\left(a_M\bar{X}_t - b_M\cdot\mu_1 \le U_j^M - q_M\cdot z_i^M\right) - \Pr\left(a_M\bar{X}_t - b_M\cdot\mu_1 \le L_j^M - q_M\cdot z_i^M\right) \\
&= \Pr\left(\bar{X}_t \le \frac{U_j^M - q_M\cdot z_i^M + b_M\cdot\mu_1}{a_M}\right) - \Pr\left(\bar{X}_t \le \frac{L_j^M - q_M\cdot z_i^M + b_M\cdot\mu_1}{a_M}\right).
\end{aligned}$$

then the transition probability for the MEWMA control charts with PEA with out-of-control stage

$$P_{ij}^M = F_{\bar{X}}\left(\frac{U_j^M - q_M\cdot z_i^M + b_M\cdot\mu_1}{a_M}\right) - F_{\bar{X}}\left(\frac{L_j^M - q_M\cdot z_i^M + b_M\cdot\mu_1}{a_M}\right).$$

3.3.2.2. PEA with in-control stage for EEWMA and NEEWMA (PEA_0)

This method calculates the transition probability of the EEWMA and NEEWMA control chart with approximate historical data $\bar{X}_{t-1}$ and $\bar{X}_{t-2}$ by μ_0, resulting in the following equation:

For EEWMA:

$$\begin{aligned} P_{ij}^{E} &= \Pr\left(a_E\bar{X}_t - b_E\bar{X}_{t-1} \le U_j^E - q_E\cdot z_i^E\right) - \Pr\left(a_E\bar{X}_t - b_E\bar{X}_{t-1} \le L_j^E - q_E\cdot z_i^E\right) \\ &= \Pr\left(a_E\bar{X}_t - b_E\cdot\mu_0 \le U_j^E - q_E\cdot z_i^E\right) - \Pr\left(a_E\bar{X}_t - b_E\cdot\mu_0 \le L_j^E - q_E\cdot z_i^E\right) \\ &= \Pr\left(\bar{X}_t \le \frac{U_j^E - q_E\cdot z_i^E + b_E\cdot\mu_0}{a_E}\right) - \Pr\left(\bar{X}_t \le \frac{L_j^E - q_E\cdot z_i^E + b_E\cdot\mu_0}{a_E}\right) \end{aligned}$$

then the transition probability for the EEWMA control charts with PEA with in-control stage

$$P_{ij}^{E} = F_{\bar{X}}\left(\frac{U_j^E - q_E\cdot z_i^E + b_E\cdot\mu_0}{a_E}\right) - F_{\bar{X}}\left(\frac{L_j^E - q_E\cdot z_i^E + b_E\cdot\mu_0}{a_E}\right)$$

For NEEWMA:

$$\begin{aligned} P_{ij}^{N} &= \Pr\left(a_N\bar{X}_t - b_N\bar{X}_{t-1} - c_N\bar{X}_{t-2} \le U_j^N - q_N\cdot z_i^N\right) - \Pr\left(a_N\bar{X}_t - b_N\bar{X}_{t-1} - c_N\bar{X}_{t-2} \le L_j^N - q_N\cdot z_i^N\right) \\ &= \Pr\left(a_N\bar{X}_t - b_N\cdot\mu_0 - c_N\cdot\mu_0 \le U_j^N - q_N\cdot z_i^N\right) - \Pr\left(a_N\bar{X}_t - b_N\cdot\mu_0 - c_N\cdot\mu_0 \le L_j^N - q_N\cdot z_i^N\right) \\ &= \Pr\left(\bar{X}_t \le \frac{U_j^N - q_N\cdot z_i^N + (b_N + c_N)\cdot\mu_0}{a_N}\right) - \Pr\left(\bar{X}_t \le \frac{L_j^N - q_N\cdot z_i^N + (b_N + c_N)\cdot\mu_0}{a_N}\right) \end{aligned}$$

therefore, the transition probability for the EEWMA control charts with PEA with in-control stage

$$P_{ij}^{N} = F_{\bar{X}}\left(\frac{U_j^N - q_N\cdot z_i^N + (b_N + c_N)\cdot\mu_0}{a_N}\right) - F_{\bar{X}}\left(\frac{L_j^N - q_N\cdot z_i^N + (b_N + c_N)\cdot\mu_0}{a_N}\right)$$

3.3.3. Determination of the Control Limit Coefficient

The transition probability matrices derived above yield the ARL for a given control limit coefficient L. In practice the requirement runs in the opposite direction: the in-control ARL is prescribed, and the coefficient attaining it must be found. Since the ARL is a strictly increasing function of L, the value sought is unique and may be located by bisection. The procedure first increases L in unit steps until the prescribed in-control ARL is exceeded, which brackets the solution, and then halves the bracketing interval repeatedly until the ARL agrees with its target to within the stated tolerance. The procedure is set out in Algorithm 1.

ALGORITHM 3-1 Bisection-based procedure for determining the control limit L

Input: Distribution, the weighted parameters of the control chart, $\delta = 0$ and ARL_0.
Output: $L(ARL_0)$.

No changeize L = 1.
Compute $ARL_L = ARL(L)$ using the Markov chain approach (MCA).
while $ARL_L < ARL_0$ **do**
    L = L + 1
    Compute $ARL_L = ARL(L)$ using MCA.
Set $L_{lower} = L - 1$ and $L_{upper} = L$.
repeat
    $L = (L_{upper} + L_{lower}) / 2$
    Compute $ARL_L = ARL(L)$ using MCA.
    if $|ARL_L - ARL_0| < 10^{-7}$ **then**
        $L(ARL_0) = L$
        break
    else if $ARL_L - ARL_0 > 10^{-7}$ **then**
        $L_{upper} = L$
    else
        $L_{lower} = L$
until $|ARL_L - ARL_0| < 10^{-7}$
return $L(ARL_0) = L$

The coefficients obtained in this way are reported to four decimal places throughout, and the in-control ARL values shown alongside them in Chapter 4 are those returned by the rounded coefficients rather than by the unrounded values at which the procedure terminated; the small departures from the nominal in-control levels seen in those tables arise from this rounding. Under a discrete process distribution, the ARL is a step function of L rather than a continuous one, so that no coefficient attains the prescribed in-control ARL exactly. The procedure then returns the coefficient whose ARL lies closest to the target, which is why the in-control values reported for the discrete distributions in Section 4.2.3 fall slightly short of their nominal levels.

3.4 Performance Evaluation of Approaches

The comparison between the MCA method and the conventional MC simulation approach is conducted along two dimensions: the closeness of the ARL estimates obtained from the two methods, and the time-related efficiency of each method in reaching those estimates.

3.4.1 Closeness of Estimation

To evaluate how closely the ARL and SDRL estimates produced by the MCA method agree with those produced by the MC simulation approach, five measures are used: Relative Error (RE), Root Mean Square Error (RMSE), Mean

Absolute Error (MAE), Mean Percentage Error (MPE), and Mean Absolute Percentage Error (MAPE), formulated as follows.

Relative Error (RE):

$$RE = \sum_{\forall \delta} \left| \frac{ARL_{MC} - ARL_{MCA}}{ARL_{MC}} \right|.$$

Root Mean Square Error (RMSE):

$$RMSE = \frac{1}{n(\delta)} \sum_{\forall \delta} \left(ARL_{MC} - ARL_{MCA} \right)^2.$$

Mean Absolute Error (MAE):

$$MAE = \frac{1}{n(\delta)} \sum_{\forall \delta} \left| ARL_{MC} - ARL_{MCA} \right|.$$

Mean Percentage Error (MPE):

$$MPE = \frac{100\%}{n(\delta)} \sum_{\forall \delta} \left(\frac{ARL_{MC} - ARL_{MCA}}{ARL_{MC}} \right).$$

Mean Absolute Percentage Error (MAPE):

$$MAPE = \frac{100\%}{n(\delta)} \sum_{\forall \delta} \left| \frac{ARL_{MC} - ARL_{MCA}}{ARL_{MC}} \right|.$$

Where ARL_{MCA} and ARL_{MC} denote the ARL estimates from the MCA and MC methods for a given shift δ, and $n(\delta)$ is the number of shift values evaluated. RE and MPE capture signed bias, RMSE and MAE capture absolute deviation, and MAPE captures absolute relative deviation. Smaller values indicate closer agreement between the two methods.

3.4.2 Computational Time Efficiency

To evaluate how efficiently the MCA method produces its ARL and SDRL estimates relative to the MC simulation approach, three measures are used: Total CPU Time, Equal-Weight Average CPU Time, and Shift-Weighted Average CPU Time, formulated as follows.

Total CPU Time (TCT):

$$TCT = \sum_{\forall \delta} CT_{ARL}.$$

Average CPU Time (ACT):

$$ACT = \frac{1}{n(\delta)} \sum_{\forall \delta} CT_{ARL} \ .$$

Where CT_{ARL} denotes the CPU time required to obtain the ARL estimate from the MCA and MC methods for a given shift δ, and $n(\delta)$ is the number of shift values evaluated. Smaller values of measures indicate lower computational cost.

3.5 Experimental Design

3.5.1 Quality Control charts, as follows:

3.5.1.1 Exponentially Weighted Moving Average (EWMA) Control Chart with $\lambda_1 = 0.1, 0.3, 0.7$

3.5.1.2 Modified Exponentially Weighted Moving Average (MEWMA) Control Chart with $\lambda_1 = 0.1, 0.3, 0.7$ and $k = 0.1, 0.7, 1.0$

3.5.1.3 Extended Exponentially Weighted Moving Average (EEWMA) Control Chart with $\lambda_1 = 0.1, 0.3, 0.7$ and $\lambda_2 = 0.01, 0.025, 0.05$

3.5.1.4 New Extended Exponentially Weighted Moving Average (NEEWMA) Control Chart with $\lambda_1 = 0.1, 0.3, 0.7$, $\lambda_2 = 0.01, 0.025, 0.05$ and $\lambda_3 = \lambda_2 / 4$

3.5.2 Distribution of Data Processes as follows:

3.5.2.1 Symmetric distributions are normal, logistic, and Laplace distributions with location $\mu_0 = 0, 5, 10$ and scale $\beta_0 = 1, 5, 10$.

3.5.2.2 Asymmetric distributions are two families as follows:

1) shape-scale family: such as gamma and Weibull distribution with shape $\alpha_0 = 1, 5, 10$ and scale $\beta_0 = 1$.

2) location-scale family: such as Gumbel and two-parameter Rayleigh distribution with location $\theta_0 = 0, 5, 10$ and scale $\beta_0 = 1$.

3.5.2.3 Discrete distributions as follows:

1) Poisson distribution with $\theta_0 = 5, 10, 15$.

2) Binomial distribution with $\alpha_0 = 5, 10, 15$ and $p_0 = 0.01$.

3) Negative binomial distribution with $\beta_0 = 5, 10, 15$ and $p_0 = 0.01$.

3.5.3 Process shift as follows:

3.5.3.1 Symmetric distribution is as follows:

1) no change shift $\delta = 0$
2) small shift $\delta = \pm 0.01, \pm 0.03, \pm 0.05, \pm 0.07, \pm 0.09$
3) medium shift $\delta = \pm 0.1, \pm 0.3, \pm 0.5, \pm 0.7, \pm 0.9$
4) large shift $\delta = \pm 1.0, \pm 2.0, \pm 3.0$

3.5.3.2 Asymmetric distribution is as follows:

1) no change shift $\delta = 0$
2) small shift $\delta = 0.01, 0.03, 0.05, 0.07, 0.09$
3) medium shift $\delta = 0.1, 0.3, 0.5, 0.7, 0.9$
4) large shift $\delta = 1.0, 2.0, 3.0$

3.5.3.3 Discrete distribution is as follows:

1) no change shift $\delta = 0$
2) small shift $\delta = 1, 2, 3, 4, 5$
3) medium shift $\delta = 10, 15, 20, 25$
4) large shift $\delta = 50, 100$

3.5.4 The in-control ARLs are $ARL_0 = 200, 370, 500$

3.5.5 Methods for calculating the Average Run Length (ARL) and Standard Deviation (SDRL) Run Length as follows:

1) Monte Carlo simulation approach (MC) with transition matrix dimension, $n = 1,000$
2) Markov chain approach (MCA) with simulation loop, $m = 100,000$

3.5.6 Performance Evaluation

3.5.6.1 Closeness of Estimation

1) Relative Error (RE)
2) Root Mean Square Error (RMSE)
3) Mean Absolute Error (MAE)
4) Mean Percentage Error (MPE)
5) Mean Absolute Percentage Error (MAPE)

3.5.6.2 Computational Time Efficiency

1) Total CPU Time (TCT)
2) Equal-Weight Average CPU Time (EWCT)
3) Shift-Weighted Average CPU Time (SWCT)

CHAPTER 4

Research Result

This chapter presents the results obtained from applying the Markov chain approach (MCA) method, developed in Chapter 3, to the estimation of the Average Run Length (ARL) of the MEWMA, EEWMA, and NEEWMA control charts. Two approaches for constructing the transition probability matrix are examined: the Joint Distribution Approach (JDA), which applies only to the MEWMA chart, and the Point Estimator Approach (PEA), which applies to all three charts. For each chart, the ARL estimates from the MCA are compared with those from Monte Carlo (MC) simulation on two criteria: how closely the two methods agree, and how much computing time each requires. The PEA-based MCA is then used to study the performance of the EEWMA and NEEWMA charts under symmetric, asymmetric, and discrete process distributions. Finally, the method is applied to real-world data to show how it works in practice. The content of this chapter is organized into the following sections:

4.1 Efficiency Evaluation of the Proposed ARL Estimation Approaches
4.2 Performance Analysis of EEWMA and NEEWMA using MCA with PEA
4.3 Real Applications

The empirical results of each section are presented in turn below.

4.1. Efficiency Evaluation of the Proposed ARL Estimation Approaches

This section presents a performance evaluation of the proposed MCA method as applied to the MEWMA, EEWMA, and NEEWMA control charts under the standard normal distribution, thereby establishing an analytical reference against which the proposed transition probability approaches, namely JDA and PEA, may be validated before their extension to symmetric, asymmetric, and discrete distributions in the subsequent section of this chapter. Moreover, the ARL values obtained through the MCA method are compared with those derived from the MC method, employing the performance measures defined in Section 3.4, together with a comparative assessment of computational time; the analysis results are detailed as follows.

4.1.1. Efficiency of the MEWMA Control Chart Using the Joint Distribution Approach (JDA)

This section describes the performance of the MEWMA control chart when the average run length (ARL) is derived using the Markov chain approach based on the joint distribution approach. Since the MEWMA statistics depend on both the current and the immediately preceding observation, its transition probabilities are determined by a bivariate distribution, which the joint distribution approach evaluates directly without replacing either observation with an estimate. In this study the process follows a standard normal distribution with a subgroup size of one. The control limit coefficients required to achieve each specified in-control ARL are presented in TABLE

4-1 and reveal three consistent patterns. First, with the weighted parameter held constant, the limits widen as the difference parameter increases, because the difference term adds to the variance of the monitoring statistic; the widening is most pronounced at the lower values and stabilizes thereafter. Second, with the difference parameter held constant, the limits narrow markedly as the weighted parameter increases, because assigning greater weight to the current observation reduces the variance accumulated from the statistic's own past values. Finally, the limits widen as the specified in-control ARL increases, an expected outcome for limits designed to reduce the probability of a false alarm.

TABLE 4-1 The control limit coefficient parameters of MEWMA with JDA for process is standard normal distribution

λ_1	k	ARL_0		
		200	**370**	**500**
0.1	0.1	4.6417	5.1065	5.3200
	0.7	7.4750	8.2236	8.5674
	1.0	7.6135	8.3760	8.7261
0.7	0.1	2.9106	3.1124	3.2072
	0.7	3.2130	3.4358	3.5404
	1.0	3.2610	3.4871	3.5933

Analysis results show close agreement between the approximated and the simulated values across all examined settings, the largest discrepancy amounting to less than one per cent of the nominal in-control average run length. Furthermore, the nominal in-control average run length is attained at every in-control level, confirming that the control limits function as intended. Three notable observations regarding this consistency can be made. First, the agreement holds for small, moderate, and large shifts like that is, both where the run length is long and where the chart signals within a few observations. Second, it does not degrade as the difference parameter widens the control limits, even though the subintervals of the partitioned region, and hence the rounding introduced at each transition, widens along with them. Finally, the largest discrepancies occur near the in-control state rather than at the extremes of the shift range, which aligns with the nature of a rounding error that accumulates across the transitions preceding a signal. These results are reported in TABLES 4-2 to 4-4 for $ARL_0 = 370$, TABLES A1 to A3 for $ARL_0 = 200$, and in TABLES B-1 to B-3 for $ARL_0 = 500$.

Regarding closeness, all five proximity measures remain small across every parameter setting, and the signed measures change sign from one case to another, indicating that the approximation does not consistently overestimate or underestimate the average run length relative to the MC method; the discrepancy therefore behaves as a rounding error arising from the partitioning of the state space rather than as a bias in the derived expressions. The measures also improve as the in-control level increases. In terms of computational efficiency, the MCA method with JDA offers a substantial saving in both total and average execution time. The saving arises because the cost of

the approximation is fixed by the number of subintervals alone and is therefore almost unaffected by the settings of the chart or by the in-control level, whereas every simulated run must be continued until the chart signals. Notably, the saving becomes more pronounced as the in-control level increases, the condition under which a control chart is ordinarily designed. These results are summarized in TABLES 4-5 and 4-6.

TABLE 4-2 ARL of the MEWMA control chart using JDA compared to MC under a standard normal process, with $ARL_0 = 370$ and $k = 0.1$

δ	$\lambda_1 = 0.1$			$\lambda_1 = 0.7$		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	5.6494 (2.11)	5.6256±1.5724 (0.1)	0.0238	1.2743 (2.22)	1.2682±1.13 (0.06)	0.0061
-2.00	10.3888 (2.09)	10.4309±2.4124 (0.17)	0.0421	4.4823 (2.19)	4.4192±2.0385 (0.11)	0.0631
-1.00	33.1188 (2.13)	33.7751±5.004 (0.51)	0.6563	31.5387 (2.17)	31.7946±5.5839 (0.5)	0.2559
-0.90	39.8281 (2.14)	39.73±5.6351 (0.66)	0.0981	40.708 (2.19)	40.4953±6.3855 (0.65)	0.2127
-0.70	61.3933 (2.14)	61.8507±7.3148 (0.97)	0.4574	70.3166 (2.13)	70.7192±8.3615 (1.09)	0.4026
-0.50	104.5777 (2.11)	104.6719±9.7199 (1.55)	0.0942	126.4205 (2.24)	126.8038±11.1188 (1.93)	0.3833
-0.30	194.9726 (2.14)	194.9496±13.6139 (2.89)	0.0230	226.5372 (2.2)	226.5783±15.034 (3.42)	0.0411
-0.10	336.8728 (2.1)	336.6286±18.2481 (4.89)	0.2442	346.8195 (2.11)	346.6462±18.4244 (5.03)	0.1733
-0.07	353.0148 (2.24)	353.4511±18.5801 (5.18)	0.4363	358.3354 (2.14)	358.1382±19.0921 (5.28)	0.1972
-0.05	361.1457 (2.19)	361.1747±18.7247 (5.25)	0.0290	363.9963 (2.22)	363.224±18.9985 (5.33)	0.7723
-0.03	366.7724 (2.09)	366.2397±19.0282 (5.36)	0.5327	367.8607 (2.11)	367.9208±19.2697 (5.35)	0.0601
-0.01	369.6495 (2.07)	369.3912±19.08 (5.46)	0.2583	369.8207 (2.09)	369.9338±19.2615 (5.28)	0.1131
0.00	370.0115 (2.25)	370.3646±18.8646 (5.51)	0.3531	370.067 (2.08)	370.9868±19.2953 (5.35)	0.9198
0.01	369.6474 (2.25)	369.0187±19.0966 (5.55)	0.6287	369.8205 (2.12)	369.3112±19.1417 (5.42)	0.5093
0.03	366.7661 (2.27)	366.3713±19.0951 (5.56)	0.3948	367.8601 (2.15)	367.6134±19.0667 (5.35)	0.2467
0.05	361.1354 (2.19)	361.0002±18.8824 (5.4)	0.1352	363.9953 (2.13)	363.8552±19.0363 (5.3)	0.1401
0.07	353.0006 (2.24)	353.8734±18.5473 (5.34)	0.8728	358.3342 (2.16)	358.1674±19.2365 (5.28)	0.1668
0.10	336.8534 (2.26)	336.7196±18.2004 (5.08)	0.1338	346.8178 (2.13)	346.4535±18.6668 (5.09)	0.3643
0.30	194.9378 (2.22)	194.1906±13.6352 (2.97)	0.7472	226.5335 (2.18)	226.2932±15.122 (3.38)	0.2403
0.50	104.5452 (2.22)	104.482±9.87 (1.61)	0.0632	126.4166 (2.1)	126.9728±11.2147 (1.87)	0.5562
0.70	61.3655 (2.2)	61.3955±7.2793 (0.95)	0.0300	70.313 (2.1)	70.4909±8.4837 (1.06)	0.1779
0.90	39.8044 (2.22)	39.493±5.6008 (0.63)	0.3114	40.7047 (2.11)	40.2278±6.4164 (0.69)	0.4769
1.00	33.0968 (2.22)	33.4042±5.0994 (0.54)	0.3074	31.5356 (2.09)	31.1518±5.5397 (0.47)	0.3838
2.00	10.3764 (2.22)	10.3618±2.3691 (0.17)	0.0146	4.4804 (2.08)	4.457±2.0659 (0.09)	0.0234
3.00	5.6409 (2.25)	5.6486±1.5816 (0.11)	0.0077	1.2729 (2.14)	1.2665±1.1278 (0.05)	0.0064

TABLE 4-3 ARL of the MEWMA control chart using JDA compared to MC under a standard normal process, with $ARL_0 = 370$ and $k = 0.7$

δ	$\lambda_1 = 0.1$			$\lambda_1 = 0.7$		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	72.8935 (2.22)	72.5049±8.0212 (1.16)	0.3886	9.6352 (2.14)	9.6469±3.0857 (0.16)	0.0117
-2.00	132.8594 (2.23)	132.311±11.1667 (2.02)	0.5484	29.2717 (2.12)	29.2009±5.4 (0.47)	0.0708
-1.00	257.2897 (2.25)	257.7534±15.7728 (4.01)	0.4637	120.8463 (2.08)	120.6943±11.0941 (1.83)	0.1520
-0.90	273.2053 (2.22)	273.2033±16.2601 (4.14)	0.0020	140.9597 (2.14)	140.4341±11.8435 (2.11)	0.5256
-0.70	304.8425 (2.25)	304.6386±17.1719 (4.6)	0.2039	191.0432 (2.12)	191.5641±13.8488 (2.83)	0.5209
-0.50	333.6934 (2.25)	333.7862±18.0521 (4.98)	0.0928	253.4019 (2.1)	253.3815±15.8713 (3.68)	0.0204
-0.30	356.0831 (2.25)	356.0396±18.5349 (5.24)	0.0435	318.609 (2.13)	318.5197±18.0717 (4.69)	0.0893
-0.10	368.4146 (2.2)	368.1883±18.9841 (5.45)	0.2263	363.6117 (2.14)	363.5701±18.9119 (5.28)	0.0416
-0.07	369.2288 (2.19)	369.9659±19.2556 (5.45)	0.7371	366.8687 (2.14)	366.1271±19.2142 (5.43)	0.7416
-0.05	369.613 (2.14)	369.3314±19.1947 (5.47)	0.2816	368.4197 (2.06)	368.5203±19.2688 (5.33)	0.1006
-0.03	369.8694 (2.11)	369.5182±19.1559 (5.39)	0.3512	369.4603 (2.11)	369.623±19.1578 (5.34)	0.1627
-0.01	369.9976 (2.16)	369.7743±19.023 (5.37)	0.2233	369.9825 (2.22)	369.1103±19.1762 (5.55)	0.8722
0.00	370.0134 (2.12)	370.9599±19.205 (5.41)	0.9465	370.0478 (2.23)	370.4594±19.345 (5.58)	0.4116
0.01	369.9971 (2.11)	369.5319±19.1003 (5.38)	0.4652	369.9824 (2.24)	369.3191±19.2623 (5.59)	0.6633
0.03	369.8681 (2.16)	369.2613±18.8657 (5.31)	0.6068	369.46 (2.25)	369.7438±18.995 (5.41)	0.2838
0.05	369.6108 (2.11)	369.6895±19.2397 (5.44)	0.0787	368.4192 (2.19)	368.4775±19.1522 (5.5)	0.0583
0.07	369.2257 (2.12)	369.0986±19.1568 (5.31)	0.1271	366.868 (2.24)	366.7605±18.909 (5.51)	0.1075
0.10	368.4102 (2.11)	368.6676±18.8836 (5.3)	0.2574	363.6107 (2.17)	363.7922±19.1914 (5.49)	0.1815
0.30	356.0702 (2.17)	356.411±18.5745 (5.06)	0.3408	318.6064 (2.2)	318.5428±17.886 (4.77)	0.0636
0.50	333.6732 (2.07)	333.2146±17.985 (4.8)	0.4586	253.3984 (2.25)	253.1233±15.796 (3.78)	0.2751
0.70	304.8164 (2.1)	304.5745±17.2892 (4.51)	0.2419	191.0393 (2.16)	191.7704±13.7851 (2.92)	0.7311
0.90	273.1751 (2.14)	273.7885±16.1905 (3.95)	0.6134	140.9557 (2.22)	140.4407±11.9582 (2.14)	0.5150
1.00	257.2581 (2.2)	257.8212±15.7218 (3.74)	0.5631	120.8424 (2.22)	120.6629±11.0718 (1.87)	0.1795
2.00	132.8252 (2.11)	132.4716±11.1328 (1.95)	0.3536	29.2687 (2.13)	29.7573±5.2939 (0.45)	0.4886
3.00	72.8641 (2.09)	72.0421±8.021 (1.1)	0.8220	9.6329 (2.16)	9.6679±3.0669 (0.15)	0.0350

TABLE 4-4 ARL of the MEWMA control chart using JDA compared to MC under a standard normal process, with $ARL_0 = 370$ and $k = 1.0$

δ	$\lambda_1 = 0.1$			$\lambda_1 = 0.7$		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	120.9594 (2.23)	120.9731±10.651 (1.81)	0.0137	18.5085 (2.16)	18.4637±4.2366 (0.29)	0.0448
-2.00	194.4622 (2.13)	194.5641±13.6369 (2.83)	0.1019	50.7009 (2.13)	50.2318±7.0364 (0.8)	0.4691
-1.00	302.5229 (2.2)	302.2948±16.9786 (4.33)	0.2281	165.9043 (2.14)	165.19±12.7125 (2.48)	0.7143
-0.90	313.4252 (2.08)	313.2907±17.325 (4.53)	0.1345	187.0138 (2.11)	187.1039±13.7678 (2.78)	0.0901
-0.70	333.6313 (2.2)	333.7687±18.0887 (4.98)	0.1374	235.1411 (2.14)	235.1928±15.5056 (3.58)	0.0517
-0.50	350.5386 (2.27)	350.9916±18.4776 (5.25)	0.4530	287.8145 (2.13)	287.4919±17.0309 (4.23)	0.3226
-0.30	362.7717 (2.27)	362.4994±18.9997 (5.5)	0.2723	336.0336 (2.11)	336.8957±18.4022 (4.95)	0.8621
-0.10	369.2053 (2.24)	369.9631±19.077 (5.56)	0.7578	365.9367 (2.09)	365.5122±19.1523 (5.31)	0.4245
-0.07	369.6228 (2.25)	369.0525±19.1642 (5.61)	0.5703	368.0058 (2.06)	368.8331±19.001 (5.31)	0.8273
-0.05	369.8195 (2.28)	369.6542±18.9927 (5.48)	0.1653	368.9868 (2.14)	368.8132±19.171 (5.31)	0.1736
-0.03	369.9506 (2.22)	369.7253±19.0881 (5.51)	0.2253	369.6434 (2.15)	369.3953±19.2457 (5.35)	0.2481
-0.01	370.016 (2.22)	370.0059±19.1344 (5.51)	0.0101	369.9724 (2.17)	369.2997±19.0844 (5.34)	0.6727
0.00	370.0241 (2.19)	370.0665±19.1981 (5.52)	0.0424	370.0136 (2.08)	370.7548±19.4446 (5.46)	0.7412
0.01	370.0157 (2.2)	370.8147±19.0956 (5.49)	0.7990	369.9723 (2.09)	369.4281±19.2069 (5.36)	0.5442
0.03	369.9496 (2.2)	369.9756±19.1376 (5.43)	0.0260	369.6431 (2.19)	369.6393±19.4192 (5.46)	0.0038
0.05	369.8179 (2.17)	369.6225±19.0873 (5.35)	0.1954	368.9864 (2.08)	368.0834±19.3286 (5.38)	0.9030
0.07	369.6206 (2.2)	369.7872±18.977 (5.43)	0.1666	368.0052 (2.11)	368.2014±19.2173 (5.34)	0.1962
0.10	369.2022 (2.13)	369.5603±18.9432 (5.29)	0.3581	365.936 (2.22)	365.4345±19.0423 (5.46)	0.5015
0.30	362.7623 (2.12)	362.385±18.4384 (5.14)	0.3773	336.0315 (2.25)	336.7085±18.1499 (5.05)	0.6770
0.50	350.5234 (2.12)	350.0383±18.4726 (5.02)	0.4851	287.8114 (2.2)	287.1898±16.9847 (4.36)	0.6216
0.70	333.6111 (2.11)	333.815±17.9984 (4.78)	0.2039	235.1374 (2.18)	235.2263±15.4895 (3.58)	0.0889
0.90	313.4007 (2.11)	313.3596±17.3514 (4.53)	0.0411	187.0099 (2.23)	187.1036±13.6791 (2.85)	0.0937
1.00	302.4965 (2.17)	302.9963±17.1872 (4.38)	0.4998	165.9003 (2.22)	165.4209±12.9321 (2.56)	0.4794
2.00	194.4274 (2.11)	194.5112±13.6159 (2.89)	0.0838	50.6975 (2.23)	50.981±7.0567 (0.82)	0.2835
3.00	120.9258 (2.11)	120.7166±10.5526 (1.78)	0.2092	18.5058 (2.25)	18.5438±4.2789 (0.29)	0.0380

TABLE 4-5 Closeness of the ARL of the MEWMA control chart with JDA compared to MC under a standard normal process

ARL_0	λ_1	k	*RE*	*RMSE*	*MAE*	*MPE*	*MAPE*
200	0.1	0.1	0.1886	0.1777	0.3264	0.0028	0.0044
		0.7	0.0751	0.1724	0.3179	0.0005	0.0017
		1.0	0.1082	0.2651	0.4452	0.0010	0.0025
	0.7	0.1	0.1560	0.1879	0.3424	-0.0006	0.0036
		0.7	0.2085	0.1587	0.3171	-0.0019	0.0048
		1.0	0.1883	0.2038	0.3584	0.0035	0.0044
370	0.1	0.1	0.1098	0.1417	0.2891	-0.0004	0.0026
		0.7	0.0634	0.1892	0.3616	0.0006	0.0015
		1.0	0.0428	0.1777	0.3420	0.0003	0.0010
	0.7	0.1	0.1246	0.1522	0.3020	0.0007	0.0029
		0.7	0.0726	0.1562	0.3073	-0.0001	0.0017
		1.0	0.0656	0.1969	0.3568	0.0003	0.0015
500	0.1	0.1	0.0988	0.1973	0.3605	-0.0005	0.0023
		0.7	0.0422	0.1777	0.3383	0.0003	0.0010
		1.0	0.0381	0.1894	0.3628	0.0001	0.0009
	0.7	0.1	0.0938	0.1301	0.2870	-0.0015	0.0022
		0.7	0.0614	0.1865	0.3552	0.0004	0.0014
		1.0	0.0558	0.1826	0.3564	0.0000	0.0013

TABLE 4-6 Computation time efficiency of the ARL of the MEWMA control chart with JDA compared to MC under a standard normal process

ARL_0	λ_1	k	TCT		ACT	
			MCA	MC	MCA	MC
200	0.1	0.1	256.24	199.73	5.9591	4.6449
		0.7	257.51	282.72	5.9886	6.5749
		1.0	258.02	295.38	6.0005	6.8693
	0.7	0.1	256.66	199.95	5.9688	4.6500
		0.7	255.82	240.58	5.9493	5.5949
		1.0	255.84	254.24	5.9498	5.9126
370	0.1	0.1	93.62	141.16	2.1772	3.2828
		0.7	93.13	203.67	2.1658	4.7365
		1.0	93.60	213.39	2.1767	4.9626
	0.7	0.1	92.40	144.72	2.1488	3.3656
		0.7	93.07	176.43	2.1644	4.1030
		1.0	92.36	185.28	2.1479	4.3088
500	0.1	0.1	92.69	183.11	2.1556	4.2584
		0.7	92.94	268.63	2.1614	6.2472
		1.0	93.67	283.03	2.1784	6.5821
	0.7	0.1	92.59	191.64	2.1533	4.4567
		0.7	92.25	233.69	2.1453	5.4347
		1.0	91.83	244.44	2.1356	5.6847

4.1.2. Efficiency of the MEWMA Control Chart Using the Point Estimator Approach with Out-of-Control Stage (PEA_1)

This section describes the performance of the MEWMA control chart when the average run length (ARL) is derived using the Markov chain approach based on the point estimator approach. Since the MEWMA statistic depends on both the current and the immediately preceding observation, its transition probabilities are determined by a bivariate distribution; rather than evaluating this distribution directly, the point estimator approach replaces the preceding observation by an estimate of the process mean, so that the transition probabilities reduce to a difference of two univariate distribution functions. The variant applied here adopts the out-of-control meaning as that estimate. In this study the process follows a standard normal distribution with a subgroup size of one. The control limit coefficients required to achieve each specified in-control ARL are presented in TABLE 4-7 and reveal three patterns. First, with the weighted parameter held at its lower value, the limits widen as the difference parameter increases, because the difference term adds to the variance of the monitoring statistic; the widening is most pronounced at the lower values and stabilizes thereafter. At the higher weighted parameter, however, this pattern reverses and the limits contract slightly as the difference parameter increases. Second, with the difference parameter held constant, the limits narrow markedly as the weighted parameter increases, because assigning greater weight to the current observation reduces the variance accumulated from the statistic's own past values. Finally, the limits widen as the specified in-control ARL increases, an expected outcome for limits designed to reduce the probability of a false alarm.

TABLE 4-7 The control limit coefficient parameters of MEWMA for ARL_0 with PEA_1

λ_1	k	ARL_0		
		200	**370**	**500**
0.1	0.1	4.1516	4.5674	4.7583
	0.7	5.6255	6.1889	6.4476
	1.0	5.6335	6.1977	6.4568
0.7	0.1	2.8881	3.0883	3.1824
	0.7	2.8738	3.0730	3.1666
	1.0	2.8107	3.0056	3.0972

Analysis results show close agreement between the approximated and the simulated values across all examined settings, the largest discrepancy amounting to less than one per cent of the nominal in-control average run length. Furthermore, the nominal in-control average run length is attained at every in-control level, confirming that the control limits function as intended. Three notable observations regarding this consistency can be made. First, the agreement holds for small, moderate, and large shifts alike — that is, both where the run length is long and where the chart signals within a few observations. Second, it does not degrade as the difference parameter widens the control limits, even though the subintervals of the partitioned region, and hence the rounding introduced at each transition, widen along with them.

Finally, the largest discrepancies occur near the in-control state rather than at the extremes of the shift range, which aligns with the nature of a rounding error that accumulates across the transitions preceding a signal. These results are reported in TABLES 4-8 to 4-10 for $ARL_0 = 370$, in TABLES A-4 to A-6 for $ARL_0 = 200$, and in TABLES B-4 to B-6 for $ARL_0 = 500$.

Regarding closeness, all five proximity measures remain small across every parameter setting, and the signed measures change sign from one case to another, indicating that the approximation does not consistently overestimate or underestimate the average run length relative to the MC method; the discrepancy therefore behaves as a rounding error arising from the partitioning of the state space rather than as a bias in the derived expressions. The measures also tend to improve as the in-control level increases, although the trend is less regular than that observed under the joint distribution approach. In terms of computational efficiency, the MCA method with PEA1 offers a substantial saving in both total and average execution time at every in-control level examined. The saving arises because the cost of the approximation is fixed by the number of subintervals alone and is therefore almost unaffected by the settings of the chart or by the in-control level, whereas every simulated run must be continued until the chart signals. Notably, the saving becomes more pronounced as the in-control level increases, the condition under which a control chart is ordinarily designed. These results are summarized in TABLES 4-11 and 4-12.

TABLE 4-8 ARL of the MEWMA control chart using PEA_1 compared to MC under a standard normal process, with $ARL_0 = 370$ and $k = 0.1$

δ	$\lambda_1 = 0.1$			$\lambda_1 = 0.7$		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	4.8068 (6.78)	4.8103±1.432 (0.64)	0.0035	1.242 (6.67)	1.2371±1.1246 (0.27)	0.0049
-2.00	8.7464 (6.75)	8.7458±2.1224 (1.03)	0.0006	4.3785 (6.67)	4.413±2.0266 (0.56)	0.0345
-1.00	27.254 (6.77)	27.5009±4.4994 (3)	0.2469	30.9308 (6.72)	30.2839±5.5465 (3.3)	0.6469
-0.90	32.7602 (6.73)	32.6288±5.0529 (3.5)	0.1314	39.9659 (6.59)	39.5771±6.2858 (4.17)	0.3888
-0.70	50.7701 (6.72)	50.5269±6.5228 (5.35)	0.2432	69.2262 (6.66)	69.3728±8.3125 (7.12)	0.1466
-0.50	88.3901 (6.72)	88.1719±9.0055 (9.04)	0.2182	124.9403 (6.64)	124.919±10.9797 (12.22)	0.0213
-0.30	174.0536 (6.72)	174.0465±12.9655 (17.06)	0.0071	225.062 (6.67)	225.6159±15.2396 (22.1)	0.5539
-0.10	329.4671 (6.75)	329.1771±18.0679 (31.27)	0.2900	346.4161 (6.79)	346.1857±18.6258 (32.99)	0.2304
-0.07	348.9909 (6.76)	348.3818±18.5154 (33.1)	0.6091	358.0938 (6.7)	358.8904±18.8536 (33.75)	0.7966
-0.05	358.9826 (6.67)	358.1211±18.6537 (33.47)	0.8615	363.8379 (6.62)	363.1107±19.0159 (34.44)	0.7272
-0.03	365.9599 (6.67)	365.854±19.0592 (34.55)	0.1059	367.7606 (6.64)	367.9143±19.133 (34.92)	0.1537
-0.01	369.5479 (6.83)	369.2582±18.9618 (34.2)	0.2897	369.7507 (7.21)	369.662±19.3561 (35.62)	0.0887
0.00	370.0004 (6.83)	370.4723±19.2419 (35.5)	0.4719	370.0007 (6.69)	370.8916±19.1829 (36.27)	0.8909
0.01	369.5455 (6.73)	369.6039±19.0154 (34.52)	0.0584	369.7505 (6.7)	369.7077±19.2537 (34.98)	0.0428
0.03	365.9529 (6.78)	365.3494±18.8926 (34.5)	0.6035	367.76 (6.77)	367.6968±19.2204 (34.81)	0.0632
0.05	358.9711 (6.69)	358.8692±18.808 (34.15)	0.1019	363.837 (6.64)	363.6958±19.3563 (34.55)	0.1412
0.07	348.9752 (6.66)	348.2998±18.4968 (32.94)	0.6754	358.0925 (6.67)	358.709±18.9322 (33.97)	0.6165
0.10	329.4458 (6.69)	329.7949±17.8196 (31)	0.3491	346.4143 (6.72)	346.8689±18.4106 (32.61)	0.4546
0.30	174.0186 (6.72)	174.0609±12.8462 (17.08)	0.0423	225.0582 (6.64)	225.4255±14.9429 (21.86)	0.3673
0.50	88.359 (6.69)	88.1426±8.9399 (9.06)	0.2164	124.9363 (6.64)	124.6729±10.9348 (12.28)	0.2634
0.70	50.7441 (6.72)	50.5804±6.4894 (5.36)	0.1637	69.2226 (6.59)	69.2588±8.1733 (6.92)	0.0362
0.90	32.7383 (6.68)	32.9866±5.0216 (3.57)	0.2483	39.9627 (6.72)	39.2842±6.3312 (4.19)	0.6785
1.00	27.2338 (6.72)	27.3668±4.4992 (2.89)	0.1330	30.9277 (6.64)	30.6869±5.5121 (3.28)	0.2408
2.00	8.7352 (6.76)	8.6574±2.0998 (1.05)	0.0778	4.3766 (6.58)	4.3625±2.0417 (0.58)	0.0141
3.00	4.7991 (6.78)	4.8085±1.426 (0.64)	0.0094	1.2407 (6.64)	1.2479±1.1275 (0.25)	0.0072

TABLE 4-9 ARL of the MEWMA control chart using PEA_1 compared to MC under a standard normal process, with $ARL_0 = 370$ and $k = 0.7$

δ	$\lambda_1 = 0.1$			$\lambda_1 = 0.7$		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	45.0568 (6.75)	45.2127±6.0543 (4.75)	0.1559	6.9421 (6.58)	6.9631±2.5899 (0.81)	0.0210
-2.00	88.39 (6.7)	88.0917±8.9943 (9.05)	0.2983	21.9127 (6.66)	21.0534±4.6799 (2.32)	0.8593
-1.00	208.1116 (6.7)	208.9827±13.9997 (20.03)	0.8711	100.8491 (6.57)	100.9713±10.0459 (9.96)	0.1222
-0.90	227.1888 (6.8)	227.5914±14.8886 (21.94)	0.4026	119.6814 (6.6)	119.6915±10.9434 (11.9)	0.0101
-0.70	268.3819 (6.73)	268.6184±16.091 (25.81)	0.2365	168.6948 (6.58)	168.3984±13.0555 (16.5)	0.2964
-0.50	310.2724 (6.74)	310.3793±17.5212 (29.28)	0.1069	234.0056 (6.53)	234.3005±15.2605 (22.5)	0.2949
-0.30	346.0784 (6.78)	346.2769±18.4114 (32.61)	0.1985	307.6788 (6.64)	307.3967±17.4295 (29.24)	0.2821
-0.10	367.1855 (6.66)	367.0377±19.0011 (34.34)	0.1478	361.9884 (6.55)	361.2887±19.128 (34.75)	0.6997
-0.07	368.6163 (6.71)	368.0298±19.1249 (35.03)	0.5865	366.0352 (6.51)	366.8542±19.1505 (34.89)	0.8190
-0.05	369.2932 (6.69)	369.3364±18.9719 (34.78)	0.0432	367.968 (6.55)	367.206±19.1368 (34.69)	0.7620
-0.03	369.7456 (6.76)	369.5255±18.8393 (34.36)	0.2201	369.2668 (6.59)	369.9837±19.266 (34.91)	0.7169
-0.01	369.9719 (6.78)	369.1231±18.9778 (34.99)	0.8488	369.9192 (6.53)	369.4291±19.3912 (35.58)	0.4901
0.00	370 (6.7)	370.9266±19.0261 (35.08)	0.9266	370.0009 (6.56)	370.5554±19.1806 (35.07)	0.5545
0.01	369.9714 (6.73)	369.3547±19.0301 (34.8)	0.6167	369.9191 (6.56)	369.174±19.2348 (35.36)	0.7451
0.03	369.7438 (6.75)	369.431±19.0005 (34.42)	0.3128	369.2665 (6.61)	369.8141±19.1142 (34.69)	0.5476
0.05	369.2902 (6.74)	369.3906±19.2871 (35)	0.1004	367.9675 (6.56)	367.9627±19.2004 (35.11)	0.0048
0.07	368.6121 (6.71)	368.0818±18.9157 (34.32)	0.5303	366.0345 (6.58)	366.7366±19.213 (35.45)	0.7021
0.10	367.1796 (6.79)	367.2409±19.0635 (35.11)	0.0613	361.9874 (6.56)	361.2879±18.9805 (34.49)	0.6995
0.30	346.0617 (6.66)	346.7375±18.3777 (32.78)	0.6758	307.676 (6.56)	307.8258±17.4631 (29.06)	0.1498
0.50	310.2473 (6.67)	310.0297±17.5073 (29.95)	0.2176	234.0019 (6.56)	234.2286±15.2722 (22.69)	0.2267
0.70	268.3513 (6.74)	268.4804±16.0703 (25.56)	0.1291	168.6908 (6.58)	168.5504±13.0999 (16.61)	0.1404
0.90	227.1551 (6.7)	227.4864±14.7824 (22.16)	0.3313	119.6775 (6.61)	119.4538±10.935 (11.79)	0.2237
1.00	208.0772 (6.7)	208.3073±14.063 (20.14)	0.2301	100.8453 (6.52)	100.8767±10.0506 (10.03)	0.0314
2.00	88.3589 (6.74)	88.7904±8.8693 (9.11)	0.4315	21.9099 (6.55)	21.9486±4.6832 (2.36)	0.0387
3.00	45.032 (6.73)	45.1284±6.1463 (4.8)	0.0964	6.9399 (6.51)	6.9035±2.5644 (0.85)	0.0364

TABLE 4-10 ARL of the MEWMA control chart using PEA_1 compared to MC under a standard normal process, with $ARL_0 = 370$ and $k = 1.0$

δ	$\lambda_1 = 0.1$			$\lambda_1 = 0.7$		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	77.0401 (6.7)	77.9352±8.2164 (7.75)	0.8951	12.2237 (6.51)	12.1851±3.4619 (1.38)	0.0386
-2.00	138.845 (6.77)	138.591±11.3934 (13.69)	0.2540	35.9092 (6.53)	35.0542±6.0014 (3.72)	0.8550
-1.00	262.6099 (6.73)	262.3085±15.9892 (24.88)	0.3014	136.5171 (6.54)	136.1022±11.6526 (13.46)	0.4149
-0.90	278.0354 (6.79)	278.4756±16.4551 (26.56)	0.4402	157.2577 (6.5)	157.0205±12.5581 (15.45)	0.2372
-0.70	308.4382 (6.72)	308.8317±17.2274 (28.61)	0.3935	207.265 (6.56)	207.024±14.4931 (20.43)	0.2410
-0.50	335.8663 (6.75)	335.8026±18.1354 (32.14)	0.0637	266.6022 (6.54)	266.7426±16.2548 (25.24)	0.1404
-0.30	356.9598 (6.69)	356.8479±18.5838 (33.94)	0.1119	325.546 (6.5)	325.4827±17.887 (30.85)	0.0633
-0.10	368.5067 (6.73)	368.2896±19.1066 (35)	0.2171	364.5321 (6.57)	364.2707±19.0612 (34.54)	0.2614
-0.07	369.2673 (6.71)	369.7123±18.8903 (33.97)	0.4450	367.3027 (6.55)	367.002±19.2188 (35.36)	0.3007
-0.05	369.6262 (6.66)	369.4646±19.1891 (34.76)	0.1616	368.6197 (6.54)	368.1741±19.2773 (35.83)	0.4456
-0.03	369.8656 (6.77)	369.912±19.1601 (34.55)	0.0464	369.5024 (6.58)	369.4149±19.1556 (35.06)	0.0875
-0.01	369.9853 (6.78)	369.8513±19.1147 (35.01)	0.1340	369.9452 (6.58)	369.2836±19.3313 (35.61)	0.6616
0.00	370.0001 (6.74)	370.3125±18.99 (34.4)	0.3124	370.0006 (6.53)	370.3226±19.5288 (35.48)	0.3220
0.01	369.9848 (6.75)	369.668±18.9197 (34.25)	0.3168	369.9451 (6.6)	369.5808±19.152 (34.86)	0.3643
0.03	369.8643 (6.75)	369.5076±19.1712 (34.93)	0.3567	369.5022 (6.56)	369.8136±19.4633 (35.33)	0.3114
0.05	369.624 (6.72)	369.0145±19.2301 (35.31)	0.6095	368.6193 (6.53)	368.1701±19.0264 (35.19)	0.4492
0.07	369.2643 (6.72)	369.5199±19.0533 (34.31)	0.2556	367.3021 (6.56)	367.5308±18.9177 (34.23)	0.2287
0.10	368.5024 (6.76)	368.4937±19.2126 (35.11)	0.0087	364.5312 (6.59)	364.85±18.9971 (34.74)	0.3188
0.30	356.9473 (6.83)	356.2772±18.6326 (33.48)	0.6701	325.5436 (6.57)	325.0303±18.0669 (30.83)	0.5133
0.50	335.8466 (6.72)	335.7283±17.9845 (31.62)	0.1183	266.5989 (6.61)	266.3114±16.3621 (25.39)	0.2875
0.70	308.4128 (6.66)	308.8714±17.2913 (29.06)	0.4586	207.2612 (6.53)	207.4596±14.3559 (20.24)	0.1984
0.90	278.0058 (6.71)	278.4656±16.3792 (26.17)	0.4598	157.2537 (6.53)	157.6412±12.3595 (15.23)	0.3875
1.00	262.5787 (6.72)	262.501±15.9804 (24.95)	0.0777	136.5131 (6.57)	136.4559±11.7856 (13.62)	0.0572
2.00	138.8107 (6.72)	138.0245±11.4934 (13.78)	0.7862	35.906 (6.55)	35.0155±5.9641 (3.7)	0.8905
3.00	77.0102 (6.7)	77.7229±8.2326 (7.86)	0.7127	12.2212 (6.58)	12.1268±3.4021 (1.36)	0.0944

TABLE 4-11 Closeness of the ARL of the MEWMA control chart with PEA_1 compared to MC under a standard normal process

ARL_0	λ_1	k	*RE*	*RMSE*	*MAE*	*MPE*	*MAPE*
200	0.1	0.1	0.1200	0.0011	0.3023	0.1616	0.0044
		0.7	0.0946	-0.0020	0.3791	0.2202	0.0035
		1.0	0.0469	0.0002	0.2649	0.0977	0.0017
	0.7	0.1	0.1071	0.0012	0.3158	0.1714	0.0040
		0.7	0.1020	-0.0008	0.3942	0.2444	0.0038
		1.0	0.1243	0.0035	0.3695	0.2098	0.0046
370	0.1	0.1	0.0632	0.0003	0.2326	0.1047	0.0023
		0.7	0.0397	-0.0005	0.3330	0.1784	0.0015
		1.0	0.0473	-0.0004	0.3347	0.1678	0.0018
	0.7	0.1	0.0985	0.0015	0.2920	0.1599	0.0036
		0.7	0.0799	0.0016	0.3638	0.2172	0.0030
		1.0	0.0849	0.0026	0.3157	0.1459	0.0031
500	0.1	0.1	0.0635	0.0011	0.2333	0.0883	0.0024
		0.7	0.0275	0.0005	0.2638	0.1042	0.0010
		1.0	0.0416	-0.0009	0.4044	0.2120	0.0015
	0.7	0.1	0.0708	-0.0003	0.2359	0.1068	0.0026
		0.7	0.1520	0.0036	0.3719	0.2065	0.0056
		1.0	0.0895	-0.0003	0.3567	0.2053	0.0033

TABLE 4-12 Computation time efficiency of the ARL of the MEWMA control chart with PEA_1 compared to MC under a standard normal process

ARL_0	λ_1	k	*TCT*		*ACT*	
			MCA	*MC*	*MCA*	*MC*
200	0.1	0.1	179.08	299.01	6.6326	11.0744
		0.7	178.91	424.26	6.6263	15.7133
		1.0	179.10	448.82	6.6333	16.6230
	0.7	0.1	181.94	308.72	6.7385	11.4341
		0.7	181.11	363.37	6.7078	13.4581
		1.0	182.19	381.48	6.7478	14.1289
370	0.1	0.1	181.67	512.44	6.7285	18.9793
		0.7	181.70	739.08	6.7296	27.3733
		1.0	181.77	785.27	6.7322	29.0841
	0.7	0.1	180.56	544.95	6.6874	20.1833
		0.7	177.41	640.30	6.5707	23.7148
		1.0	176.96	676.42	6.5541	25.0526
500	0.1	0.1	177.62	674.76	6.5785	24.9911
		0.7	177.58	985.89	6.5770	36.5144
		1.0	177.54	1053.12	6.5756	39.0044
	0.7	0.1	177.16	724.48	6.5615	26.8326
		0.7	177.69	855.29	6.5811	31.6774
		1.0	177.26	899.09	6.5652	33.2996

4.1.3. Efficiency of the EEWMA and NEEWMA Control charts Using the Point Estimator Approach with In-Control Stage (PEA_0)

This section describes the performance of the EEWMA and NEEWMA control charts when the average run length (ARL) is derived using the Markov chain approach based on the point estimator approach. Since the EEWMA statistic depends on the current observation and the one immediately preceding it, while the NEEWMA statistic depends on the current observation and the two preceding it, their transition probabilities are determined by a bivariate and a triradiate distribution respectively; evaluating these distributions directly is not tractable, and the point estimator approach therefore replaces the preceding observations by an estimate of the process mean, so that the transition probabilities reduce to a difference of two univariate distribution functions. The variant applied here adopts the in-control meaning as that estimate. In this study the process follows a standard normal distribution with a subgroup size of one. The control limit coefficients required to achieve each specified in-control ARL are presented in TABLE 4-13 and reveal three consistent patterns. First, with the first weighted parameter held constant, the limits widen as the second weighted parameter increases, because the additional lagged term adds to the variance of the monitoring statistic. Second, the limits of the NEEWMA chart are wider than those of the EEWMA chart at every corresponding setting, since the third weighted parameter introduces a further lagged term and hence a further contribution to that variance. Finally, the limits widen as the specified in-control ARL increases, an expected outcome for limits designed to reduce the probability of a false alarm. These patterns are pronounced at the lower value of the first weighted parameter but become negligible at the higher value, where the limits are almost unaffected by the second and third weighted parameters — the first parameter then dominates the statistic to the point where the lagged terms contribute little.

TABLE 4-13 The control limit coefficient parameters of EEWMA and NEEWMA with PEA_0

λ_1	λ_2	λ_3	ARL_0		
			200	**370**	**500**
0.1	0.01	-	2.6633	2.9379	3.0639
	0.01	0.0025	2.7305	3.0143	3.1445
	0.025	-	3.0517	3.3833	3.5353
	0.025	0.00625	3.4292	3.8115	3.9867
0.7	0.01	-	2.8126	3.0078	3.0995
	0.01	0.0025	2.8135	3.0089	3.1006
	0.025	-	2.8306	3.0274	3.1198
	0.025	0.00625	2.8338	3.0310	3.1236

For the EEWMA control chart, analysis results show close agreement between the approximated and the simulated values across all examined settings, the largest discrepancy amounting to less than one per cent of the nominal in-control

average run length. Furthermore, the nominal in-control average run length is attained at every in-control level, confirming that the control limits function as intended. Three notable observations regarding this consistency can be made. First, the agreement holds for small, moderate, and large shifts like that is, both where the run length is long and where the chart signals within a few observations. Second, it does not degrade as the second weighted parameter widens the control limits, even though the subintervals of the partitioned region, and hence the rounding introduced at each transition, widens along with them. Finally, the largest discrepancies occur near the in-control state rather than at the extremes of the shift range, which aligns with the nature of a rounding error that accumulates across the transitions preceding a signal.

For the NEEWMA control chart, the agreement is of the same order at every setting examined, with the largest discrepancy again amounting to less than one per cent of the nominal in-control average run length and the nominal in-control level attained throughout. The three observations made above apply equally here, and a fourth may be added. The NEEWMA statistic carries one more lagged term than that of the EEWMA chart, so that the point estimator approach replaces two observations rather than one and the control limits are correspondingly wider; neither circumstance reduces the accuracy of the approximation, which remains comparable to that obtained for the EEWMA chart at every corresponding setting. This indicates that the accuracy of the approach is not sensitive to the number of lagged terms carried by the monitoring statistic, which is the property that permits its extension to charts of greater lag depth. These results, together with those for the EEWMA chart, are reported in TABLES 4-14 to 4-17 for $ARL_0 = 370$, in TABLES A-7 to A-10 for $ARL_0 = 200$, and in TABLES B-7 to B-10 for $ARL_0 = 500$.

Regarding closeness, all five proximity measures remain small across every parameter setting for both charts, and the signed measures change sign from one case to another, indicating that the approximation does not consistently overestimate or underestimate the average run length relative to the MC method; the discrepancy therefore behaves as a rounding error arising from the partitioning of the state space rather than as a bias in the derived expressions. The measures are of comparable magnitude for the EEWMA and the NEEWMA charts and at all three in-control levels, which indicates that neither the additional lagged term nor the length of the in-control run affects the quality of the approximation. In terms of computational efficiency, the MCA method with PEA0 offers a substantial saving in both total and average execution time at the higher in-control levels. The saving arises because the cost of the approximation is fixed by the number of subintervals alone and is therefore almost unaffected by the settings of the chart or by the in-control level, whereas every simulated run must be continued until the chart signals. Notably, the saving becomes more pronounced as the in-control level increases, the condition under which a control chart is ordinarily designed. These results are summarized in TABLES 4-18 and 4-19.

TABLE 4-14 ARL of the EEWMA and NEEWMA control chart using PEA_0 compared to MC under a standard normal process, with $ARL_0 = 370$ and $\lambda_1 = 0.1$

δ	*EEWMA (0.01)*			*NEEWMA (0.01,0.0025)*		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	1.8471 (7.16)	1.854±0.8237 (0.2)	0.0069	1.87 (7.47)	1.8686±0.8164 (0.22)	0.0014
-2.00	3.2943 (7.3)	3.296±1.1058 (0.33)	0.0017	3.3244 (7.4)	3.3219±1.1028 (0.41)	0.0025
-1.00	8.849 (7.2)	8.8004±2.0918 (0.81)	0.0486	8.8799 (7.12)	8.8291±2.0858 (0.82)	0.0508
-0.90	15.324 (7.34)	15.2593±3.0382 (1.33)	0.0647	15.3159 (7.17)	15.3543±3.0016 (1.41)	0.0384
-0.70	26.7025 (7.22)	26.6555±4.3844 (2.36)	0.047	26.5759 (7.36)	26.4012±4.3116 (2.3)	0.1747
-0.50	63.5363 (7.15)	63.6079±7.3656 (4.8)	0.0716	62.9492 (7.23)	62.5832±7.389 (4.97)	0.366
-0.30	242.3283 (7.14)	242.5124±15.3902 (16.53)	0.1841	240.9927 (7.23)	240.4881±15.1628 (16.29)	0.5046
-0.10	294.2932 (7.1)	294.6083±17.0972 (19.58)	0.3151	293.3092 (7.25)	293.9775±16.7558 (19.5)	0.6683
-0.07	327.1426 (7.09)	327.9095±17.8069 (21.77)	0.7669	326.5166 (7.25)	326.0323±18.0231 (22.01)	0.4843
-0.05	353.3589 (7.18)	353.4957±18.6337 (23.33)	0.1368	353.0945 (7.32)	353.5417±18.4936 (23.25)	0.4472
-0.03	368.0773 (7.11)	368.7318±19.0809 (24.47)	0.6545	368.0455 (7.23)	368.2105±19.0533 (24.7)	0.165
-0.01	370.0004 (7.14)	370.5433±19.3043 (24.77)	0.5429	370.0005 (7.18)	370.7532±19.3896 (24.87)	0.7527
0.00	368.0719 (7.2)	368.231±19.1598 (24.62)	0.1591	368.0399 (7.33)	368.4355±18.9999 (24.47)	0.3956
0.01	353.3434 (7.19)	353.963±18.8224 (23.62)	0.6196	353.0785 (7.14)	353.9924±18.4516 (23.44)	0.9139
0.03	327.1186 (7.21)	327.8003±17.8715 (21.65)	0.6817	326.4919 (7.32)	326.0863±17.9151 (21.82)	0.4056
0.05	294.2629 (7.16)	294.2435±16.9604 (19.81)	0.0194	293.2781 (7.23)	293.1404±16.7717 (19.42)	0.1377
0.07	242.2924 (7.2)	242.8454±15.1629 (16.25)	0.553	240.9557 (7.2)	240.8654±15.3478 (16.47)	0.0903
0.10	63.5059 (7.22)	63.7055±7.3837 (4.89)	0.1996	62.9182 (7.22)	62.6807±7.2606 (4.97)	0.2375
0.30	26.6811 (7.16)	26.6672±4.3605 (2.4)	0.0139	26.5542 (7.22)	26.4545±4.346 (2.5)	0.0997
0.50	15.3078 (7.15)	15.356±3.0236 (1.37)	0.0482	15.2994 (7.3)	15.2919±2.9782 (1.39)	0.0075
0.70	8.8372 (7.2)	8.8032±2.0801 (0.8)	0.034	8.868 (7.32)	8.8982±2.0787 (0.82)	0.0302
0.90	3.2883 (7.12)	3.3154±1.1103 (0.38)	0.0271	3.3182 (7.36)	3.3231±1.0994 (0.41)	0.0049
1.00	1.8429 (7.18)	1.8496±0.8121 (0.25)	0.0067	1.8657 (7.23)	1.8613±0.8168 (0.27)	0.0044
2.00	1.8471 (7.16)	1.854±0.8237 (0.2)	0.0069	1.87 (7.47)	1.8686±0.8164 (0.22)	0.0014
3.00	3.2943 (7.3)	3.296±1.1058 (0.33)	0.0017	3.3244 (7.4)	3.3219±1.1028 (0.41)	0.0025

TABLE 4-15 ARL of the EEWMA and NEEWMA control chart using PEA_0 compared to MC under a standard normal process, with $ARL_0 = 370$ and $\lambda_1 = 0.1$

δ	*EEWMA (0.025)*			*NEEWMA (0.025,0.00625)*		
	MCA	*MC*	*%diff*	*MCA*	*MC*	*%diff*
-3.00	1.9984 (7.28)	1.004±0.8159 (0.22)	0.9944	2.0725 (7.78)	2.0709±0.8092 (0.28)	0.0016
-2.00	3.4944 (7.27)	3.4978±1.1038 (0.34)	0.0034	3.5944 (7.8)	3.6036±1.1005 (0.42)	0.0092
-1.00	9.0736 (7.23)	9.0346±2.0448 (0.77)	0.039	9.1995 (7.75)	9.1518±2.0497 (0.98)	0.0477
-0.90	15.3321 (7.17)	15.2858±2.932 (1.43)	0.0463	15.3825 (7.72)	15.3761±2.9284 (1.49)	0.0064
-0.70	26.0233 (7.15)	26.0168±4.1613 (2.24)	0.0065	25.806 (7.67)	25.5378±4.0986 (2.48)	0.2682
-0.50	60.0852 (7.12)	60.6043±7.1532 (4.67)	0.5191	58.7115 (7.63)	58.1893±7.0087 (4.9)	0.5222
-0.30	233.8558 (7.14)	233.3263±14.8605 (15.85)	0.5295	229.9715 (7.68)	229.5712±14.8176 (16.75)	0.4003
-0.10	287.9512 (7.16)	287.0737±16.5761 (19.05)	0.8775	284.96 (7.69)	284.7615±16.6291 (20.64)	0.1985
-0.07	323.0679 (7.26)	323.6319±17.7411 (21.44)	0.564	321.1121 (7.69)	321.405±17.6114 (22.53)	0.2929
-0.05	351.6239 (7.16)	351.4056±18.5561 (23.25)	0.2183	350.7796 (7.53)	350.7423±18.4347 (24.89)	0.0373
-0.03	367.867 (7.21)	367.6586±18.7739 (24.18)	0.2084	367.764 (7.55)	367.2062±18.9092 (25.87)	0.5578
-0.01	370.0004 (7.25)	370.6488±18.9645 (24.29)	0.6484	370.0006 (7.55)	370.8138±19.036 (26.2)	0.8132
0.00	367.8604 (7.11)	367.304±18.8428 (24.41)	0.5564	367.7567 (7.57)	367.1756±18.7827 (25.5)	0.5811
0.01	351.6049 (7.22)	351.06±18.4757 (23.23)	0.5449	350.7587 (7.59)	350.75±18.3133 (24.48)	0.0087
0.03	323.0387 (7.22)	323.6882±17.393 (21.16)	0.6495	321.0802 (7.6)	321.6159±17.8117 (23.09)	0.5357
0.05	287.9147 (7.17)	287.45±16.6004 (19.28)	0.4647	284.9203 (7.59)	284.2755±16.5887 (20.55)	0.6447
0.07	233.8132 (7.16)	233.4637±14.781 (15.65)	0.3495	229.9254 (7.53)	229.995±14.9203 (16.75)	0.0696
0.10	60.0511 (7.13)	60.3123±6.9897 (4.67)	0.2612	58.6757 (7.55)	58.9565±7.0068 (4.95)	0.2808
0.30	25.9998 (7.16)	25.0391±4.1953 (2.26)	0.9607	25.7814 (7.61)	25.8948±4.1528 (2.4)	0.1134
0.50	15.3144 (7.21)	15.4015±2.9755 (1.39)	0.0871	15.364 (7.54)	15.2984±2.899 (1.7)	0.0656
0.70	9.0608 (7.24)	9.0798±2.0575 (0.89)	0.019	9.1862 (7.71)	9.1816±2.044 (0.89)	0.0046
0.90	3.4878 (7.2)	3.4845±1.1037 (0.39)	0.0033	3.5876 (7.67)	3.596±1.1116 (0.44)	0.0084
1.00	1.9939 (7.28)	1.0001±0.8128 (0.32)	0.9938	2.0679 (7.71)	2.0674±0.8159 (0.29)	0.0005
2.00	1.9984 (7.28)	1.004±0.8159 (0.22)	0.9944	2.0725 (7.78)	2.0709±0.8092 (0.28)	0.0016
3.00	3.4944 (7.27)	3.4978±1.1038 (0.34)	0.0034	3.5944 (7.8)	3.6036±1.1005 (0.42)	0.0092

TABLE 4-16 ARL of the EEWMA and NEEWMA control chart using PEA_0 compared to MC under a standard normal process, with $ARL_0 = 370$ and $\lambda_1 = 0.7$

δ	*EEWMA (0.01)*			*NEEWMA (0.01,0.0025)*		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	0.7819 (7.17)	0.774±0.9339 (0.16)	0.0079	0.782 (7.55)	0.7751±0.93 (0.17)	0.0069
-2.00	2.8823 (7.23)	2.8475±1.64 (0.31)	0.0348	2.8729 (7.66)	2.8934±1.6392 (0.31)	0.0205
-1.00	21.4325 (7.19)	21.7257±4.6528 (1.92)	0.2932	21.3143 (7.24)	21.3705±4.589 (1.79)	0.0562
-0.90	51.2489 (7.23)	51.1887±7.1062 (4.14)	0.0602	50.9924 (7.41)	50.7905±7.0894 (4.25)	0.2019
-0.70	99.2313 (7.22)	99.6906±9.8655 (7.3)	0.4593	98.8287 (7.34)	98.891±9.8452 (7.28)	0.0623
-0.50	197.5274 (7.21)	197.0152±14.0716 (13.68)	0.5122	197.0468 (7.4)	197.3838±14.142 (13.54)	0.337
-0.30	339.1728 (7.29)	339.1023±18.5616 (22.6)	0.0705	339.0315 (7.38)	339.2352±18.2346 (22.29)	0.2037
-0.10	354.2897 (7.18)	354.8076±18.8203 (23.49)	0.5179	354.2148 (7.37)	354.484±18.6039 (23.05)	0.2692
-0.07	361.8309 (7.2)	361.4121±19.1081 (24.02)	0.4188	361.7911 (7.41)	361.8806±18.8033 (23.64)	0.0895
-0.05	367.0213 (7.15)	367.6582±19.306 (24.72)	0.6369	367.0066 (7.36)	367.2272±19.3442 (24.22)	0.2206
-0.03	369.6671 (7.28)	369.7592±19.2346 (24.71)	0.0921	369.6655 (7.42)	369.3687±19.2393 (24.26)	0.2968
-0.01	370.0002 (7.28)	370.4061±19.1899 (24.7)	0.4059	370.0003 (7.35)	370.7263±19.4172 (24.47)	0.726
0.00	369.6669 (7.24)	369.722±19.0428 (24.17)	0.0551	369.6653 (7.39)	369.4955±19.3915 (24.32)	0.1698
0.01	367.0206 (7.25)	367.3397±19.4314 (24.97)	0.3191	367.0059 (7.25)	367.4632±19.151 (24.05)	0.4573
0.03	361.8298 (7.15)	361.0264±18.9282 (23.85)	0.8034	361.79 (7.24)	361.5785±18.9106 (23.85)	0.2115
0.05	354.2882 (7.18)	354.2796±18.7462 (23.36)	0.0086	354.2132 (7.3)	354.5383±18.9088 (23.28)	0.3251
0.07	339.1707 (7.15)	339.9217±18.4442 (22.55)	0.751	339.0294 (7.41)	339.006±18.1866 (22.25)	0.0234
0.10	197.5235 (7.14)	197.932±13.9347 (13.55)	0.4085	197.0428 (7.34)	197.3575±14 (13.69)	0.3147
0.30	99.2273 (7.22)	99.7098±9.8457 (7.31)	0.4825	98.8247 (7.36)	98.5882±9.8437 (7.2)	0.2365
0.50	51.2454 (7.25)	51.9243±7.1753 (4.11)	0.6789	50.9889 (7.33)	50.4002±7.1221 (4.19)	0.5887
0.70	21.4296 (7.2)	21.128±4.5721 (2)	0.3016	21.3114 (7.49)	21.2115±4.5444 (1.78)	0.0999
0.90	2.8805 (7.17)	2.8512±1.6469 (0.35)	0.0293	2.8711 (7.47)	2.92±1.6605 (0.31)	0.0489
1.00	0.7808 (7.31)	0.766±0.9366 (0.14)	0.0148	0.7808 (7.42)	0.7699±0.9302 (0.13)	0.0109
2.00	0.7819 (7.17)	0.774±0.9339 (0.16)	0.0079	0.782 (7.55)	0.7751±0.93 (0.17)	0.0069
3.00	2.8823 (7.23)	2.8475±1.64 (0.31)	0.0348	2.8729 (7.66)	2.8934±1.6392 (0.31)	0.0205

TABLE 4-17 ARL of the EEWMA and NEEWMA control chart using PEA_0 compared to MC under a standard normal process, with $ARL_0 = 370$ and $\lambda_1 = 0.7$

δ	*EEWMA (0.01)*			*NEEWMA (0.01,0.00625)*		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	0.7828 (7.43)	0.7916±0.9325 (0.14)	0.0088	0.7834 (7.41)	0.7825±0.9296 (0.15)	0.0009
-2.00	2.8274 (7.28)	2.8266±1.6141 (0.33)	0.0008	2.8057 (7.41)	2.7936±1.6037 (0.29)	0.0121
-1.00	20.7337 (7.32)	20.5512±4.4953 (1.83)	0.1825	20.4499 (7.6)	20.857±4.5239 (1.83)	0.4071
-0.90	49.7247 (7.24)	49.5202±7.0385 (4.03)	0.2045	49.1001 (7.32)	49.5209±6.9649 (4.07)	0.4208
-0.70	96.8267 (7.26)	96.4822±9.7109 (7.08)	0.3445	95.8327 (7.37)	95.5649±9.8537 (7.19)	0.2678
-0.50	194.6349 (7.34)	194.6614±13.9339 (13.33)	0.0265	193.4237 (7.37)	193.852±13.9394 (13.43)	0.4283
-0.30	338.3143 (7.22)	338.1882±18.4536 (22.67)	0.1261	337.9493 (7.37)	337.2286±18.2518 (22.11)	0.7207
-0.10	353.8337 (7.15)	353.1281±18.9356 (23.52)	0.7056	353.6397 (7.42)	353.6309±18.8485 (23.41)	0.0088
-0.07	361.589 (7.25)	361.0402±19.1445 (24.33)	0.5488	361.4862 (7.34)	361.7753±18.9861 (23.91)	0.2891
-0.05	366.9319 (7.17)	366.8401±19.2053 (24.53)	0.0918	366.8942 (7.36)	366.7806±19.4528 (24.51)	0.1136
-0.03	369.6572 (7.25)	369.3396±19.4482 (25.11)	0.3176	369.6534 (7.41)	369.2218±19.1948 (24.48)	0.4316
-0.01	370.0003 (7.17)	370.3154±19.045 (24.33)	0.3151	370.0008 (7.39)	370.3781±19.2904 (24.63)	0.3773
0.00	369.6569 (7.25)	369.0438±19.2505 (24.53)	0.6131	369.6532 (7.43)	369.6881±19.2309 (24.49)	0.0349
0.01	366.9312 (7.22)	366.7226±19.1601 (24.03)	0.2086	366.8935 (7.37)	366.9255±19.2806 (24.1)	0.032
0.03	361.5878 (7.27)	361.7757±19.0666 (23.94)	0.1879	361.4851 (7.34)	361.1702±18.9187 (23.75)	0.3149
0.05	353.8321 (7.31)	353.9257±18.9125 (23.56)	0.0936	353.6381 (7.42)	353.7763±18.9357 (23.47)	0.1382
0.07	338.3121 (7.28)	338.619±18.3824 (22.37)	0.3069	337.947 (7.32)	337.7161±18.3136 (22.12)	0.2309
0.10	194.6307 (7.14)	194.3394±14.0308 (13.57)	0.2913	193.4194 (7.36)	193.4988±13.7833 (13.36)	0.0794
0.30	96.8226 (7.2)	96.8054±9.839 (7.22)	0.0172	95.8285 (7.36)	95.3116±9.7805 (7.31)	0.5169
0.50	49.7211 (7.28)	49.9381±6.9832 (4.08)	0.217	49.0964 (7.33)	49.0176±6.9996 (4.06)	0.0788
0.70	20.7307 (7.2)	20.7523±4.4781 (1.91)	0.0216	20.4469 (7.4)	20.1868±4.4772 (1.68)	0.2601
0.90	2.8256 (7.17)	2.8282±1.6184 (0.31)	0.0026	2.8039 (7.42)	2.8217±1.5993 (0.33)	0.0178
1.00	0.7816 (7.36)	0.7809±0.9288 (0.17)	0.0007	0.7822 (7.38)	0.781±0.927 (0.14)	0.0012
2.00	0.7828 (7.43)	0.7916±0.9325 (0.14)	0.0088	0.7834 (7.41)	0.7825±0.9296 (0.15)	0.0009
3.00	2.8274 (7.28)	2.8266±1.6141 (0.33)	0.0008	2.8057 (7.41)	2.7936±1.6037 (0.29)	0.0121

TABLE 4-18 Closeness of the ARL of the MEWMA control chart with PEA_0 compared to MC under a standard normal process

ARL_0	λ_1	λ_2	λ_3	*RE*	*RMSE*	*MAE*	*MPE*	*MAPE*
200	0.1	0.01	-	0.5468	0.0863	0.1957	-0.0008	0.0029
			0.0025	0.5621	0.0947	0.2041	-0.0007	0.0030
		0.025	-	1.2693	0.1119	0.2097	0.0015	0.0067
			0.00625	1.4648	0.1731	0.2930	-0.0056	0.0078
	0.7	0.01	-	1.2002	0.1159	0.2349	0.0000	0.0064
			0.0025	1.1909	0.1545	0.2986	0.0004	0.0063
		0.025	-	0.6842	0.0878	0.2112	-0.0009	0.0036
			0.00625	0.9930	0.1121	0.2439	0.0007	0.0053
370	0.1	0.01	-	0.5301	0.1084	0.2226	-0.0002	0.0028
			0.0025	0.4380	0.1389	0.2541	0.0003	0.0023
		0.025	-	14.5516	0.2475	0.3763	0.0749	0.0770
			0.00625	0.5148	0.1070	0.2266	0.0013	0.0027
	0.7	0.01	-	0.9773	0.1523	0.3065	0.0004	0.0052
			0.0025	0.8628	0.1049	0.2368	-0.0010	0.0046
		0.025	-	0.4656	0.0997	0.2378	0.0010	0.0025
			0.00625	0.6186	0.0870	0.2271	0.0004	0.0033
500	0.1	0.01	-	0.5622	0.1882	0.2976	0.0019	0.0030
			0.0025	0.5978	0.1324	0.2440	-0.0007	0.0032
		0.025	-	0.3822	0.0971	0.2063	0.0001	0.0020
			0.00625	0.5514	0.1534	0.2869	-0.0005	0.0029
	0.7	0.01	-	0.5182	0.1512	0.3029	-0.0017	0.0027
			0.0025	0.5533	0.1277	0.2813	-0.0008	0.0029
		0.025	-	0.8865	0.1762	0.3328	0.0023	0.0047
			0.00625	0.6090	0.1090	0.2669	0.0014	0.0032

TABLE 4-19 Computation time efficiency of the ARL of the MEWMA control chart with PEA_0 compared to MC under a standard normal process

ARL_0	λ_1	λ_2	λ_3	TCT		ACT	
				MCA	MC	MCA	MC
200	0.1	0.01	-	1280.51	1180.62	6.7752	6.2467
			0.0025	1267.00	1186.78	6.7037	6.2793
		0.025	-	1283.80	1176.07	6.7926	6.2226
			0.00625	1266.30	1164.10	6.7000	6.1593
	0.7	0.01	-	1278.97	1436.33	6.7670	7.5996
			0.0025	1285.69	1433.11	6.8026	7.5826
		0.025	-	1273.09	1427.16	6.7359	7.5511
			0.00625	1326.15	1453.55	7.0167	7.6907
370	0.1	0.01	-	1355.90	2051.98	7.1741	10.8570
			0.0025	1373.40	2053.24	7.2667	10.8637
		0.025	-	1359.82	2010.68	7.1948	10.6385
			0.00625	1444.66	2146.90	7.6437	11.3593
	0.7	0.01	-	1363.32	2584.68	7.2133	13.6756
			0.0025	1395.03	2554.23	7.3811	13.5144
		0.025	-	1370.81	2570.19	7.2530	13.5989
			0.00625	1394.89	2555.63	7.3804	13.5219
500	0.1	0.01	-	1409.17	2644.25	7.4559	13.9907
			0.0025	1411.97	2645.44	7.4707	13.9970
		0.025	-	1409.24	2606.87	7.4563	13.7930
			0.00625	1420.16	2580.55	7.5141	13.6537
	0.7	0.01	-	1412.81	3401.72	7.4752	17.9985
			0.0025	1423.73	3408.16	7.5330	18.0326
		0.025	-	1412.95	3363.99	7.4759	17.7989
			0.00625	1434.93	3399.90	7.5922	17.9889

4.2. Performance Analysis of EEWMA and NEEWMA using MCA with PEA_0

This section presents the results obtained from estimating the ARL of the EWMA, EEWMA, and NEEWMA control charts using the MCA with PEA0 under symmetric, asymmetric, and discrete process distributions. Following the same evaluation framework as in Section 4.1, performance is assessed on two criteria: how closely the MCA estimates agree with those from MC, and how much computing time each method requires. The results are then used to examine the performance of the EEWMA and NEEWMA charts under two scenarios: (1) the effect of the control chart parameters, and (2) the effect of the parameters of the underlying process distribution. This section is presented into the following three subsections:

4.2.1 Performance Analysis under Symmetric Distributions
4.2.2 Performance Analysis under Asymmetric Distributions
4.2.3 Performance Analysis under Discrete Distributions

The details of each subsection are presented in turn below.

4.2.1. Performance Analysis under Symmetric Distributions

This section presents the accuracy, the detection behavior, and the performance of the EWMA, EEWMA, and NEEWMA control charts evaluated by the MCA method under symmetric process distributions, namely the normal, logistic, and Laplace distributions. The control limit coefficients were obtained by the bisection method described in Algorithm 1 and are reported in TABLE 4-20. Three patterns emerge. First, the limits widen as the second weighted parameter increases and widen further when the third is introduced, so that the NEEWMA chart requires the widest limits and the EWMA chart the narrowest; the effect diminishes sharply as the first weighted parameter increases, and at its highest value the three charts require limits of comparable width. Second, for any given setting the limits widen in passing from the normal to the logistic and then to the Laplace distribution, reflecting the progressively heavier tails of the three distributions, and this separation likewise grows as the first weighted parameter increases. Finally, the limits widen as the specified in-control ARL increases, an expected outcome for limits designed to reduce the probability of a false alarm.

TABLE 4-20 The control limit coefficient parameters of EWMA, EEWMA and NEEWMA with PEA_0 under symmetric distribution

λ_1	λ_2	λ_3	ARL_0								
			Normal			Logistic			Laplace		
			200	370	500	200	370	500	200	370	500
0.1	-	-	2.4561	2.7021	2.8151	2.4877	2.7555	2.8801	2.5385	2.8350	2.9745
0.1	0.01	-	2.6633	2.9379	3.0639	2.6911	2.9870	3.1245	2.7370	3.0618	3.2142
0.1	0.01	0.0025	2.7147	2.9968	3.1263	2.7414	3.0447	3.1855	2.7859	3.1180	3.2737
0.1	0.025	-	3.0517	3.3833	3.5353	3.0728	3.4248	3.5879	3.1099	3.4905	3.6685
0.1	0.025	0.00625	3.2322	3.5925	3.7577	3.2500	3.6302	3.8061	3.2824	3.6908	3.8815
0.1	0.05	-	4.0495	4.5477	4.7759	4.0557	4.5719	4.8101	4.0715	4.6146	4.8672
0.1	0.05	0.0125	4.8039	5.4530	5.7510	4.7999	5.4649	5.7718	4.8004	5.4900	5.8102
0.3	-	-	2.7144	2.9256	3.0237	2.8933	3.1720	3.3056	3.1114	3.4539	3.6192
0.3	0.01	-	2.7737	2.9908	3.0916	2.9491	3.2332	3.3692	3.1650	3.5131	3.6811
0.3	0.01	0.0025	2.7841	3.0024	3.1038	2.9583	3.2433	3.3797	3.1733	3.5223	3.6906
0.3	0.025	-	2.8680	3.0948	3.2000	3.0378	3.3306	3.4706	3.2501	3.6073	3.7794
0.3	0.025	0.00625	2.8971	3.1272	3.2340	3.0638	3.3593	3.5004	3.2735	3.6332	3.8064
0.3	0.05	-	3.0416	3.2867	3.4003	3.2012	3.5106	3.6579	3.4061	3.7803	3.9601
0.3	0.05	0.0125	3.1122	3.3655	3.4829	3.2650	3.5811	3.7315	3.4638	3.8444	4.0272
0.7	-	-	2.8010	2.9952	3.0864	3.2301	3.5559	3.7151	3.6381	4.0543	4.2580
0.7	0.01	-	2.8126	3.0078	3.0995	3.2392	3.5656	3.7250	3.6463	4.0632	4.2671
0.7	0.01	0.0025	2.8134	3.0088	3.1005	3.2391	3.5654	3.7248	3.6457	4.0624	4.2663
0.7	0.025	-	2.8306	3.0274	3.1198	3.2533	3.5806	3.7404	3.6590	4.0768	4.2812
0.7	0.025	0.00625	2.8332	3.0304	3.1229	3.2534	3.5805	3.7403	3.6577	4.0752	4.2795
0.7	0.05	-	2.8625	3.0622	3.1559	3.2782	3.6070	3.7676	3.6810	4.1006	4.3058
0.7	0.05	0.0125	2.8693	3.0698	3.1639	3.2798	3.6083	3.7686	3.6797	4.0987	4.3036

4.2.1.1. Accuracy of ARL Evaluation Method

This section compares the MCA and the MC methods for calculating the ARL and the SDRL of the EWMA, EEWMA, and NEEWMA control charts under a symmetrically distributed process with location $\mu_0 = 0$ and scale $\sigma_0 = 1$. Three symmetric distributions are considered, namely the normal, the logistic, and the Laplace, and the chart parameters are fixed at $\lambda_1 = 0.1$, $\lambda_2 = 0.05$, and $\lambda_3 = \lambda_2/4$ throughout, the corresponding control limits being taken from TABLE 4-20. The agreement between the two methods is measured by the MPE and the MAPE, and the results are as follows.

For the normal distribution, comparing the MCA and the MC methods for calculating the ARL and the SDRL at $ARL_0 = 370$, the values obtained by the two methods are close at every shift level and for all three charts, and the in-control average run length is attained in each case. The analysis results are shown in TABLE 4-21. For $ARL_0 = 200$ and 500, the MCA reproduces the shape of the ARL curve obtained by simulation, as evidenced by the similar trends between the two methods, as shown in TABLES A-11 and B-11 respectively.

For the logistic distribution, comparing the MCA and the MC methods for calculating the ARL and the SDRL at $ARL_0 = 370$, the values obtained by the two methods are close at every shift level and for all three charts, and the in-control average run length is attained in each case. The analysis results are shown in TABLE 4-22. For $ARL_0 = 200$ and 500, the MCA reproduces the shape of the ARL curve obtained by simulation, as evidenced by the similar trends between the two methods, as shown in TABLES A-12 and B-12 respectively.

For the Laplace distribution, comparing the MCA and the MC methods for calculating the ARL and the SDRL at $ARL_0 = 370$, the values obtained by the two methods are close at every shift level and for all three charts, and the in-control average run length is attained in each case. The analysis results are shown in TABLE 4-23. For $ARL_0 = 200$ and 500, the MCA reproduces the shape of the ARL curve obtained by simulation, as evidenced by the similar trends between the two methods, as shown in TABLES A-13 and B-13 respectively.

Turning to the overall comparison, TABLE 4-24 summarizes the closeness of the two methods and TABLE 4-25 their computation times. In point of closeness, the relative error lies between 6.23 and 6.58 under the normal distribution, between 4.13 and 4.77 under the logistic, and between 4.86 and 5.54 under the Laplace, so that the agreement is closest under the logistic distribution and widest under the normal, while the MPE and the MAPE remain below one per cent throughout and are almost unchanged across the three in-control levels. The MPE moreover equals the MAPE in magnitude in every row, which can occur only if the error carries the same sign at every shift level, so that the approximation lies uniformly below the

simulation rather than scattering about it. In point of computation time, the MCA requires between 3.02 and 3.34 seconds per shift level under all three distributions and is therefore effectively invariant to the distribution, to the chart, and to the in-control level, since the size of the matrix it inverts is fixed by the number of subintervals alone, whereas the MC simulation requires between 66.56 and 197.03 seconds and rises steadily with the in-control level because every run must be continued until the chart signals. The approximation is thus between twenty-two and sixty-three times faster per shift level, a saving exceeding 95 per cent over all shift levels combined.



TABLE 4-21 ARL of the EEWMA and NEEWMA control chart using PEA_0 compared to MC under a Normal distribution, with $ARL_0 = 370$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MCA	MCA	MCA	MCA	MCA
-1.00	ARL	8.7464	8.7879	8.8490	8.8303	8.8799	8.8856
	SDRL	4.4865	4.4334	4.3798	4.3095	4.3542	4.3612
	CPU Time	2.86	7.85	2.81	8.04	2.89	8.33
-0.70	ARL	15.3887	15.4942	15.3240	15.2500	15.3159	15.3625
	SDRL	9.5833	9.8079	9.2072	9.1648	9.1155	9.1195
	CPU Time	3.07	12.86	2.81	12.33	3.12	13.22
-0.50	ARL	27.2540	27.5936	26.7025	26.6108	26.5759	26.7463
	SDRL	20.0502	20.2854	19.0417	19.9933	18.7926	18.0652
	CPU Time	3.14	22.06	2.81	20.81	3.41	22.88
-0.10	ARL	247.3983	247.7902	242.3283	242.7912	240.9925	240.9834
	SDRL	239.5225	239.8907	233.6217	233.2451	232.0541	232.0381
	CPU Time	3.01	173.66	3.17	169.33	3.31	180.22
-0.07	ARL	297.9774	297.0163	294.2932	294.6766	293.3090	293.6066
	SDRL	290.5444	290.0685	286.0746	286.7983	284.8708	284.4786
	CPU Time	3.31	210.43	3.25	202.67	3.39	213.03
-0.05	ARL	329.4668	329.9990	327.1426	327.7976	326.5164	326.4627
	SDRL	322.3239	322.4430	319.2514	319.0550	318.4161	318.3078
	CPU Time	3.36	226.46	3.27	227.33	3.44	235.15
-0.01	ARL	368.1944	368.7168	368.0773	368.3821	368.0452	368.5102
	SDRL	361.4221	361.2412	360.6123	360.3479	360.3869	360.8881
	CPU Time	3.36	251.64	3.20	256.13	3.52	263.88
0	ARL	370.0001	370.7120	370.0004	370.5771	370.0002	370.9577
	SDRL	363.2473	363.2864	362.5579	362.5097	362.3653	362.9099
	CPU Time	3.36	253.12	3.25	257.80	3.42	264.63
0.01	ARL	368.1896	368.9850	368.0719	368.4225	368.0396	368.6461
	SDRL	361.4220	361.8812	360.6122	360.6070	360.3868	360.3743
	CPU Time	3.30	255.40	3.21	253.09	3.38	262.11
0.05	ARL	329.4456	329.7065	327.1186	327.7412	326.4917	326.5210
	SDRL	322.3235	322.0081	319.2509	319.1308	318.4156	318.4291
	CPU Time	3.33	229.68	3.22	224.60	3.53	234.67
0.07	ARL	297.9503	297.0804	294.2629	294.4241	293.2778	293.9026
	SDRL	290.5439	290.1783	286.0740	286.7827	284.8702	284.0819
	CPU Time	3.34	208.91	3.25	209.85	3.44	211.49
0.10	ARL	247.3659	247.7596	242.2924	242.5768	240.9556	240.8700
	SDRL	239.5218	239.5142	233.6209	233.0310	232.0533	232.6443
	CPU Time	3.31	175.33	3.28	172.01	3.51	175.30
0.50	ARL	27.2338	27.1963	26.6811	26.6613	26.5542	26.2945
	SDRL	20.0490	20.5197	19.0403	19.1203	18.7911	18.6321
	CPU Time	3.25	21.46	3.24	20.72	3.43	22.05
0.70	ARL	15.3733	15.3263	15.3078	15.3458	15.2994	15.2415
	SDRL	9.5821	9.5244	9.2059	9.1971	9.1141	9.1825
	CPU Time	3.25	12.50	3.12	12.79	3.47	13.45
1.00	ARL	8.7352	8.8307	8.8372	8.8085	8.8680	8.8409
	SDRL	4.4855	4.5564	4.3786	4.3441	4.3530	4.3445
	CPU Time	3.14	8.10	2.94	8.02	3.36	8.45

TABLE 4-22 ARL of the EEWMA and NEEWMA control chart using PEA_0 compared to MC under a Logistic distribution, with $ARL_0 = 370$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MCA	MCA	MCA	MCA	MCA
	ARL	24.6103	24.6199	23.9268	23.9208	23.7728	23.8990
-1.00	SDRL	17.4473	17.4607	16.3707	16.3139	16.1119	16.0377
	CPU Time	2.80	20.06	2.75	19.35	2.72	20.05
	ARL	46.7383	46.5362	44.6674	44.6025	44.1763	44.0901
-0.70	SDRL	38.4307	38.6204	35.7632	35.0517	35.1123	35.7402
	CPU Time	2.78	35.72	3.05	34.05	3.22	35.21
	ARL	83.2208	83.3633	79.0336	79.1798	78.0112	78.0450
-0.50	SDRL	74.5105	74.2818	69.5506	69.3975	68.3188	68.4945
	CPU Time	3.14	62.75	3.18	57.94	3.28	59.30
	ARL	327.8490	327.0131	324.6910	324.3467	323.8432	323.0856
-0.10	SDRL	321.3890	321.0788	317.4522	317.5333	316.3866	316.2314
	CPU Time	3.18	227.58	3.22	229.08	3.33	235.59
	ARL	348.1221	348.0838	346.3670	346.5759	345.8926	345.6959
-0.07	SDRL	341.9112	341.5499	339.4127	339.7634	338.7304	338.0538
	CPU Time	3.19	235.23	3.17	243.55	3.28	250.98
	ARL	358.5188	358.7141	357.5665	357.2895	357.3081	357.4943
-0.05	SDRL	352.4372	352.1966	350.7612	350.3744	350.3005	350.0800
	CPU Time	3.20	247.29	3.15	249.09	3.35	256.24
	ARL	369.5282	369.9778	369.4879	369.0736	369.4768	369.0857
-0.01	SDRL	363.5865	363.9086	362.8447	362.0497	362.6377	362.3630
	CPU Time	3.19	251.83	3.25	254.99	3.24	266.48
	ARL	370.0001	370.8339	370.0002	370.9009	370.0002	370.8229
0	SDRL	364.0653	364.8023	363.3651	363.5052	363.1695	363.8294
	CPU Time	3.20	260.29	3.27	260.41	3.29	264.23
	ARL	369.5259	369.1957	369.4852	369.7036	369.4740	369.3157
0.01	SDRL	363.5864	363.1598	362.8446	362.7047	362.6377	362.7937
	CPU Time	3.28	257.41	3.17	257.97	3.36	262.28
	ARL	358.5076	358.9679	357.5536	357.1239	357.2947	357.4841
0.05	SDRL	352.4370	352.4917	350.7609	350.7138	350.3002	350.4888
	CPU Time	3.20	249.12	3.22	250.02	3.23	255.05
	ARL	348.1068	348.3723	346.3495	346.0045	345.8745	345.7198
0.07	SDRL	341.9109	341.3086	339.4123	339.7646	338.7300	338.8211
	CPU Time	3.22	237.84	3.19	243.59	3.26	249.37
	ARL	327.8284	327.8526	324.6676	324.9693	323.8189	323.6913
0.10	SDRL	321.3886	321.5161	317.4517	317.4468	316.3860	316.4362
	CPU Time	3.14	227.90	3.16	223.06	3.22	232.09
	ARL	83.1911	83.2953	79.0017	79.6392	77.9788	77.1463
0.50	SDRL	74.5095	74.8236	69.5494	69.2370	68.3175	68.4699
	CPU Time	3.16	62.78	3.17	60.00	3.30	61.08
	ARL	46.7136	46.8817	44.6413	44.1645	44.1497	44.2778
0.70	SDRL	38.4296	38.6233	35.7619	35.0716	35.1110	35.6694
	CPU Time	3.00	36.40	3.15	35.22	3.27	34.80
	ARL	24.5913	24.4806	23.9067	23.8348	23.7524	23.6858
1.00	SDRL	17.4461	17.3110	16.3693	16.1725	16.1105	16.2196
	CPU Time	3.11	19.73	3.10	19.44	3.25	19.74

TABLE 4-23 ARL of the EEWMA and NEEWMA control chart using PEA_0 compared to MC under a Laplace distribution, with $ARL_0 = 370$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MCA	MCA	MCA	MCA	MCA
-1.00	ARL	17.2774	17.2935	16.7952	16.6397	16.6911	16.7516
	SDRL	10.7540	10.7304	10.0233	10.8479	9.8533	9.9248
	CPU Time	2.74	15.58	2.69	14.84	3.22	16.55
-0.70	ARL	33.3506	33.8975	31.6455	31.6825	31.2528	31.3312
	SDRL	25.3177	25.0397	23.1535	23.0875	22.6405	22.1192
	CPU Time	2.92	29.26	3.14	28.29	2.84	28.50
-0.50	ARL	62.0354	62.3638	58.0884	58.5946	57.1482	57.4493
	SDRL	53.3078	53.4591	48.6624	48.4325	47.5369	47.6668
	CPU Time	3.07	53.08	2.95	51.27	3.20	50.59
-0.10	ARL	314.8317	314.4664	309.9400	309.6324	308.6231	308.3128
	SDRL	308.9050	308.6124	303.1992	303.3120	301.6536	301.2663
	CPU Time	3.21	245.70	3.12	238.43	3.30	247.62
-0.07	ARL	340.8661	340.4594	338.0395	338.2269	337.2709	337.6053
	SDRL	335.3328	335.9134	331.7503	331.9369	330.7692	330.7101
	CPU Time	3.24	268.85	3.22	262.66	3.30	269.35
-0.05	ARL	354.5733	354.0621	353.0078	353.1409	352.5802	352.7137
	SDRL	349.2492	349.4572	346.9622	346.9683	346.3318	346.9431
	CPU Time	3.13	273.42	3.16	274.11	3.27	286.19
-0.01	ARL	369.3604	369.8113	369.2916	369.3259	369.2736	369.1835
	SDRL	364.2653	364.8557	363.5153	363.7599	363.3060	363.8387
	CPU Time	3.29	288.96	3.22	283.17	3.30	294.63
0	ARL	370.0008	370.6386	370.0000	370.0297	370.0007	370.0934
	SDRL	364.9168	364.9654	364.2367	364.1068	364.0467	364.3319
	CPU Time	3.12	291.46	3.12	289.25	3.34	295.20
0.01	ARL	369.3578	369.0995	369.2886	369.5035	369.2705	369.3746
	SDRL	364.2653	364.9410	363.5152	363.4626	363.3060	363.8434
	CPU Time	3.17	287.62	3.10	291.67	3.34	296.18
0.05	ARL	354.5610	354.7294	352.9936	352.8129	352.5654	352.8627
	SDRL	349.2490	349.9506	346.9619	346.5569	346.3315	346.8262
	CPU Time	3.06	277.66	3.15	279.74	3.30	281.31
0.07	ARL	340.8494	340.4386	338.0203	338.5885	337.2510	337.0453
	SDRL	335.3325	335.0675	331.7499	331.8408	330.7688	330.9333
	CPU Time	3.14	265.66	3.16	261.84	3.31	275.93
0.10	ARL	314.8096	314.0462	309.9147	309.5513	308.5970	308.4216
	SDRL	308.9045	308.1779	303.1986	303.5163	301.6529	301.2722
	CPU Time	3.11	247.06	3.15	244.77	3.28	250.56
0.50	ARL	62.0092	62.6488	58.0604	58.4512	57.1198	57.2555
	SDRL	53.3068	53.6714	48.6613	48.3830	47.5356	47.8642
	CPU Time	3.14	53.86	3.14	49.54	3.34	51.06
0.70	ARL	33.3295	33.4014	31.6232	31.7240	31.2302	31.7669
	SDRL	25.3167	25.8735	23.1522	23.2870	22.6392	22.4480
	CPU Time	3.15	29.88	3.14	28.65	3.34	28.09
1.00	ARL	17.2614	17.1361	16.7785	16.7542	16.6742	16.6057
	SDRL	10.7529	10.5070	10.0220	10.1880	9.8519	9.8989
	CPU Time	3.00	15.26	2.78	15.11	3.29	16.31

TABLE 4-24 Closeness of the ARL of the MEWMA control chart with PEA_0 compared to MC under a symmetric distribution.

Distribution	ARL_0	Charts	RE	RMSE	MAE	MPE	MAPE
Normal	200	EWMA	6.2892	29.1636	5.0713	-0.2329	0.2329
		EEWMA	6.5807	35.2734	5.5496	-0.2437	0.2437
		NEEWMA	6.5678	34.4275	5.4878	-0.2433	0.2433
	370	EWMA	6.2302	41.6045	6.0577	-0.2307	0.2307
		EEWMA	6.5223	51.1941	6.6857	-0.2416	0.2416
		NEEWMA	6.5840	52.1486	6.7527	-0.2439	0.2439
	500	EWMA	6.2576	48.4964	6.5443	-0.2318	0.2318
		EEWMA	6.4502	57.0322	7.0683	-0.2389	0.2389
		NEEWMA	6.5791	61.7662	7.3360	-0.2437	0.2437
Logistic	200	EWMA	4.4669	31.4513	5.4470	-0.1654	0.1654
		EEWMA	4.7020	36.6477	5.8770	-0.1741	0.1741
		NEEWMA	4.7677	37.5127	5.9347	-0.1766	0.1766
	370	EWMA	4.2099	44.3918	6.4620	-0.1559	0.1559
		EEWMA	4.4857	52.3563	7.0087	-0.1661	0.1661
		NEEWMA	4.5583	53.4976	7.0991	-0.1688	0.1688
	500	EWMA	4.1308	52.4049	7.0086	-0.1530	0.1530
		EEWMA	4.3827	61.4318	7.6011	-0.1623	0.1623
		NEEWMA	4.4547	64.2907	7.7507	-0.1650	0.1650
Laplace	200	EWMA	5.1718	26.4862	4.9091	-0.1915	0.1915
		EEWMA	5.4441	30.5771	5.3069	-0.2016	0.2016
		NEEWMA	5.5397	32.2366	5.4536	-0.2052	0.2052
	370	EWMA	4.9892	39.2407	5.9606	-0.1848	0.1848
		EEWMA	5.2757	45.1722	6.4225	-0.1954	0.1954
		NEEWMA	5.3704	47.6654	6.5992	-0.1989	0.1989
	500	EWMA	4.8624	45.4633	6.4294	-0.1801	0.1801
		EEWMA	5.1820	52.1164	6.8772	-0.1919	0.1919
		NEEWMA	5.2618	55.5356	7.1135	-0.1949	0.1949

TABLE 4-25 Computation time efficiency of the ARL of the MEWMA control chart with PEA_0 compared to MC under a symmetric distribution.

Distribution	ARL_0	Charts	TCA		ACT	
			MCA	MC	MCA	MC
Normal	200	EWMA	86.28	1807.77	3.20	66.95
		EEWMA	82.58	1797.17	3.06	66.56
		NEEWMA	90.29	1881.21	3.34	69.67
	370	EWMA	85.97	3057.26	3.18	113.23
		EEWMA	82.78	3037.80	3.07	112.51
		NEEWMA	90.27	3143.91	3.34	116.44
	500	EWMA	87.26	3984.42	3.23	147.57
		EEWMA	83.58	3932.65	3.10	145.65
		NEEWMA	88.94	4093.01	3.29	151.59
Logistic	200	EWMA	83.40	2179.09	3.09	80.71
		EEWMA	83.94	2174.48	3.11	80.54
		NEEWMA	84.10	2235.61	3.11	82.80
	370	EWMA	83.17	3712.43	3.08	137.50
		EEWMA	83.97	3713.57	3.11	137.54
		NEEWMA	85.84	3814.55	3.18	141.28
	500	EWMA	83.41	4875.52	3.09	180.57
		EEWMA	83.07	4839.71	3.08	179.25
		NEEWMA	86.07	4976.79	3.19	184.33
Laplace	200	EWMA	82.73	2330.65	3.06	86.32
		EEWMA	81.97	2317.29	3.04	85.83
		NEEWMA	85.02	2359.85	3.15	87.40
	370	EWMA	82.38	4000.86	3.05	148.18
		EEWMA	82.38	3958.04	3.05	146.59
		NEEWMA	87.61	4062.46	3.24	150.46
	500	EWMA	83.84	5250.12	3.11	194.45
		EEWMA	81.51	5172.28	3.02	191.57
		NEEWMA	86.81	5319.77	3.22	197.03

4.2.1.2. Efficiency of Control charts

This section applies the MCA method to compute the ARL and the SDRL and compares the performance of the EWMA, EEWMA, and NEEWMA control charts, so as to show that the MCA method can be used to measure performance. The processes are defined by a symmetric distribution with location $\mu_0 = 0$ and scale $\sigma_0 = 1$, namely the normal, logistic, and Laplace distributions, and the results are as follows.

For the normal distribution, with $ARL_0 = 370$, it was found that as λ_1 increases the performance in detecting small and moderate shifts decreases, but conversely the performance in detecting large shifts increases. When λ_1 is held constant, increasing λ_2 improves the performance in detecting small and moderate shifts while reducing it for large shifts, and with $\lambda_3 = \lambda_2/4$ an increase in λ_3 produces the same effect. Furthermore, the gain obtained in the small-shift region exceeds the loss incurred in the large-shift region, so that the extended charts are preferable wherever small shifts are the principal concern. The results are shown in TABLE 4-26, and those for $ARL_0 = 200$ and 500 in TABLES A-14 and B-14 respectively.

For the logistic distribution, with $ARL_0 = 370$, the same relationships hold: an increase in λ_1 reduces the efficiency in detecting small and moderate shifts while improving it for large shifts, whereas increases in λ_2 and λ_3 act in the opposite direction, and the gain obtained in the small-shift region again exceeds the loss incurred in the large-shift region. The run lengths under this distribution are longer than under the normal at every out-of-control shift, so that a shift of a given magnitude requires more observations before it is signalled, but the direction and the relative magnitude of the parameter effects are unaltered. The results are shown in TABLE 4-27, and those for $ARL_0 = 200$ and 500 in TABLES A-15 and B-15 respectively.

For the Laplace distribution, with $ARL_0 = 370$, the relationships are likewise unchanged, except that the superiority of a higher λ_1 emerges only once the shift is sufficiently large; for moderate shifts the lower value of λ_1 retains its advantage over a wider range than under the other two distributions. Increases in λ_2 and λ_3 again improve the detection of small and moderate shifts at the expense of large ones, with the gain exceeding the loss. The results are shown in TABLE 4-28, and those for $ARL_0 = 200$ and 500 in TABLES A-16 and B-16 respectively.

The three symmetric distributions therefore yield the same qualitative pattern at all three in-control levels, which indicates that the influence of the weighted parameters is a property of the monitoring statistics themselves rather than of any particular process distribution. The chart is more effective at detecting small and moderate shifts when λ_1 is low and λ_2 and λ_3 are high, and more effective

against large shifts when the reverse holds, since a small λ_1 accumulates information over a longer history while the second and third weighted parameters extend the memory of the statistic in the same way that a reduction in λ_1 does..

TABLE 4-26 Effect of control charts parameters on ARL under a Normal distribution, with PEA_0 and $ARL_0 = 370$.

δ	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
Large	-3.00	1.76	1.09	0.78	1.85	2.00	2.35	1.87	2.07	2.61
	-2.00	3.18	2.39	2.92	3.29	3.49	3.98	3.32	3.59	4.36
	-1.00	8.75	9.90	21.91	8.85	9.07	9.75	8.88	9.20	10.34
Medium	-0.90	10.29	12.51	28.81	10.36	10.56	11.21	10.39	10.67	11.82
	-0.70	15.39	22.00	52.28	15.32	15.33	15.77	15.32	15.38	16.35
	-0.50	27.25	45.59	100.85	26.70	26.02	25.49	26.58	25.81	25.70
	-0.30	65.92	114.81	199.44	63.54	60.09	54.97	62.95	58.71	53.02
	-0.10	247.40	302.47	339.73	242.33	233.86	216.82	240.99	229.97	206.60
Small	-0.09	264.15	313.47	345.13	259.43	251.46	235.12	258.18	247.78	225.07
	-0.07	297.98	333.76	354.59	294.29	287.95	274.39	293.31	284.96	265.61
	-0.05	329.47	350.64	361.99	327.14	323.07	314.01	326.52	321.11	307.86
	-0.03	354.33	362.81	367.08	353.36	351.62	347.64	353.09	350.78	344.83
	-0.01	368.19	369.19	369.67	368.08	367.87	367.38	368.05	367.76	367.02
No change	0.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00
Small	0.01	368.19	369.19	369.67	368.07	367.86	367.37	368.04	367.76	367.01
	0.03	354.32	362.80	367.08	353.34	351.60	347.61	353.08	350.76	344.80
	0.05	329.45	350.64	361.99	327.12	323.04	313.96	326.49	321.08	307.80
	0.07	297.95	333.75	354.58	294.26	287.91	274.33	293.28	284.92	265.55
	0.09	264.12	313.46	345.13	259.39	251.42	235.06	258.14	247.73	225.00
Medium	0.10	247.37	302.46	339.73	242.29	233.81	216.76	240.96	229.93	206.53
	0.30	65.89	114.80	199.44	63.51	60.05	54.93	62.92	58.68	52.97
	0.50	27.23	45.58	100.85	26.68	26.00	25.46	26.55	25.78	25.67
	0.70	15.37	21.99	52.28	15.31	15.31	15.75	15.30	15.36	16.33
	0.90	10.28	12.51	28.81	10.35	10.54	11.19	10.38	10.66	11.80
Large	1.00	8.74	9.90	21.91	8.84	9.06	9.73	8.87	9.19	10.32
	2.00	3.18	2.39	2.92	3.29	3.49	3.98	3.32	3.59	4.35
	3.00	1.76	1.09	0.78	1.84	1.99	2.35	1.87	2.07	2.61

TABLE 4-27 Effect of control charts parameters on ARL under a Logistic distribution, with PEA_0 and $ARL_0 = 370$.

δ	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
Large	-3.00	4.29	4.16	12.85	4.39	4.59	5.11	4.42	4.69	5.53
	-2.00	7.80	10.62	45.18	7.88	8.06	8.64	7.90	8.16	9.17
	-1.00	24.61	56.36	169.19	23.93	23.11	22.48	23.77	22.86	22.69
Medium	-0.90	29.72	70.09	191.31	28.73	27.49	26.26	28.50	27.07	26.24
	-0.70	46.74	110.71	240.59	44.67	41.90	38.41	44.18	40.88	37.44
	-0.50	83.22	176.54	292.65	79.03	73.07	64.36	78.01	70.73	60.93
	-0.30	168.67	270.06	338.59	161.64	150.79	132.18	159.86	146.18	122.98
	-0.10	327.85	355.86	366.27	324.69	319.20	307.25	323.84	316.59	299.38
Small	-0.09	335.13	358.47	366.98	332.45	327.77	317.46	331.73	325.53	310.58
	-0.07	348.12	362.95	368.17	346.37	343.27	336.31	345.89	341.77	331.54
	-0.05	358.52	366.37	369.06	357.57	355.87	352.00	357.31	355.05	349.30
	-0.03	365.79	368.69	369.66	365.43	364.79	363.31	365.33	364.48	362.26
	-0.01	369.53	369.85	369.96	369.49	369.42	369.25	369.48	369.38	369.13
No change	0.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00
Small	0.01	369.53	369.85	369.96	369.49	369.41	369.24	369.47	369.38	369.12
	0.03	365.78	368.69	369.66	365.42	364.78	363.30	365.32	364.47	362.24
	0.05	358.51	366.37	369.06	357.55	355.86	351.98	357.29	355.03	349.26
	0.07	348.11	362.95	368.17	346.35	343.25	336.27	345.87	341.75	331.50
	0.09	335.11	358.47	366.98	332.43	327.74	317.42	331.71	325.50	310.52
Medium	0.10	327.83	355.85	366.27	324.67	319.17	307.20	323.82	316.55	299.32
	0.30	168.63	270.05	338.59	161.61	150.75	132.13	159.82	146.14	122.91
	0.50	83.19	176.53	292.65	79.00	73.04	64.32	77.98	70.69	60.88
	0.70	46.71	110.70	240.59	44.64	41.87	38.37	44.15	40.84	37.40
	0.90	29.70	70.08	191.31	28.71	27.46	26.23	28.48	27.05	26.20
Large	1.00	24.59	56.35	169.19	23.91	23.09	22.46	23.75	22.84	22.66
	2.00	7.79	10.61	45.18	7.87	8.04	8.63	7.89	8.15	9.15
	3.00	4.28	4.15	12.85	4.38	4.58	5.10	4.41	4.69	5.52

TABLE 4-28 Effect of control charts parameters on ARL under a Laplace distribution, with PEA_0 and $ARL_0 = 370$.

δ	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
	-3.00	3.11	2.93	11.38	3.19	3.35	3.75	3.21	3.42	4.08
Large	-2.00	5.59	7.80	43.41	5.66	5.82	6.31	5.68	5.91	6.74
	-1.00	17.28	48.13	167.66	16.80	16.28	16.06	16.69	16.14	16.39
	-0.90	20.90	61.02	189.89	20.17	19.32	18.69	20.00	19.06	18.89
	-0.70	33.35	100.12	239.48	31.65	29.50	27.16	31.25	28.76	26.75
Medium	-0.50	62.04	165.73	291.93	58.09	52.75	45.78	57.15	50.77	43.48
	-0.30	138.50	262.50	338.28	130.43	118.43	99.52	128.42	113.53	91.22
	-0.10	314.83	354.54	366.23	309.94	301.39	282.93	308.62	297.32	271.14
	-0.09	324.08	357.39	366.95	319.87	312.46	296.17	318.73	308.90	285.54
	-0.07	340.87	362.28	368.15	338.04	332.98	321.51	337.27	330.53	313.71
Small	-0.05	354.57	366.03	369.05	353.01	350.17	343.56	352.58	348.78	338.91
	-0.03	364.30	368.56	369.66	363.71	362.62	360.03	363.54	362.08	358.16
	-0.01	369.36	369.84	369.96	369.29	369.17	368.87	369.27	369.11	368.65
No change	0.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00
	0.01	369.36	369.84	369.96	369.29	369.16	368.86	369.27	369.10	368.64
	0.03	364.30	368.56	369.66	363.70	362.61	360.01	363.54	362.07	358.14
Small	0.05	354.56	366.02	369.05	352.99	350.16	343.53	352.57	348.76	338.87
	0.07	340.85	362.28	368.15	338.02	332.96	321.47	337.25	330.50	313.66
	0.09	324.06	357.38	366.94	319.85	312.43	296.12	318.71	308.87	285.48
	0.10	314.81	354.53	366.23	309.91	301.36	282.88	308.60	297.29	271.08
	0.30	138.47	262.49	338.28	130.40	118.39	99.47	128.38	113.48	91.15
Medium	0.50	62.01	165.71	291.93	58.06	52.72	45.75	57.12	50.73	43.44
	0.70	33.33	100.11	239.48	31.62	29.47	27.13	31.23	28.73	26.72
	0.90	20.89	61.01	189.89	20.15	19.30	18.66	19.98	19.04	18.86
	1.00	17.26	48.12	167.66	16.78	16.26	16.04	16.67	16.12	16.36
Large	2.00	5.59	7.79	43.41	5.65	5.81	6.30	5.68	5.90	6.73
	3.00	3.10	2.93	11.37	3.18	3.34	3.74	3.21	3.42	4.07

4.2.1.3. Behavior of Detection

This section applies the MCA method to compute the ARL and the SDRL in order to study the effect of the location and scale parameters of a symmetrically distributed process on the performance of the EWMA, EEWMA, and NEEWMA control charts under the normal, logistic, and Laplace distributions. The chart parameters were fixed at $\lambda_1 = 0.1$, $\lambda_2 = 0.05$, and $\lambda_3 = \lambda_2/4$, and the results are presented separately for the two process parameters.

For the location parameter, with $ARL_0 = 370$ and the process scale held at $\sigma_0 = 1$, it was found that the ARL values at each level of the location parameter did not differ across all three symmetric distributions and at all shift magnitudes. This holds for the EWMA control chart, for the EEWMA control chart, and for the NEEWMA control chart alike, demonstrating that the location parameter of a symmetric distribution does not affect the performance of any of the three charts. The result follows from the charts being constructed on deviations from the target rather than on the observations themselves, so that a shift in the centre of the process distribution leaves the run-length behaviour unchanged. The analysis results are shown in TABLES 4-29 to 4-31 for the normal, logistic, and Laplace distributions respectively. For $ARL_0 = 200$ and 500 the location parameter likewise did not affect the performance of any of the three charts, as shown in TABLES A-17 to A-19 for $ARL_0 = 200$ and TABLES B-17 to B-19 for $ARL_0 = 500$.

For the scale parameter, with $ARL_0 = 370$ and the process location held at $\mu_0 = 0$, it was found that the performance of all three control charts decreased as the process took a higher scale value, indicating that the scale parameter of a symmetric distribution has an inverse effect on the performance of the EWMA, EEWMA, and NEEWMA control charts. The effect is substantial: under the normal distribution a shift of one standard deviation is detected after roughly nine observations when the scale is unity but after more than two hundred when the scale is ten, since a more dispersed process conceals a shift of any given size for longer. The analysis results are shown in TABLES 4-32 to 4-34 for the normal, logistic, and Laplace distributions respectively. For $ARL_0 = 200$ and 500 the scale parameter likewise resulted in decreased performance for all three charts, as shown in TABLES A-20 to A-22 for $ARL_0 = 200$ and TABLES B-20 to B-22 for $ARL_0 = 500$.

Furthermore, a comparison of the performance of the control charts across the three distributions revealed that the charts were best at detecting shifts occurring in processes with symmetric distributions. The ordering holds for all three charts and at every shift magnitude, and its size is not negligible: under the EWMA chart a shift of one standard deviation is signaled after roughly nine observations when the process is normal, but after seventeen under the Laplace and twenty-five under the logistic distribution. The same ordering is evident in the control

limits reported in TABLE 4-20, the heavier-tailed distributions requiring wider limits so that a shift must displace the monitoring statistic further before a signal is produced.

TABLE 4-29 Effect of the location parameters in process on the ARL of control charts under a Normal distribution, with PEA_0 and $ARL_0 = 370$.

δ	Charts	EMWA			EEWMA			NEEWMA		
	μ_0	0	5	10	0	5	10	0	5	10
	-3.00	1.76	1.76	1.76	1.85	1.85	1.85	1.87	1.87	1.76
Large	-2.00	3.18	3.18	3.18	3.29	3.29	3.29	3.32	3.32	3.18
	-1.00	8.75	8.75	8.75	8.85	8.85	8.85	8.88	8.88	8.75
	-0.90	10.29	10.29	10.29	10.36	10.36	10.36	10.39	10.39	10.29
	-0.70	15.39	15.39	15.39	15.32	15.32	15.32	15.32	15.32	15.39
Medium	-0.50	27.25	27.25	27.25	26.70	26.70	26.70	26.58	26.58	27.25
	-0.30	65.92	65.92	65.92	63.54	63.54	63.54	62.95	62.95	65.92
	-0.10	247.40	247.40	247.40	242.33	242.33	242.33	240.99	240.99	247.40
	-0.09	264.15	264.15	264.15	259.43	259.43	259.43	258.18	258.18	264.15
	-0.07	297.98	297.98	297.98	294.29	294.29	294.29	293.31	293.31	297.98
Small	-0.05	329.47	329.47	329.47	327.14	327.14	327.14	326.52	326.52	329.47
	-0.03	354.33	354.33	354.33	353.36	353.36	353.36	353.09	353.09	354.33
	-0.01	368.19	368.19	368.19	368.08	368.08	368.08	368.05	368.05	368.19
No change	0.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00
	0.01	368.19	368.19	368.19	368.07	368.07	368.07	368.04	368.04	368.19
	0.03	354.32	354.32	354.32	353.34	353.34	353.34	353.08	353.08	354.32
Small	0.05	329.45	329.45	329.45	327.12	327.12	327.12	326.49	326.49	329.45
	0.07	297.95	297.95	297.95	294.26	294.26	294.26	293.28	293.28	297.95
	0.09	264.12	264.12	264.12	259.39	259.39	259.39	258.14	258.14	264.12
	0.10	247.37	247.37	247.37	242.29	242.29	242.29	240.96	240.96	247.37
	0.30	65.89	65.89	65.89	63.51	63.51	63.51	62.92	62.92	65.89
Medium	0.50	27.23	27.23	27.23	26.68	26.68	26.68	26.55	26.55	27.23
	0.70	15.37	15.37	15.37	15.31	15.31	15.31	15.30	15.30	15.37
	0.90	10.28	10.28	10.28	10.35	10.35	10.35	10.38	10.38	10.28
	1.00	8.74	8.74	8.74	8.84	8.84	8.84	8.87	8.87	8.74
Large	2.00	3.18	3.18	3.18	3.29	3.29	3.29	3.32	3.32	3.18
	3.00	1.76	1.76	1.76	1.84	1.84	1.84	1.87	1.87	1.76

TABLE 4-30 Effect of the location parameters in process on the ARL of control charts under a Logistic distribution, with PEA_0 and $ARL_0 = 370$.

δ	Charts	EMWA			EEWMA			NEEWMA		
	μ_0	0	5	10	0	5	10	0	5	10
Large	-3.00	1.87	4.29	4.29	4.29	4.39	4.39	4.39	4.42	4.42
	-2.00	3.32	7.80	7.80	7.80	7.88	7.88	7.88	7.90	7.90
	-1.00	8.88	24.61	24.61	24.61	23.93	23.93	23.93	23.77	23.77
Medium	-0.90	10.39	29.72	29.72	29.72	28.73	28.73	28.73	28.50	28.50
	-0.70	15.32	46.74	46.74	46.74	44.67	44.67	44.67	44.18	44.18
	-0.50	26.58	83.22	83.22	83.22	79.03	79.03	79.03	78.01	78.01
	-0.30	62.95	168.67	168.67	168.67	161.64	161.64	161.64	159.86	159.86
	-0.10	240.99	327.85	327.85	327.85	324.69	324.69	324.69	323.84	323.84
Small	-0.09	258.18	335.13	335.13	335.13	332.45	332.45	332.45	331.73	331.73
	-0.07	293.31	348.12	348.12	348.12	346.37	346.37	346.37	345.89	345.89
	-0.05	326.52	358.52	358.52	358.52	357.57	357.57	357.57	357.31	357.31
	-0.03	353.09	365.79	365.79	365.79	365.43	365.43	365.43	365.33	365.33
	-0.01	368.05	369.53	369.53	369.53	369.49	369.49	369.49	369.48	369.48
No change	0.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00
Small	0.01	368.04	369.53	369.53	369.53	369.49	369.49	369.49	369.47	369.47
	0.03	353.08	365.78	365.78	365.78	365.42	365.42	365.42	365.32	365.32
	0.05	326.49	358.51	358.51	358.51	357.55	357.55	357.55	357.29	357.29
	0.07	293.28	348.11	348.11	348.11	346.35	346.35	346.35	345.87	345.87
	0.09	258.14	335.11	335.11	335.11	332.43	332.43	332.43	331.71	331.71
Medium	0.10	240.96	327.83	327.83	327.83	324.67	324.67	324.67	323.82	323.82
	0.30	62.92	168.63	168.63	168.63	161.61	161.61	161.61	159.82	159.82
	0.50	26.55	83.19	83.19	83.19	79.00	79.00	79.00	77.98	77.98
	0.70	15.30	46.71	46.71	46.71	44.64	44.64	44.64	44.15	44.15
	0.90	10.38	29.70	29.70	29.70	28.71	28.71	28.71	28.48	28.48
Large	1.00	8.87	24.59	24.59	24.59	23.91	23.91	23.91	23.75	23.75
	2.00	3.32	7.79	7.79	7.79	7.87	7.87	7.87	7.89	7.89
	3.00	1.87	4.28	4.28	4.28	4.38	4.38	4.38	4.41	4.41

TABLE 4-31 Effect of the location parameters in process on the ARL of control charts under a Laplace distribution, with PEA_0 and $ARL_0 = 370$.

δ	Charts	EMWA			EEWMA			NEEWMA		
	μ_0	0	5	10	0	5	10	0	5	10
Large	-3.00	3.11	3.11	3.11	3.19	3.19	3.19	3.21	3.21	3.21
	-2.00	5.59	5.59	5.59	5.66	5.66	5.66	5.68	5.68	5.68
	-1.00	17.28	17.28	17.28	16.80	16.80	16.80	16.69	16.69	16.69
Medium	-0.90	20.90	20.90	20.90	20.17	20.17	20.17	20.00	20.00	20.00
	-0.70	33.35	33.35	33.35	31.65	31.65	31.65	31.25	31.25	31.25
	-0.50	62.04	62.04	62.04	58.09	58.09	58.09	57.15	57.15	57.15
	-0.30	138.50	138.50	138.50	130.43	130.43	130.43	128.42	128.42	128.42
	-0.10	314.83	314.83	314.83	309.94	309.94	309.94	308.62	308.62	308.62
Small	-0.09	324.08	324.08	324.08	319.87	319.87	319.87	318.73	318.73	318.73
	-0.07	340.87	340.87	340.87	338.04	338.04	338.04	337.27	337.27	337.27
	-0.05	354.57	354.57	354.57	353.01	353.01	353.01	352.58	352.58	352.58
	-0.03	364.30	364.30	364.30	363.71	363.71	363.71	363.54	363.54	363.54
	-0.01	369.36	369.36	369.36	369.29	369.29	369.29	369.27	369.27	369.27
No change	0.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00
Small	0.01	369.36	369.36	369.36	369.29	369.29	369.29	369.27	369.27	369.27
	0.03	364.30	364.30	364.30	363.70	363.70	363.70	363.54	363.54	363.54
	0.05	354.56	354.56	354.56	352.99	352.99	352.99	352.57	352.57	352.57
	0.07	340.85	340.85	340.85	338.02	338.02	338.02	337.25	337.25	337.25
	0.09	324.06	324.06	324.06	319.85	319.85	319.85	318.71	318.71	318.71
Medium	0.10	314.81	314.81	314.81	309.91	309.91	309.91	308.60	308.60	308.60
	0.30	138.47	138.47	138.47	130.40	130.40	130.40	128.38	128.38	128.38
	0.50	62.01	62.01	62.01	58.06	58.06	58.06	57.12	57.12	57.12
	0.70	33.33	33.33	33.33	31.62	31.62	31.62	31.23	31.23	31.23
	0.90	20.89	20.89	20.89	20.15	20.15	20.15	19.98	19.98	19.98
Large	1.00	17.26	17.26	17.26	16.78	16.78	16.78	16.67	16.67	16.67
	2.00	5.59	5.59	5.59	5.65	5.65	5.65	5.68	5.68	5.68
	3.00	3.10	3.10	3.10	3.18	3.18	3.18	3.21	3.21	3.21

TABLE 4-32 Effect of the scale parameters in process on the ARL of control charts under a Normal distribution, with PEA_0 and $ARL_0 = 370$.

δ	Charts	EMWA			EEWMA			NEEWMA		
	β_0	1	5	10	1	5	10	1	5	10
Large	-3.00	1.76	19.91	65.92	1.85	19.68	63.54	1.87	19.63	62.95
	-2.00	3.18	40.26	122.45	3.29	39.09	118.01	3.32	38.81	116.88
	-1.00	8.75	122.45	247.40	8.85	118.01	242.33	8.88	116.88	240.99
Medium	-0.90	10.29	140.62	264.15	10.36	135.75	259.43	10.39	134.50	258.18
	-0.70	15.39	186.99	297.98	15.32	181.52	294.29	15.32	180.11	293.31
	-0.50	27.25	247.40	329.47	26.70	242.33	327.14	26.58	240.99	326.52
	-0.30	65.92	314.26	354.33	63.54	311.23	353.36	62.95	310.42	353.09
	-0.10	247.40	362.87	368.19	242.33	362.42	368.08	240.99	362.30	368.05
Small	-0.09	264.15	364.21	368.54	259.43	363.84	368.44	258.18	363.74	368.42
	-0.07	297.98	366.48	369.11	294.29	366.25	369.06	293.31	366.19	369.04
	-0.05	329.47	368.19	369.55	327.14	368.08	369.52	326.52	368.05	369.51
	-0.03	354.33	369.35	369.84	353.36	369.31	369.83	353.09	369.29	369.82
	-0.01	368.19	369.93	369.98	368.08	369.92	369.98	368.05	369.92	369.98
No change	0.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00
Small	0.01	368.19	369.93	369.98	368.07	369.92	369.98	368.04	369.92	369.98
	0.03	354.32	369.35	369.84	353.34	369.30	369.83	353.08	369.29	369.82
	0.05	329.45	368.19	369.55	327.12	368.07	369.52	326.49	368.04	369.51
	0.07	297.95	366.47	369.11	294.26	366.24	369.05	293.28	366.18	369.04
	0.09	264.12	364.20	368.53	259.39	363.83	368.44	258.14	363.73	368.41
Medium	0.10	247.37	362.87	368.19	242.29	362.41	368.07	240.96	362.28	368.04
	0.30	65.89	314.24	354.32	63.51	311.21	353.34	62.92	310.39	353.08
	0.50	27.23	247.37	329.45	26.68	242.29	327.12	26.55	240.96	326.49
	0.70	15.37	186.96	297.95	15.31	181.48	294.26	15.30	180.07	293.28
	0.90	10.28	140.59	264.12	10.35	135.71	259.39	10.38	134.47	258.14
Large	1.00	8.74	122.41	247.37	8.84	117.97	242.29	8.87	116.85	240.96
	2.00	3.18	40.24	122.41	3.29	39.06	117.97	3.32	38.78	116.85
	3.00	1.76	19.90	65.89	1.84	19.66	63.51	1.87	19.61	62.92

TABLE 4-33 Effect of the scale parameters in process on the ARL of control charts under a Logistic distribution, with PEA_0 and $ARL_0 = 370$.

δ	Charts	EMWA			EEWMA			NEEWMA		
	β_0	1	5	10	1	5	10	1	5	10
Large	-3.00	4.29	61.31	168.67	4.39	58.34	161.64	4.42	57.63	159.86
	-2.00	7.80	116.90	243.27	7.88	111.21	236.51	7.90	109.80	234.75
	-1.00	24.61	243.27	327.85	23.93	236.51	324.69	23.77	234.75	323.84
Medium	-0.90	29.72	260.44	335.13	28.73	254.13	332.45	28.50	252.47	331.73
	-0.70	46.74	295.27	348.12	44.67	290.30	346.37	44.18	288.98	345.89
	-0.50	83.22	327.85	358.52	79.03	324.69	357.57	78.01	323.84	357.31
	-0.30	168.67	353.68	365.79	161.64	352.35	365.43	159.86	351.99	365.33
	-0.10	327.85	368.12	369.53	324.69	367.96	369.49	323.84	367.91	369.48
Small	-0.09	335.13	368.47	369.62	332.45	368.34	369.59	331.73	368.31	369.58
	-0.07	348.12	369.08	369.77	346.37	369.00	369.75	345.89	368.97	369.74
	-0.05	358.52	369.53	369.88	357.57	369.49	369.87	357.31	369.48	369.87
	-0.03	365.79	369.83	369.96	365.43	369.82	369.95	365.33	369.81	369.95
	-0.01	369.53	369.98	370.00	369.49	369.98	370.00	369.48	369.98	370.00
No change	0.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00
Small	0.01	369.53	369.98	370.00	369.49	369.98	369.99	369.47	369.98	369.99
	0.03	365.78	369.83	369.96	365.42	369.81	369.95	365.32	369.81	369.95
	0.05	358.51	369.53	369.88	357.55	369.49	369.87	357.29	369.47	369.87
	0.07	348.11	369.07	369.77	346.35	368.99	369.75	345.87	368.97	369.74
	0.09	335.11	368.47	369.62	332.43	368.34	369.58	331.71	368.30	369.57
Medium	0.10	327.83	368.11	369.53	324.67	367.95	369.49	323.82	367.91	369.47
	0.30	168.63	353.67	365.78	161.61	352.33	365.42	159.82	351.97	365.32
	0.50	83.19	327.83	358.51	79.00	324.67	357.55	77.98	323.82	357.29
	0.70	46.71	295.25	348.11	44.64	290.27	346.35	44.15	288.95	345.87
	0.90	29.70	260.41	335.11	28.71	254.10	332.43	28.48	252.44	331.71
Large	1.00	24.59	243.24	327.83	23.91	236.48	324.67	23.75	234.72	323.82
	2.00	7.79	116.87	243.24	7.87	111.18	236.48	7.89	109.76	234.72
	3.00	4.28	61.28	168.63	4.38	58.31	161.61	4.41	57.60	159.82

TABLE 4-34 Effect of the scale parameters in process on the ARL of control charts under a Laplace distribution, with PEA_0 and $ARL_0 = 370$.

δ	Charts	EMWA			EEWMA			NEEWMA		
	β_0	1	5	10	1	5	10	1	5	10
Large	-3.00	3.11	44.49	138.50	3.19	41.89	130.43	3.21	41.28	128.42
	-2.00	5.59	90.70	215.33	5.66	84.85	206.40	5.68	83.43	204.09
	-1.00	17.28	215.33	314.83	16.80	206.40	309.94	16.69	204.09	308.62
Medium	-0.90	20.90	234.41	324.08	20.17	225.79	319.87	20.00	223.55	318.73
	-0.70	33.35	274.83	340.87	31.65	267.59	338.04	31.25	265.67	337.27
	-0.50	62.04	314.83	354.57	58.09	309.94	353.01	57.15	308.62	352.58
	-0.30	138.50	348.16	364.30	130.43	345.99	363.71	128.42	345.40	363.54
	-0.10	314.83	367.45	369.36	309.94	367.18	369.29	308.62	367.11	369.27
Small	-0.09	324.08	367.93	369.48	319.87	367.71	369.43	318.73	367.65	369.41
	-0.07	340.87	368.75	369.69	338.04	368.61	369.65	337.27	368.58	369.64
	-0.05	354.57	369.36	369.84	353.01	369.29	369.82	352.58	369.27	369.82
	-0.03	364.30	369.77	369.94	363.71	369.75	369.94	363.54	369.74	369.94
	-0.01	369.36	369.98	369.99	369.29	369.97	369.99	369.27	369.97	369.99
No change	0.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00
Small	0.01	369.36	369.97	369.99	369.29	369.97	369.99	369.27	369.97	369.99
	0.03	364.30	369.77	369.94	363.70	369.74	369.94	363.54	369.74	369.93
	0.05	354.56	369.36	369.84	352.99	369.29	369.82	352.57	369.27	369.82
	0.07	340.85	368.74	369.69	338.02	368.61	369.65	337.25	368.57	369.64
	0.09	324.06	367.93	369.48	319.85	367.71	369.42	318.71	367.65	369.41
Medium	0.10	314.81	367.44	369.36	309.91	367.17	369.29	308.60	367.10	369.27
	0.30	138.47	348.15	364.30	130.40	345.98	363.70	128.38	345.38	363.54
	0.50	62.01	314.81	354.56	58.06	309.91	352.99	57.12	308.60	352.57
	0.70	33.33	274.80	340.85	31.62	267.56	338.02	31.23	265.64	337.25
	0.90	20.89	234.38	324.06	20.15	225.76	319.85	19.98	223.51	318.71
Large	1.00	17.26	215.30	314.81	16.78	206.36	309.91	16.67	204.05	308.60
	2.00	5.59	90.67	215.30	5.65	84.82	206.36	5.68	83.39	204.05
	3.00	3.10	44.47	138.47	3.18	41.87	130.40	3.21	41.26	128.38

4.2.2. Performance of MCA under Asymmetric Distributions

This section presents the accuracy, the detection behavior, and the performance of the EWMA, EEWMA, and NEEWMA control charts evaluated by the MCA method under asymmetric process distributions. Two families are considered, so that the effects of the shape parameter and of the location parameter may be examined separately: the gamma and Weibull distributions from the shape–scale family, and the Gumbel and two-parameter Rayleigh distributions from the location–scale family. The control limit coefficients were obtained by the bisection method described in Algorithm 1, with the scale parameter fixed at unity, and are reported in TABLES 4-35 to 4-38, one table for each distribution in the order named above. Three patterns emerge. First, the limits widen as the second weighted parameter increases and widen further when the third is introduced, so that the NEEWMA chart requires the widest limits and the EWMA chart the narrowest; the effect is more pronounced than under the symmetric distributions and diminishes sharply as the first weighted parameter increases. Second, the two families differ in how the remaining process parameters. Under the location–scale family the limits of the EWMA chart are unaffected by the location parameter, taking identical values at every level of it, under the shape–scale family the limits of the EWMA chart change only slightly with the shape parameter, while those of the extended charts again grow substantially. Finally, the limits widen as the specified in-control ARL increases, an expected outcome for limits designed to reduce the probability of a false alarm.

TABLE 4-35 The control limit coefficient parameters of EWMA, EEWMA and NEEWMA with PEA_0 under Gamma distribution with scale 1.

λ_1	λ_2	λ_3	ARL_0								
			$\alpha_0 = 1$			$\alpha_0 = 5$			$\alpha_0 = 10$		
			200	370	500	200	370	500	200	370	500
0.1	0	0	2.495	2.911	3.114	2.457	2.732	2.865	2.456	2.716	2.839
0.1	0.01	0	3.613	4.106	4.333	4.659	5.076	5.261	5.539	5.942	6.118
0.1	0.01	0.0025	3.943	4.452	4.684	5.361	5.794	5.985	6.520	6.938	7.121
0.1	0.025	0	6.094	6.701	6.973	9.924	10.459	10.689	12.890	13.414	13.636
0.1	0.025	0.00625	7.498	8.165	8.461	12.953	13.552	13.806	17.136	17.726	17.972
0.1	0.05	0	14.642	15.624	16.039	28.486	29.429	29.805	38.957	39.908	40.279
0.1	0.05	0.0125	23.920	25.335	25.903	49.091	50.555	51.096	68.061	69.584	70.128
0.3	0	0	3.380	3.877	4.116	2.882	3.221	3.383	2.789	3.074	3.212
0.3	0.01	0	3.593	4.100	4.343	3.295	3.648	3.814	3.322	3.641	3.790
0.3	0.01	0.0025	3.643	4.152	4.395	3.399	3.754	3.920	3.466	3.788	3.937
0.3	0.025	0	3.946	4.468	4.718	3.991	4.359	4.531	4.275	4.609	4.764
0.3	0.025	0.00625	4.088	4.615	4.867	4.290	4.663	4.837	4.690	5.030	5.187
0.3	0.05	0	4.637	5.188	5.450	5.373	5.768	5.951	6.173	6.534	6.700
0.3	0.05	0.0125	5.001	5.564	5.831	6.141	6.548	6.737	7.241	7.614	7.786
0.7	0	0	4.140	4.733	5.022	3.306	3.678	3.857	3.092	3.414	3.568
0.7	0.01	0	4.187	4.781	5.071	3.402	3.776	3.955	3.225	3.549	3.703
0.7	0.01	0.0025	4.196	4.789	5.079	3.424	3.798	3.977	3.257	3.580	3.735
0.7	0.025	0	4.260	4.855	5.146	3.553	3.928	4.108	3.433	3.759	3.914
0.7	0.025	0.00625	4.283	4.878	5.168	3.611	3.986	4.166	3.516	3.842	3.997
0.7	0.05	0	4.388	4.986	5.278	3.820	4.198	4.379	3.803	4.131	4.288
0.7	0.05	0.0125	4.440	5.038	5.330	3.947	4.326	4.508	3.985	4.314	4.471

TABLE 4-36 The control limit coefficient parameters of EWMA, EEWMA and NEEWMA with PEA_0 under Weibull distribution with scale 1.

λ_1	λ_2	λ_3	ARL_0								
			$\alpha_0 = 1$			$\alpha_0 = 5$			$\alpha_0 = 10$		
			200	370	500	200	370	500	200	370	500
0.1	0	0	2.4948	2.9112	3.1138	3.6436	3.6598	3.6665	3.9686	3.9741	3.9763
0.1	0.01	0	3.6128	4.1064	4.3328	3.2111	3.2296	3.2371	3.5939	3.6001	3.6026
0.1	0.01	0.0025	3.9428	4.4515	4.6840	3.0690	3.0881	3.0958	3.4679	3.4743	3.4768
0.1	0.025	0	6.0935	6.7013	6.9735	2.1699	2.1928	2.2020	2.6752	2.6829	2.6859
0.1	0.025	0.00625	7.4976	8.1653	8.4610	1.5103	1.5350	1.5450	2.0820	2.0904	2.0936
0.1	0.05	0	14.6419	15.6236	16.0390	2.2090	2.2464	2.2603	1.1142	1.1249	1.1287
0.1	0.05	0.0125	23.9200	25.3349	25.9026	7.2589	7.3316	7.3546	5.7205	5.7470	5.7543
0.3	0	0	3.3795	3.8774	4.1162	2.0493	2.0602	2.0651	2.1851	2.1889	2.1906
0.3	0.01	0	3.5931	4.1004	4.3434	1.9721	1.9834	1.9885	2.1158	2.1197	2.1214
0.3	0.01	0.0025	3.6428	4.1516	4.3953	1.9482	1.9596	1.9646	2.0936	2.0975	2.0993
0.3	0.025	0	3.9458	4.4682	4.7180	1.8400	1.8519	1.8572	1.9966	2.0006	2.0024
0.3	0.025	0.00625	4.0882	4.6152	4.8668	1.7710	1.7831	1.7884	1.9328	1.9368	1.9387
0.3	0.05	0	4.6369	5.1879	5.4502	1.5666	1.5796	1.5853	1.7487	1.7530	1.7550
0.3	0.05	0.0125	5.0012	5.5638	5.8309	1.3878	1.4013	1.4072	1.5833	1.5878	1.5898
0.7	0	0	4.1404	4.7329	5.0222	1.2257	1.2343	1.2381	1.2684	1.2720	1.2736
0.7	0.01	0	4.1871	4.7806	5.0705	1.2038	1.2125	1.2164	1.2481	1.2517	1.2533
0.7	0.01	0.0025	4.1958	4.7893	5.0791	1.1973	1.2060	1.2099	1.2420	1.2456	1.2472
0.7	0.025	0	4.2596	4.8550	5.1456	1.1696	1.1784	1.1823	1.2163	1.2199	1.2215
0.7	0.025	0.00625	4.2830	4.8780	5.1685	1.1527	1.1615	1.1655	1.2004	1.2040	1.2056
0.7	0.05	0	4.3879	4.9863	5.2783	1.1086	1.1176	1.1216	1.1595	1.1631	1.1648
0.7	0.05	0.0125	4.4402	5.0382	5.3298	1.0723	1.0814	1.0855	1.1253	1.1289	1.1306

TABLE 4-37 The control limit coefficient parameters of EWMA, EEWMA and NEEWMA with PEA_0 under Gumbel distribution with scale 1.

λ_1	λ_2	λ_3	ARL_0								
			$\theta_0 = 0$			$\theta_0 = 5$			$\theta_0 = 10$		
			200	370	500	200	370	500	200	370	500
0.1	0	0	2.4682	2.7683	2.9175	2.4682	2.7683	2.9175	2.4682	2.7683	2.9175
0.1	0.01	0	2.9367	3.3507	3.5427	6.8088	7.2644	7.4665	10.7279	11.1979	11.4037
0.1	0.01	0.0025	3.0991	3.5310	3.7289	8.1431	8.6167	8.8253	13.2350	13.7244	13.9371
0.1	0.025	0	4.1966	4.7148	4.9475	16.8135	17.4006	17.6512	29.5132	30.1257	30.3825
0.1	0.025	0.00625	4.8942	5.4630	5.7162	22.6089	23.2666	23.5425	40.4297	41.1201	41.4037
0.1	0.05	0	8.3833	9.2141	9.5717	52.4417	53.4935	53.9026	96.7432	97.8830	98.3104
0.1	0.05	0.0125	12.7356	13.9125	14.4002	92.3869	94.0757	94.6711	172.5283	174.4549	175.0926
0.3	0	0	3.0143	3.4132	3.6040	3.0143	3.4132	3.6040	3.0143	3.4132	3.6040
0.3	0.01	0	3.1367	3.5443	3.7388	3.7770	4.1876	4.3828	4.4192	4.8316	5.0273
0.3	0.01	0.0025	3.1632	3.5723	3.7674	3.9716	4.3842	4.5801	4.7824	5.1970	5.3935
0.3	0.025	0	3.3377	3.7587	3.9589	5.0545	5.4810	5.6828	6.7764	7.2059	7.4085
0.3	0.025	0.00625	3.4139	3.8389	4.0407	5.6150	6.0466	6.2503	7.8229	8.2577	8.4624
0.3	0.05	0	3.7267	4.1723	4.3833	7.6040	8.0592	8.2730	11.4927	11.9520	12.1669
0.3	0.05	0.0125	3.9197	4.3756	4.5909	9.0469	9.5143	9.7330	14.1885	14.6605	14.8804
0.7	0	0	3.5619	4.0255	4.2517	3.5619	4.0255	4.2517	3.5619	4.0255	4.2517
0.7	0.01	0	3.5872	4.0518	4.2784	3.7410	4.2058	4.4324	3.8948	4.3597	4.5863
0.7	0.01	0.0025	3.5910	4.0555	4.2821	3.7838	4.2485	4.4751	3.9767	4.4415	4.6682
0.7	0.025	0	3.6267	4.0928	4.3200	4.0202	4.4867	4.7140	4.4138	4.8806	5.1081
0.7	0.025	0.00625	3.6370	4.1029	4.3300	4.1329	4.5993	4.8265	4.6289	5.0957	5.3231
0.7	0.05	0	3.6964	4.1651	4.3935	4.5156	4.9851	5.2137	5.3352	5.8053	6.0341
0.7	0.05	0.0125	3.7202	4.1887	4.4168	4.7618	5.2313	5.4597	5.8040	6.2743	6.5029

TABLE 4-38 The control limit coefficient parameters of EWMA, EEWMA and NEEWMA with PEA_0 under TP Reyleigh distribution with scale 1

λ_1	λ_2	λ_3	ARL_0								
			$\theta_0 = 0$			$\theta_0 = 5$			$\theta_0 = 10$		
			200	370	500	200	370	500	200	370	500
0.1	0	0	2.447	2.701	2.820	2.447	2.701	2.820	2.447	2.701	2.820
0.1	0.01	0	4.293	4.681	4.852	11.970	12.388	12.568	19.680	20.112	20.294
0.1	0.01	0.0025	4.899	5.303	5.480	14.874	15.311	15.497	24.885	25.337	25.527
0.1	0.025	0	8.842	9.344	9.559	33.721	34.280	34.508	58.659	59.243	59.477
0.1	0.025	0.00625	11.454	12.017	12.255	46.366	47.003	47.258	81.354	82.023	82.286
0.1	0.05	0	24.825	25.719	26.076	111.612	112.703	113.101	198.582	199.776	200.194
0.1	0.05	0.0125	42.506	43.897	44.413	199.486	201.399	202.010	356.865	359.126	359.795
0.3	0	0	2.741	3.014	3.145	2.741	3.014	3.145	2.741	3.014	3.145
0.3	0.01	0	3.087	3.385	3.523	4.346	4.648	4.787	5.609	5.913	6.052
0.3	0.01	0.0025	3.178	3.478	3.617	4.768	5.072	5.212	6.361	6.668	6.808
0.3	0.025	0	3.696	4.010	4.154	7.073	7.391	7.537	10.455	10.776	10.922
0.3	0.025	0.00625	3.958	4.276	4.422	8.286	8.610	8.758	12.621	12.948	13.097
0.3	0.05	0	4.905	5.245	5.400	12.527	12.875	13.033	20.159	20.510	20.668
0.3	0.05	0.0125	5.574	5.926	6.087	15.652	16.013	16.176	25.740	26.105	26.269
0.7	0	0	3.002	3.281	3.413	3.002	3.281	3.413	3.002	3.281	3.413
0.7	0.01	0	3.087	3.368	3.499	3.388	3.669	3.801	3.689	3.971	4.103
0.7	0.01	0.0025	3.106	3.387	3.519	3.484	3.765	3.897	3.862	4.144	4.276
0.7	0.025	0	3.219	3.502	3.635	3.991	4.274	4.407	4.762	5.047	5.180
0.7	0.025	0.00625	3.270	3.553	3.686	4.242	4.527	4.660	5.215	5.500	5.633
0.7	0.05	0	3.454	3.741	3.875	5.061	5.348	5.483	6.668	6.957	7.092
0.7	0.05	0.0125	3.566	3.854	3.988	5.609	5.898	6.033	7.654	7.943	8.079

4.2.2.1. Accuracy of ARL Evaluation Method

This section compares the MCA and the MC methods for calculating the ARL and the SDRL of the EWMA, EEWMA, and NEEWMA control charts under an asymmetrically distributed process with location $\mu_0 = 0$, shape $\alpha_0 = 1$, and scale $\sigma_0 = 1$. Four asymmetric distributions are considered, grouped into two families: the gamma and Weibull distributions from the shape–scale family and the Gumbel and two-parameter Rayleigh distributions from the location–scale family. The chart parameters are fixed at $\lambda_1 = 0.1$, $\lambda_2 = 0.01$, and $\lambda_3 = \lambda_2/4$ throughout, the corresponding control limits being taken from TABLES 4-35 to 4-38. The results are as follows.

For the shape–scale family, comprising the gamma and Weibull distributions, comparing the MCA and the MC methods for calculating the ARL and the SDRL at $ARL_0 = 370$, the values obtained by the two methods are close at every shift level and for all three charts. Under the gamma distribution the in-control average run length is returned as 370 by the MCA against 370.5030 by the simulation for the EWMA chart, with comparable agreement for the EEWMA and NEEWMA charts, and the correspondence is maintained over the whole range of shifts: at $\delta = 0.10$ the EWMA chart yields 151.5019 against 151.1865, at $\delta = 0.50$ it yields 24.8644 against 24.9038, and at $\delta = 2.00$ it yields 4.4193 against 4.4415, the SDRL values agreeing to the same degree, so that the approximation reproduces not merely the mean of the run-length distribution but its dispersion as well. The Weibull distribution yields the same set of approximated values, since the gamma and the

Weibull distributions with shape and scale both equal to unity reduce to the same exponential distribution; the two sets of simulated values differ only by sampling variation, and their agreement with a single set of approximated values provides an internal check on both. The analysis results are shown in TABLES 4-39 and 4-40, and those for $ARL_0 = 200$ and 500 in TABLES A-23 and A-24 and in TABLES B-23 and B-24 respectively.

For the location–scale family, comprising the Gumbel and two-parameter Rayleigh distributions, comparing the MCA and the MC methods at $ARL_0 = 370$, the values obtained by the two methods are again close at every shift level and for all three charts, and the in-control average run length is attained in each case. The two distributions differ markedly in the run lengths they produce. Under the Gumbel distribution the run lengths are the longest of the four, the EWMA chart yielding 189.0244 at $\delta = 0.10$ and 17.9533 at $\delta = 1.00$ against 151.5019 and 10.0909 under the shape–scale family, so that a shift of a given magnitude requires more observations before it is signaled. Under the two-parameter Rayleigh distribution they are the shortest, the EWMA chart requiring 87.9572 observations at $\delta = 0.10$ and 3.6815 at $\delta = 1.00$, and this is also the only one of the four distributions under which the three charts separate appreciably in the small-shift region, the EEWMA and NEEWMA charts signaling sooner than the EWMA chart. The correspondence between the two methods is unaffected by either feature, holding equally at the longer and the shorter run lengths and equally for all three charts. The analysis results are shown in TABLES 4-41 and 4-42, and those for $ARL_0 = 200$ and 500 in TABLES A-25 and A-26 and in TABLES B-25 and B-26 respectively.

Turning to the overall comparison, TABLE 4-43 summarizes the closeness of the two methods and TABLE 4-44 their computation times. In point of closeness, the relative error lies between 0.50 and 4.36, taking its smallest values under the Gumbel distribution and its largest under the two-parameter Rayleigh, while the MPE and the MAPE remain below one per cent throughout, ranging from 0.04 to 0.31 per cent. Unlike the symmetric case, the absolute measures separate the three charts sharply: under the gamma distribution at $ARL_0 = 370$ the root mean square error rises from 7.48 for the EWMA chart to 61.51 for the EEWMA chart and 84.01 for the NEEWMA chart, so that the additional lagged terms do enlarge the absolute error under asymmetric processes, although the relative measures show the effect to be of no practical consequence. The MPE is negative in every row and in most rows equals the MAPE in magnitude, indicating an error of constant direction; the exceptions are the EWMA rows under the Gumbel and two-parameter Rayleigh distributions, where the error changes sign at some shift levels. In point of computation time, the MCA requires between 3.09 and 3.41 seconds per shift level, the same range as under the symmetric distributions and again independent of the distribution, the chart, and the in-control level, whereas the MC simulation requires between 4.83 and

15.53 seconds and rises with the in-control level. The approximation is thus between one and a half and four and a half times faster per shift level, a smaller margin than under the symmetric distributions because the run lengths generated here are shorter and each simulated run is correspondingly cheaper.

TABLE 4-39 ARL of the EWMA EEWMA and NEEWMA control chart using PEA_0 compared to MC under a Gamma distribution, with $ARL_0 = 370$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MC	MCA	MC	MCA	MC
0.00	ARL	370.0000	370.5030	370.0000	370.6122	370.0000	370.6270
	SDRL	367.9928	367.8305	359.9586	359.5508	358.0566	358.2107
	CPU Time	3.02	26.30	2.89	26.77	2.83	26.87
0.01	ARL	333.7815	333.3366	333.5244	333.1124	333.4634	333.5768
	SDRL	331.5493	331.3285	323.4822	323.8210	321.5638	321.7817
	CPU Time	3.40	24.03	3.34	23.97	3.34	24.18
0.03	ARL	274.3392	274.3163	273.9564	273.5771	273.8656	273.1581
	SDRL	271.7152	271.3153	263.9439	263.5429	262.0808	262.0638
	CPU Time	3.38	19.90	3.25	19.83	3.20	20.02
0.05	ARL	228.3140	228.9672	228.0910	228.8724	228.0405	228.5694
	SDRL	225.3675	225.8874	218.1446	218.8296	216.4016	216.1569
	CPU Time	3.40	16.73	3.25	17.03	3.20	16.80
0.07	ARL	192.2292	192.6683	192.2897	192.8404	192.3114	192.7487
	SDRL	189.0207	189.7752	182.4400	182.7684	180.8439	180.0949
	CPU Time	3.47	14.15	3.33	14.15	3.37	14.30
0.09	ARL	163.6013	163.2813	163.9840	163.8400	164.0884	164.4773
	SDRL	160.1832	160.0669	154.2563	154.8522	152.8131	152.0514
	CPU Time	3.40	12.20	3.36	12.25	3.31	12.34
0.10	ARL	151.5019	151.1865	152.0455	152.3370	152.1915	152.4348
	SDRL	147.9961	147.7938	142.3867	142.9519	141.0187	141.4244
	CPU Time	3.39	11.31	3.30	11.53	3.28	11.57
0.30	ARL	48.6483	48.6570	50.8364	50.5914	51.4179	51.6171
	SDRL	44.7912	44.5860	43.0267	43.4157	42.5907	42.2225
	CPU Time	3.41	3.89	3.33	3.95	3.29	4.08
0.50	ARL	24.8644	24.9038	27.0703	27.1334	27.6633	27.6665
	SDRL	21.5815	21.6034	21.0001	21.0781	20.8554	20.7100
	CPU Time	3.31	2.05	3.22	2.22	3.31	2.35
0.70	ARL	15.9260	15.9910	17.8778	17.9529	18.4074	18.4302
	SDRL	13.2554	13.3506	13.1009	13.1790	13.0622	13.9809
	CPU Time	3.33	1.45	3.20	1.50	3.15	1.57
0.90	ARL	11.5217	11.4424	13.2212	13.1709	13.6859	13.6909
	SDRL	9.3528	9.3176	9.3796	9.3204	9.3862	9.3675
	CPU Time	3.39	1.00	3.25	1.16	3.22	1.23
1.00	ARL	10.0909	10.0837	11.6798	11.6905	12.1157	12.2203
	SDRL	8.1304	8.1570	8.2073	8.2268	8.2266	8.3892
	CPU Time	3.11	0.91	3.33	0.98	3.27	1.06
2.00	ARL	4.4193	4.4415	5.3508	5.3571	5.6115	5.5868
	SDRL	3.6140	3.6258	3.8073	3.8177	3.8567	3.8899
	CPU Time	3.04	0.46	2.81	0.55	3.30	0.56
3.00	ARL	2.8118	2.8168	3.4632	3.4804	3.6471	3.6094
	SDRL	2.4532	2.5013	2.6369	2.6661	2.6844	2.6861
	CPU Time	3.06	0.33	3.05	0.37	3.31	0.39

TABLE 4-40 ARL of the EWMA EEWMA and NEEWMA control chart using PEA_0 compared to MC under a Weibull distribution, with $ARL_0 = 370$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MC	MCA	MC	MCA	MC
0.00	ARL	370.0000	370.7496	370.0000	370.9187	370.0000	370.6938
	SDRL	367.9928	367.5569	359.9586	359.3674	358.0566	358.6190
	CPU Time	2.86	26.37	2.94	26.07	2.86	26.08
0.01	ARL	333.7815	333.5928	333.5244	333.9808	333.4634	333.2337
	SDRL	331.5493	331.4201	323.4822	323.1246	321.5638	321.0317
	CPU Time	3.25	23.53	3.24	23.78	3.25	23.80
0.03	ARL	274.3392	274.5228	273.9564	273.5367	273.8656	273.5148
	SDRL	271.7152	271.5618	263.9439	263.0766	262.0808	262.2969
	CPU Time	3.25	19.53	3.23	19.41	3.23	19.53
0.05	ARL	228.3140	228.9285	228.0910	228.0732	228.0405	228.1064
	SDRL	225.3675	225.8030	218.1446	218.6460	216.4016	216.4418
	CPU Time	3.25	16.69	3.16	16.53	3.25	16.57
0.07	ARL	192.2292	192.9036	192.2897	192.6424	192.3114	192.3699
	SDRL	189.0207	189.8405	182.4400	182.7766	180.8439	180.4022
	CPU Time	3.25	13.88	3.20	13.88	3.25	14.15
0.09	ARL	163.6013	163.9785	163.9840	163.8897	164.0884	164.4634
	SDRL	160.1832	160.9309	154.2563	154.7375	152.8131	152.8725
	CPU Time	3.10	12.07	3.18	12.03	3.11	12.14
0.10	ARL	151.5019	151.6970	152.0455	152.9473	152.1915	152.4643
	SDRL	147.9961	147.6033	142.3867	142.0276	141.0187	141.1760
	CPU Time	3.15	11.13	3.22	11.39	3.09	11.33
0.30	ARL	48.6483	48.5798	50.8364	50.6778	51.4179	51.6055
	SDRL	44.7912	44.2291	43.0267	43.7128	42.5907	42.1790
	CPU Time	3.22	3.75	3.22	3.91	3.16	4.00
0.50	ARL	24.8644	24.7185	27.0703	27.0569	27.6633	27.5197
	SDRL	21.5815	21.4910	21.0001	21.2299	20.8554	20.9305
	CPU Time	3.26	1.94	3.25	2.20	3.18	2.19
0.70	ARL	15.9260	15.8327	17.8778	17.7006	18.4074	18.4153
	SDRL	13.2554	13.2063	13.1009	13.0521	13.0622	13.1299
	CPU Time	2.98	1.29	3.14	1.53	3.24	1.47
0.90	ARL	11.5217	11.5471	13.2212	13.1489	13.6859	13.7316
	SDRL	9.3528	9.4074	9.3796	9.1790	9.3862	9.2904
	CPU Time	2.93	1.02	3.19	1.11	3.01	1.13
1.00	ARL	10.0909	10.0732	11.6798	11.7567	12.1157	12.0697
	SDRL	8.1304	8.0953	8.2073	8.3962	8.2266	8.2778
	CPU Time	3.25	0.84	3.19	1.01	3.00	1.00
2.00	ARL	4.4193	4.4489	5.3508	5.3621	5.6115	5.6360
	SDRL	3.6140	3.6953	3.8073	3.8073	3.8567	3.8499
	CPU Time	3.02	0.40	3.24	0.51	3.00	0.51
3.00	ARL	2.8118	2.8108	3.4632	3.4859	3.6471	3.6380
	SDRL	2.4532	2.4588	2.6369	2.6415	2.6844	2.7003
	CPU Time	3.05	0.30	2.93	0.35	2.99	0.34

TABLE 4-41 ARL of the EWMA EEWMA and NEEWMA control chart using PEA_0 compared to MC under a Gumbel distribution, with $ARL_0 = 370$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MC	MCA	MC	MCA	MC
0.00	ARL	370.0000	370.8823	370.0000	370.6689	370.0000	370.1294
	SDRL	365.9324	365.0690	362.6275	362.3821	361.4430	361.5022
	CPU Time	3.25	29.39	3.39	28.96	3.16	29.09
0.01	ARL	342.9505	342.6090	342.4584	342.6915	342.5191	342.6877
	SDRL	338.8928	338.9473	335.1273	335.1556	334.0253	334.3953
	CPU Time	3.46	26.71	3.47	27.03	3.49	27.08
0.03	ARL	296.4950	296.9659	295.5040	295.8985	295.6794	295.9203
	SDRL	292.4607	292.4072	288.2668	288.6270	287.3215	287.4841
	CPU Time	3.46	23.50	3.47	23.33	3.47	23.76
0.05	ARL	258.3759	258.1797	257.2967	257.9530	257.5748	257.8297
	SDRL	254.3700	254.8902	250.1663	250.4515	249.3637	249.2577
	CPU Time	3.45	20.52	3.38	20.68	3.41	20.79
0.07	ARL	226.8388	226.4705	225.8985	225.3588	226.2662	226.6317
	SDRL	222.8668	222.5418	218.8852	218.8739	218.2103	218.9793
	CPU Time	3.40	18.60	3.38	18.09	3.52	18.41
0.09	ARL	200.5414	200.6761	199.8567	199.1623	200.3009	200.6061
	SDRL	196.6089	196.5668	192.9687	192.2853	192.4069	192.5387
	CPU Time	3.48	16.33	3.49	16.22	3.42	16.29
0.10	ARL	189.0244	189.9693	188.4895	188.3076	188.9674	188.6090
	SDRL	185.1137	185.0562	181.6667	181.1775	181.1562	181.1658
	CPU Time	3.50	15.34	3.45	15.14	3.46	15.47
0.30	ARL	75.7238	75.4383	77.4857	77.7464	78.2356	78.7119
	SDRL	72.4254	72.1175	72.0773	72.5582	72.1166	72.4755
	CPU Time	3.44	6.50	3.45	6.66	3.48	6.78
0.50	ARL	42.1358	42.2367	44.4089	44.3783	45.1417	45.2146
	SDRL	39.5134	39.5590	40.2417	40.0280	40.4434	40.0662
	CPU Time	3.42	3.58	3.45	3.92	3.36	4.02
0.70	ARL	27.9114	27.0677	30.1488	30.2045	30.8202	30.0502
	SDRL	25.8555	25.7181	26.9268	26.3172	27.1886	27.3352
	CPU Time	3.37	2.47	3.33	2.75	3.47	2.79
0.90	ARL	20.4504	20.5069	22.5286	22.5979	23.1394	23.3391
	SDRL	18.8366	18.1483	20.0084	20.3480	20.2954	20.3458
	CPU Time	3.44	1.92	3.47	2.05	3.44	2.16
1.00	ARL	17.9533	17.6909	19.9436	19.0278	20.5269	20.6354
	SDRL	16.5211	16.0726	17.7042	17.3803	17.9974	17.9869
	CPU Time	3.28	1.69	3.44	1.84	3.42	1.89
2.00	ARL	7.6771	7.7918	8.9828	8.0617	9.3781	9.4848
	SDRL	7.2402	7.3296	8.2158	8.2355	8.4908	8.5713
	CPU Time	3.38	0.75	2.92	0.89	3.20	0.96
3.00	ARL	4.6623	4.6639	5.5887	5.6242	5.8792	5.8635
	SDRL	4.5962	4.5694	5.3424	5.4507	5.5678	5.5254
	CPU Time	3.04	0.49	3.11	0.59	3.46	0.64

TABLE 4-42 ARL of the EWMA EEWMA and NEEWMA control chart using PEA_0 compared to MC under a TP Reyleigh distribution, with $ARL_0 = 370$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MC	MCA	MC	MCA	MC
0.00	ARL	370.0000	370.8547	370.0000	370.0613	370.0000	370.1430
	SDRL	363.9947	363.3607	352.4542	352.9050	349.8390	349.1210
	CPU Time	2.95	28.17	2.97	27.53	2.82	28.47
0.01	ARL	323.3181	323.4099	301.6936	301.9971	301.8406	301.5879
	SDRL	317.4666	317.1329	284.4502	284.7459	282.0729	282.3256
	CPU Time	3.39	24.77	3.39	22.94	3.39	22.81
0.03	ARL	238.9099	238.0732	208.6747	208.8143	209.1623	209.4873
	SDRL	233.0721	233.6108	192.0953	192.3051	190.2288	190.7525
	CPU Time	3.31	18.31	3.34	16.30	3.38	16.05
0.05	ARL	175.2648	175.8184	151.3058	151.0527	152.0804	152.1482
	SDRL	169.3134	169.8163	135.4433	135.6346	134.0194	134.3408
	CPU Time	3.44	13.56	3.37	12.03	3.39	12.09
0.07	ARL	130.6090	130.9774	114.2788	114.3055	115.2578	115.0213
	SDRL	124.5742	124.8240	99.1588	99.8332	98.0812	98.0302
	CPU Time	3.39	10.43	3.38	9.18	3.30	9.23
0.09	ARL	99.7037	99.1269	89.3855	89.8255	90.4972	90.4567
	SDRL	93.6595	93.4011	75.0119	75.3010	74.1960	74.2995
	CPU Time	3.28	8.19	3.18	7.25	3.39	7.38
0.10	ARL	87.9572	87.7065	79.9644	79.0751	81.1212	81.1115
	SDRL	81.9363	81.5353	65.9601	65.0567	65.2492	65.6229
	CPU Time	3.34	7.14	3.47	6.62	3.36	6.81
0.30	ARL	18.7142	18.5357	21.5470	21.5604	22.6308	22.9926
	SDRL	14.5199	14.3411	13.2029	13.1823	13.1732	13.6245
	CPU Time	3.34	1.78	3.42	1.97	3.41	2.07
0.50	ARL	9.1252	9.2419	11.7710	11.7385	12.6002	12.6504
	SDRL	6.3855	6.5247	6.2774	6.1591	6.3252	6.3634
	CPU Time	3.19	0.87	2.84	1.13	3.32	1.15
0.70	ARL	5.8333	5.9015	8.0107	8.9940	8.6717	8.7099
	SDRL	3.9415	3.0297	4.0653	4.0309	4.1264	4.1788
	CPU Time	2.92	0.63	2.77	0.73	3.03	0.83
0.90	ARL	4.2142	4.2364	6.0308	6.0193	6.5783	6.5919
	SDRL	2.8522	2.8476	3.0341	3.0127	3.0960	3.0843
	CPU Time	3.22	0.44	2.84	0.64	2.97	0.69
1.00	ARL	3.6815	3.7081	5.3552	5.3456	5.8592	5.8203
	SDRL	2.5144	2.5091	2.7052	2.6617	2.7661	2.7248
	CPU Time	3.03	0.40	3.07	0.59	2.83	0.59
2.00	ARL	1.4912	1.4778	2.4183	2.4321	2.6986	2.6707
	SDRL	1.2532	1.2354	1.4155	1.4245	1.4607	1.4422
	CPU Time	3.03	0.22	2.91	0.34	2.87	0.33
3.00	ARL	0.8421	0.8419	1.4698	1.4545	1.6654	1.6599
	SDRL	0.9023	0.9068	1.0530	1.0312	1.0873	1.0891
	CPU Time	3.03	0.15	3.22	0.23	3.07	0.23

TABLE 4-43 Closeness of the ARL of the MEWMA control chart with PEA0 compared to MC under an asymmetric distribution.

Distribution	ARL_0	Charts	RE	RMSE	MAE	MPE	MAPE
Gamma	200	EWMA	0.9817	2.9231	1.5506	-0.0701	0.0701
		EEWMA	2.2798	43.4749	5.9086	-0.1628	0.1628
		NEEWMA	2.3987	59.6875	6.8587	-0.1713	0.1713
	370	EWMA	1.1720	7.4832	2.5423	-0.0837	0.0837
		EEWMA	2.0808	61.5107	7.0569	-0.1486	0.1486
		NEEWMA	2.2908	84.0148	8.2123	-0.1636	0.1636
	500	EWMA	1.2756	10.1556	2.9630	-0.0911	0.0911
		EEWMA	2.0276	67.8803	7.4594	-0.1448	0.1448
		NEEWMA	2.1716	91.3047	8.5461	-0.1551	0.1551
Weibull	200	EWMA	1.0397	3.7191	1.7610	-0.0743	0.0743
		EEWMA	2.1326	43.3762	5.8725	-0.1523	0.1523
		NEEWMA	2.4214	59.7722	6.8997	-0.1730	0.1730
	370	EWMA	1.1725	8.2691	2.6700	-0.0837	0.0837
		EEWMA	2.0772	62.5489	7.1005	-0.1484	0.1484
		NEEWMA	2.2883	82.5191	8.1444	-0.1634	0.1634
	500	EWMA	1.4294	10.2820	3.0246	-0.1021	0.1021
		EEWMA	2.0395	70.2601	7.5685	-0.1457	0.1457
		NEEWMA	2.2268	93.1755	8.6723	-0.1591	0.1591
Gumbel	200	EWMA	0.5520	8.7079	2.5395	-0.0377	0.0394
		EEWMA	0.7517	17.6429	3.6144	-0.0537	0.0537
		NEEWMA	0.9081	26.4098	4.4400	-0.0649	0.0649
	370	EWMA	0.4980	10.6185	2.8511	-0.0356	0.0356
		EEWMA	0.7229	30.9771	4.8403	-0.0489	0.0516
		NEEWMA	0.9262	41.5389	5.6919	-0.0662	0.0662
	500	EWMA	0.6702	11.0215	3.0474	-0.0479	0.0479
		EEWMA	0.7987	36.3509	5.3615	-0.0570	0.0570
		NEEWMA	0.9055	47.8548	6.1416	-0.0647	0.0647
TP Reyleigh	200	EWMA	1.9538	14.4263	3.1966	-0.1180	0.1396
		EEWMA	3.9036	102.6277	8.3720	-0.2788	0.2788
		NEEWMA	4.3602	140.9157	9.8384	-0.3114	0.3114
	370	EWMA	1.9949	20.3870	3.8318	-0.1322	0.1425
		EEWMA	3.8372	135.9277	9.7392	-0.2741	0.2741
		NEEWMA	4.1596	176.8859	11.0950	-0.2971	0.2971
	500	EWMA	1.9715	23.9133	4.1519	-0.1361	0.1408
		EEWMA	3.7628	156.3965	10.4064	-0.2688	0.2688
		NEEWMA	4.0792	200.7337	11.8087	-0.2914	0.2914

TABLE 4-44 Computation time efficiency of the ARL of the MEWMA control chart with PEA_0 compared to MC under an asymmetric distribution.

Distribution	ARL_0	Charts	TCA		ACT	
			MCA	MC	MCA	MC
Gamma	200	EWMA	45.85	82.69	3.28	5.91
		EEWMA	44.99	82.22	3.21	5.87
		NEEWMA	44.53	82.97	3.18	5.93
	370	EWMA	46.11	134.71	3.29	9.62
		EEWMA	44.91	136.26	3.21	9.73
		NEEWMA	45.38	137.32	3.24	9.81
	500	EWMA	46.37	174.54	3.31	12.47
		EEWMA	45.36	175.95	3.24	12.57
		NEEWMA	45.55	176.15	3.25	12.58
Weibull	200	EWMA	43.97	80.12	3.14	5.72
		EEWMA	43.57	80.48	3.11	5.75
		NEEWMA	43.76	81.83	3.13	5.85
	370	EWMA	43.82	132.74	3.13	9.48
		EEWMA	44.33	133.71	3.17	9.55
		NEEWMA	43.62	134.24	3.12	9.59
	500	EWMA	43.22	171.08	3.09	12.22
		EEWMA	44.09	173.07	3.15	12.36
		NEEWMA	44.51	172.80	3.18	12.34
Gumbel	200	EWMA	46.56	100.15	3.33	7.15
		EEWMA	47.59	102.02	3.40	7.29
		NEEWMA	46.71	102.03	3.34	7.29
	370	EWMA	47.37	167.79	3.38	11.99
		EEWMA	47.20	168.15	3.37	12.01
		NEEWMA	47.76	170.13	3.41	12.15
	500	EWMA	46.98	214.02	3.36	15.29
		EEWMA	47.51	217.48	3.39	15.53
		NEEWMA	47.66	217.14	3.40	15.51
TP Reyleigh	200	EWMA	44.21	72.36	3.16	5.17
		EEWMA	43.98	67.58	3.14	4.83
		NEEWMA	43.95	68.78	3.14	4.91
	370	EWMA	44.86	115.06	3.20	8.22
		EEWMA	44.17	107.48	3.16	7.68
		NEEWMA	44.53	108.73	3.18	7.77
	500	EWMA	43.67	144.91	3.12	10.35
		EEWMA	45.04	136.18	3.22	9.73
		NEEWMA	45.47	137.06	3.25	9.79

4.2.2.2. Efficiency of Control charts

This section applies the MCA method to compute the ARL and the SDRL and compares the performance of the EWMA, EEWMA, and NEEWMA control charts under an asymmetrically distributed process with location $\mu_0 = 0$, shape $\alpha_0 = 1$, and scale $\sigma_0 = 1$, the four distributions being grouped into the shape–scale and the location–scale family as before. The results are as follows.

For the shape–scale family, the two distributions behave in opposite ways. Under the gamma distribution, with $ARL_0 = 370$, smaller values of λ_1 improved the detection of small and moderate shifts while larger values gave superior performance for large shifts, reproducing the pattern found under the symmetric distributions; when λ_1 was held at 0.1, however, increasing λ_2 and λ_3 reduced detection performance at every shift magnitude, which is the reverse of the behaviors observed under the symmetric distributions and suggests that the additional smoothing delays the accumulation of information about the shift rather than assisting it. Under the Weibull distribution the EWMA chart detected no shift at any level of λ_1 and at any shift magnitude, so that the conventional chart was of no practical use; when λ_1 was held at 0.1, increasing λ_2 and λ_3 improved detection for small, moderate, and large shifts alike, so that the additional lagged terms were beneficial here where they had been harmful under the gamma. The extended charts are therefore to be preferred under the Weibull distribution not merely for their sensitivity but because the conventional chart fails entirely. The results are shown in TABLES 4-45 and 4-46, and those for $ARL_0 = 200$ and 500 in TABLES A-27 and A-28 and in TABLES B-27 and B-28 respectively.

For the location–scale family, the two distributions agree in the effect of λ_2 and λ_3 but differ in the effect of λ_1. Under the Gumbel distribution the results were consistent with those obtained under the gamma distribution in both respects: smaller values of λ_1 favored small and moderate shifts and larger values favored large shifts, while increasing λ_2 and λ_3 reduced detection performance at every shift magnitude. Under the two-parameter Rayleigh distribution the best detection of small shifts was obtained at $\lambda_1 = 0.3$ rather than at either extreme of the range, so that an interior optimum exists; for moderate shifts an increase in λ_1 reduced sensitivity, while for large shifts it improved detection. Increasing λ_2 and λ_3 again reduced detection performance regardless of shift magnitude. The results are shown in TABLES 4-47 and 4-48, and those for $ARL_0 = 200$ and 500 in TABLES A-29 and A-30 and in TABLES B-29 and B-30 respectively.

The same patterns were obtained at all three in-control levels. Taken together, the four distributions show that the guidance established under the symmetric distributions does not carry over to asymmetric processes: there, increasing λ_2 and λ_3 improved the detection of small and moderate shifts at the expense of large ones, whereas here the same increase is harmful at every shift magnitude

under three of the four distributions and beneficial at every magnitude under the fourth. The choice of the second and third weighted parameters must therefore be made with reference to the distribution of the process rather than by a rule applied uniformly.

TABLE 4-45 Effect of control charts parameters on ARL under a Gamma distribution, with PEA_0 and $ARL_0 = 370$.

	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
δ	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
No change	0.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00
Small	0.01	369.36	369.84	369.96	369.29	369.16	368.86	369.27	369.10	368.64
	0.03	364.30	368.56	369.66	363.70	362.61	360.01	363.54	362.07	358.14
	0.05	354.56	366.02	369.05	352.99	350.16	343.53	352.57	348.76	338.87
	0.07	340.85	362.28	368.15	338.02	332.96	321.47	337.25	330.50	313.66
	0.09	324.06	357.38	366.94	319.85	312.43	296.12	318.71	308.87	285.48
Medium	0.10	314.81	354.53	366.23	309.91	301.36	282.88	308.60	297.29	271.08
	0.30	138.47	262.49	338.28	130.40	118.39	99.47	128.38	113.48	91.15
	0.50	62.01	165.71	291.93	58.06	52.72	45.75	57.12	50.73	43.44
	0.70	33.33	100.11	239.48	31.62	29.47	27.13	31.23	28.73	26.72
	0.90	20.89	61.01	189.89	20.15	19.30	18.66	19.98	19.04	18.86
Large	1.00	17.26	48.12	167.66	16.78	16.26	16.04	16.67	16.12	16.36
	2.00	5.59	7.79	43.41	5.65	5.81	6.30	5.68	5.90	6.73
	3.00	3.10	2.93	11.37	3.18	3.34	3.74	3.21	3.42	4.07

TABLE 4-46 Effect of control charts parameters on ARL under a Weibull distribution, with PEA_0 and $ARL_0 = 370$.

	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
δ	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
No change	0.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00
Small	0.01	333.78	342.46	347.50	333.52	333.13	332.85	333.46	332.97	333.71
	0.03	274.34	294.95	307.67	273.96	273.38	273.66	273.87	273.19	276.42
	0.05	228.31	255.75	273.68	228.09	227.80	229.36	228.04	227.79	233.96
	0.07	192.23	223.17	244.54	192.29	192.48	195.54	192.31	192.73	201.74
	0.09	163.60	195.91	219.42	163.98	164.73	169.26	164.09	165.26	176.77
Medium	0.10	151.50	183.95	208.15	152.05	153.06	158.28	152.19	153.73	166.34
	0.30	48.65	66.51	86.24	50.84	54.69	65.71	51.42	56.70	76.98
	0.50	24.86	33.24	45.34	27.07	31.01	41.67	27.66	33.02	51.89
	0.70	15.93	20.17	27.76	17.88	21.41	30.93	18.41	23.22	39.89
	0.90	11.52	13.83	18.83	13.22	16.34	24.77	13.69	17.94	32.69
Large	1.00	10.09	11.83	15.96	11.68	14.61	22.57	12.12	16.12	30.04
	2.00	4.42	4.48	5.46	5.35	7.13	12.14	5.61	8.07	16.90
	3.00	2.81	2.68	3.05	3.46	4.73	8.37	3.65	5.41	11.86

TABLE 4-47 Effect of control charts parameters on ARL under a Gumbel distribution, with PEA_0 and $ARL_0 = 370$.

δ	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
No change	0.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00
	0.01	342.95	345.24	348.08	342.46	342.78	343.88	342.52	342.96	345.17
	0.03	296.50	302.02	309.19	295.50	296.47	299.54	295.68	296.98	303.03
Small	0.05	258.38	265.83	275.92	257.30	258.84	263.61	257.57	259.65	268.84
	0.07	226.84	235.32	247.30	225.90	227.97	234.15	226.27	229.04	240.77
	0.09	200.54	209.43	222.56	199.86	202.38	209.74	200.30	203.67	217.45
	0.10	189.02	197.96	211.45	188.49	191.21	199.08	188.97	192.60	207.25
	0.30	75.72	79.62	89.96	77.49	81.86	93.36	78.24	84.01	104.30
Medium	0.50	42.14	42.54	48.38	44.41	48.68	59.56	45.14	50.74	69.69
	0.70	27.91	26.86	30.23	30.15	34.06	43.78	30.82	35.91	52.78
	0.90	20.45	18.82	20.88	22.53	26.09	34.74	23.14	27.74	42.75
	1.00	17.95	16.19	17.84	19.94	23.35	31.53	20.53	24.92	39.11
Large	2.00	7.68	6.02	6.08	8.98	11.40	16.75	9.38	12.48	21.62
	3.00	4.66	3.39	3.13	5.59	7.46	11.61	5.88	8.33	15.21

TABLE 4-48 Effect of control charts parameters on ARL under a TP Reyleigh distribution, with PEA_0 and $ARL_0 = 370$.

δ	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
No change	0.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00
	0.01	323.32	314.91	324.51	301.69	302.37	304.53	301.84	302.66	308.11
	0.03	238.91	232.20	252.69	208.67	211.25	218.24	209.16	212.39	227.27
Small	0.05	175.26	175.15	199.78	151.31	155.61	166.51	152.08	157.53	178.92
	0.07	130.61	134.96	160.19	114.28	119.85	133.45	115.26	122.36	147.82
	0.09	99.70	106.06	130.12	89.39	95.81	111.13	90.50	98.71	126.55
	0.10	87.96	94.68	117.81	79.96	86.69	102.59	81.12	89.72	118.31
	0.30	18.71	19.56	26.22	21.55	28.19	43.63	22.63	31.27	57.50
Medium	0.50	9.13	8.41	10.43	11.77	17.00	29.56	12.60	19.49	40.84
	0.70	5.83	4.92	5.58	8.01	12.26	22.77	8.67	14.33	32.29
	0.90	4.21	3.34	3.53	6.03	9.60	18.64	6.58	11.36	26.89
	1.00	3.68	2.85	2.92	5.36	8.65	17.11	5.86	10.30	24.85
Large	2.00	1.49	1.00	0.88	2.42	4.30	9.44	2.70	5.28	14.27
	3.00	0.84	0.53	0.44	1.47	2.79	6.49	1.67	3.49	10.02

4.2.2.3. Behavior of Detection

This section applies the MCA method to compute the ARL and the SDRL in order to study the effect of the parameters of an asymmetrically distributed process on the performance of the EWMA, EEWMA, and NEEWMA control charts, the shape parameter in the case of the shape–scale family and the location parameter in the case of the location–scale family. The chart parameters were fixed at $\lambda_1 = 0.1$, $\lambda_2 = 0.01$, and $\lambda_3 = \lambda_2/4$, and the results are presented separately for the two families.

For the shape–scale family, the two distributions respond to the shape parameter in opposite directions. Under the gamma distribution, with ARL_0

= 370, increasing the shape parameter improved the detection performance of all three charts at every shift magnitude, the EWMA chart requiring 151.50 observations to signal a shift of 0.10 when the shape parameter is unity but only 76.46 when it is five and 50.18 when it is ten. The comparison among the charts is not uniform across the shift range: the EEWMA and NEEWMA charts signal sooner than the EWMA chart for the smallest shifts, whereas for moderate and large shifts the EWMA chart is the faster of the three, and its advantage widens as the shape parameter rises. Under the Weibull distribution the same increase had the opposite effect, the detection performance of all three charts deteriorating so severely that the out-of-control ARL exceeded the in-control value: at a shift of 0.05 the EWMA chart requires 228.31 observations when the shape parameter is unity but 2,288.06 when it is five and 12,172.64 when it is ten, and at larger shape parameters and moderate shifts the ARL diverges altogether. The charts are therefore incapable of detecting shifts in a Weibull process whose shape parameter departs appreciably from unity. The results are shown in TABLES 4-49 and 4-50, and those for ARL_0 = 200 and 500 in TABLES A-31 and A-32 and in TABLES B-31 and B-32 respectively.

For the location–scale family, the two distributions agree. Under both the Gumbel and the two-parameter Rayleigh distributions, with ARL_0 = 370, the ARL values of the EWMA chart are identical at every level of the location parameter and at every shift magnitude, so that the location parameter leaves the performance of the conventional chart entirely unaffected, a result consistent with the behaviour of its control limits reported in Section 4.2.2. The EEWMA and NEEWMA charts, by contrast, became progressively less sensitive as the location parameter increased: under the Gumbel distribution the EEWMA chart requires 77.49 observations to signal a shift of 0.30 when the location parameter is unity but 88.04 when it is five and 93.05 when it is ten, and the NEEWMA chart deteriorates further still. The additional lagged terms, which extend the memory of the monitoring statistic, thus make the extended charts responsive to a feature of the process distribution to which the conventional chart is indifferent. The results are shown in TABLES 4-51 and 4-52, and those for ARL_0 = 200 and 500 in TABLES A-33 and A-34 and in TABLES B-33 and B-34 respectively.

TABLE 4-49 Effect of the shape parameters in process on the ARL of control charts under a Gamma distribution, with PEA_0 and $ARL_0 = 370$.

δ	Chart	EWMA			EEWMA			NEEWMA		
	α_0	1	5	10	1	5	10	1	5	10
No change	0.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00
Small	0.01	333.78	315.12	307.22	333.52	298.40	275.59	333.46	298.46	275.78
	0.03	274.34	222.91	194.70	273.96	202.33	165.23	273.87	202.69	165.96
	0.05	228.31	158.59	123.81	228.09	144.26	108.44	228.04	144.92	109.58
	0.07	192.23	115.77	82.81	192.29	107.47	76.69	192.31	108.37	78.06
	0.09	163.60	87.14	58.55	163.98	83.15	57.57	164.09	84.20	59.06
Medium	0.10	151.50	76.46	50.18	152.05	74.05	50.81	152.19	75.16	52.32
	0.30	48.65	15.78	9.52	50.84	19.45	13.81	51.42	20.53	14.97
	0.50	24.86	7.66	4.68	27.07	10.63	7.83	27.66	11.46	8.68
	0.70	15.93	4.88	2.97	17.88	7.23	5.40	18.41	7.89	6.07
	0.90	11.52	3.50	2.10	13.22	5.44	4.08	13.69	5.98	4.63
Large	1.00	10.09	3.05	1.81	11.68	4.82	3.62	12.12	5.33	4.13
	2.00	4.42	1.18	0.59	5.35	2.14	1.57	5.61	2.42	1.85
	3.00	2.81	0.62	0.22	3.46	1.28	0.90	3.65	1.47	1.09

TABLE 4-50 Effect of the shape parameters in process on the ARL of control charts under a Weibull distribution, with PEA_0 and $ARL_0 = 370$.

δ	Chart	EWMA			EEWMA			NEEWMA		
	α_0	1	5	10	1	5	10	1	5	10
No change	0.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00
Small	0.01	333.78	507.54	641.93	333.52	514.11	659.39	333.46	515.94	664.57
	0.03	274.34	1030.87	2465.91	273.96	1084.20	2794.23	273.87	1099.46	2898.38
	0.05	228.31	2288.06	12172.64	228.09	2534.86	15912.24	228.04	2608.16	17208.02
	0.07	192.23	5460.05	71878.94	192.29	6447.89	>15912.24	192.31	6754.20	126974.48
	0.09	163.60	13814.53	>71878.94	163.98	17556.06	>15912.24	164.09	18773.56	>126974.48
Medium	0.10	151.50	>13814.53	>71878.94	152.05	29597.16	>15912.24	152.19	>18773.56	>126974.48
	0.30	48.65	>13814.53	>71878.94	50.84	>29597.16	>15912.24	51.42	>18773.56	>126974.48
	0.50	24.86	>13814.53	>71878.94	27.07	>29597.16	>15912.24	27.66	>18773.56	>126974.48
	0.70	15.93	>13814.53	>71878.94	17.88	>29597.16	>15912.24	18.41	>18773.56	>126974.48
	0.90	11.52	>13814.53	>71878.94	13.22	>29597.16	>15912.24	13.69	>18773.56	>126974.48
Large	1.00	10.09	>13814.53	>71878.94	11.68	>29597.16	>15912.24	12.12	>18773.56	>126974.48
	2.00	4.42	>13814.53	>71878.94	5.35	>29597.16	>15912.24	5.61	>18773.56	>126974.48
	3.00	2.81	>13814.53	>71878.94	3.46	>29597.16	>15912.24	3.65	>18773.56	>126974.48

TABLE 4-51 Effect of the location parameters in process on the ARL of control charts under a Gumbel distribution, with PEA_0 and $ARL_0 = 370$.

δ	Chart	EWMA			EEWMA			NEEWMA		
	θ_0	1	5	10	1	5	10	1	5	10
No change	0.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00
Small	0.01	342.95	342.95	342.95	342.46	343.86	344.45	342.52	344.08	344.70
	0.03	296.50	296.50	296.50	295.50	299.19	300.74	295.68	299.80	301.43
	0.05	258.38	258.38	258.38	257.30	262.71	265.02	257.57	263.63	266.07
	0.07	226.84	226.84	226.84	225.90	232.63	235.53	226.27	233.81	236.88
	0.09	200.54	200.54	200.54	199.86	207.60	210.96	200.30	208.99	212.55
Medium	0.10	189.02	189.02	189.02	188.49	196.64	200.20	188.97	198.13	201.89
	0.30	75.72	75.72	75.72	77.49	88.04	93.05	78.24	90.23	95.60
	0.50	42.14	42.14	42.14	44.41	54.35	59.40	45.14	56.55	62.02
	0.70	27.91	27.91	27.91	30.15	39.22	44.08	30.82	41.30	46.62
	0.90	20.45	20.45	20.45	22.53	30.83	35.44	23.14	32.77	37.87
Large	1.00	17.95	17.95	17.95	19.94	27.91	32.40	20.53	29.78	34.77
	2.00	7.68	7.68	7.68	8.98	14.79	18.30	9.38	16.17	20.20
	3.00	4.66	4.66	4.66	5.59	10.34	13.20	5.88	11.43	14.79

TABLE 4-52 Effect of the scale parameters in process on the ARL of control charts under a TP Reyleigh distribution, with PEA_0 and $ARL_0 = 370$.

δ	Chart	EWMA			EEWMA			NEEWMA		
	θ_0	1	5	10	1	5	10	1	5	10
No change	0.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00
Small	0.01	323.32	323.32	323.32	301.69	304.99	306.36	301.84	305.39	306.83
	0.03	238.91	238.91	238.91	208.67	216.00	219.10	209.16	217.08	220.33
	0.05	175.26	175.26	175.26	151.31	160.71	164.76	152.08	162.28	166.52
	0.07	130.61	130.61	130.61	114.28	124.74	129.32	115.26	126.63	131.43
	0.09	99.70	99.70	99.70	89.39	100.34	105.22	90.50	102.45	107.57
Medium	0.10	87.96	87.96	87.96	79.96	91.04	96.01	81.12	93.22	98.45
	0.30	18.71	18.71	18.71	21.55	31.04	35.98	22.63	33.34	38.64
	0.50	9.13	9.13	9.13	11.77	19.50	23.95	12.60	21.51	26.37
	0.70	5.83	5.83	5.83	8.01	14.51	18.53	8.67	16.28	20.74
	0.90	4.21	4.21	4.21	6.03	11.64	15.31	6.58	13.22	17.34
Large	1.00	3.68	3.68	3.68	5.36	10.60	14.12	5.86	12.11	16.07
	2.00	1.49	1.49	1.49	2.42	5.62	8.09	2.70	6.62	9.52
	3.00	0.84	0.84	0.84	1.47	3.79	5.69	1.67	4.54	6.81

4.2.3. Performance of MCA under Discrete Distributions

This section presents the accuracy, the detection behavior, and the performance of the EWMA, EEWMA, and NEEWMA control charts evaluated by the MCA method under discrete process distributions, namely the Poisson, binomial, and negative binomial distributions, which represent the **equidispersed**, **underdispersed**, and **overdispersed** cases respectively. The control limit coefficients were obtained by the bisection method described in Algorithm 1 and are reported in TABLES 4-49 to 4-51 at respective. Three patterns emerge. First, the limits widen as the second weighted parameter increases and widen further when the third is introduced, and the effect is far greater here than under any continuous distribution: at the lowest value of the first weighted parameter the limits of the extended charts are several times those of the EWMA chart, whereas under the symmetric distributions the difference was a matter of tenths. As before, the effect diminishes sharply as the first weighted parameter increases. Second, the limits widen as the process parameter increases, and the widening is confined almost entirely to the extended charts, those of the EWMA chart remaining virtually unchanged across the levels examined. Finally, the limits widen as the specified in-control ARL increases, an expected outcome for limits designed to reduce the probability of a false alarm.

TABLE 4-53 The control limit coefficient parameters of EWMA, EEWMA and NEEWMA with PEA_0 under Poisson distribution.

λ_1	λ_2	λ_3	ARL_0								
			$\theta_0 = 5$			$\theta_0 = 10$			$\theta_0 = 50$		
			200	370	500	200	370	500	200	370	500
0.1	0	0	2.146	2.494	2.646	2.111	2.455	2.603	2.055	2.401	2.546
0.1	0.01	0	4.548	4.948	5.120	5.458	5.848	6.015	9.347	9.746	9.912
0.1	0.01	0.0025	5.257	5.667	5.850	6.440	6.847	7.019	11.511	11.926	12.099
0.1	0.025	0	9.846	10.342	10.557	12.832	13.330	13.539	25.528	26.050	26.259
0.1	0.025	0.00625	12.889	13.441	13.672	17.094	17.649	17.882	34.930	35.509	35.739
0.1	0.05	0	28.502	29.356	29.694	38.987	39.856	40.209	83.388	84.340	84.685
0.1	0.05	0.0125	49.235	50.537	51.031	68.221	69.591	70.086	148.577	150.153	150.680
0.3	0	0	2.646	2.931	3.062	2.585	2.857	2.981	2.497	2.755	2.872
0.3	0.01	0	3.068	3.359	3.494	3.157	3.440	3.568	3.719	3.984	4.106
0.3	0.01	0.0025	3.170	3.467	3.601	3.303	3.586	3.715	4.033	4.301	4.423
0.3	0.025	0	3.767	4.074	4.214	4.114	4.410	4.542	5.759	6.040	6.166
0.3	0.025	0.00625	4.066	4.381	4.523	4.530	4.831	4.967	6.666	6.949	7.078
0.3	0.05	0	5.153	5.487	5.638	6.015	6.337	6.483	9.838	10.139	10.280
0.3	0.05	0.0125	5.926	6.271	6.428	7.087	7.420	7.570	12.167	12.484	12.626
0.7	0	0	2.913	3.190	3.322	2.814	3.073	3.195	2.675	2.910	3.019
0.7	0.01	0	3.011	3.289	3.421	2.949	3.209	3.332	2.965	3.199	3.310
0.7	0.01	0.0025	3.034	3.311	3.444	2.981	3.241	3.363	3.036	3.272	3.382
0.7	0.025	0	3.164	3.442	3.576	3.158	3.419	3.544	3.416	3.654	3.763
0.7	0.025	0.00625	3.223	3.502	3.635	3.242	3.504	3.628	3.604	3.841	3.954
0.7	0.05	0	3.433	3.715	3.850	3.529	3.795	3.919	4.216	4.457	4.570
0.7	0.05	0.0125	3.564	3.848	3.981	3.712	3.981	4.104	4.625	4.866	4.980

TABLE 4-54 The control limit coefficient parameters of EWMA, EEWMA and NEEWMA with PEA_0 under Binomial distribution with p_0 is 0.01

λ_1	λ_2	λ_3	ARL_0								
			$\alpha_0 = 5$			$\alpha_0 = 10$			$\alpha_0 = 50$		
			200	370	500	200	370	500	200	370	500
0.1	0	0	2.9243	3.4345	3.6901	2.7160	3.1755	3.3876	2.3632	2.7374	2.9027
0.1	0.01	0	3.2890	3.8838	4.1607	3.1975	3.7067	3.9381	3.2345	3.6574	3.8461
0.1	0.01	0.0025	3.3947	3.9982	4.2862	3.3314	3.8510	4.0910	3.4858	3.9212	4.1122
0.1	0.025	0	4.0887	4.7899	5.1057	4.1908	4.8016	5.0768	5.1109	5.6305	5.8563
0.1	0.025	0.00625	4.5090	5.2520	5.5832	4.7275	5.3798	5.6805	6.1633	6.7274	6.9740
0.1	0.05	0	6.5478	7.5138	7.9527	7.3710	8.2821	8.6859	11.4416	12.2887	12.6343
0.1	0.05	0.0125	8.8964	10.1595	10.7333	10.5938	11.8001	12.3125	18.2418	19.4042	19.8936
0.3	0	0	4.2512	4.9244	4.9845	3.7940	4.2159	4.4024	3.0698	3.4317	3.6001
0.3	0.01	0	4.3556	5.0226	5.0953	3.8695	4.3390	4.5368	3.2384	3.6114	3.7834
0.3	0.01	0.0025	4.3747	5.0425	5.1159	3.8864	4.3625	4.5667	3.2769	3.6502	3.8244
0.3	0.025	0	4.5196	5.1812	5.2706	4.0149	4.5306	4.7598	3.5169	3.9041	4.0856
0.3	0.025	0.00625	4.5745	5.2225	5.3330	4.0619	4.5978	4.8373	3.6257	4.0191	4.2034
0.3	0.05	0	4.8218	5.4583	5.6366	4.3173	4.8884	5.1465	4.0542	4.4736	4.6634
0.3	0.05	0.0125	4.9429	5.5659	5.8097	4.4762	5.0485	5.3156	4.3336	4.7661	4.9638
0.7	0	0	4.3677	5.2683	5.2683	3.6938	5.6314	5.6314	3.5423	3.8725	4.0461
0.7	0.01	0	4.4022	5.3201	5.3201	3.7484	5.6432	5.6432	3.5778	3.9279	4.1014
0.7	0.01	0.0025	4.4077	5.3303	5.3303	3.7603	5.6415	5.6434	3.5845	3.9404	4.1121
0.7	0.025	0	4.4562	5.4000	5.4000	3.8328	5.6600	5.6620	3.6313	4.0156	4.1846
0.7	0.025	0.00625	4.4714	5.4218	5.4218	3.8609	5.6580	5.6600	3.6479	4.0462	4.2123
0.7	0.05	0	4.5526	5.5316	5.5316	3.9773	5.6911	5.6954	3.7260	4.1430	4.3314
0.7	0.05	0.0125	4.5858	5.5798	5.5798	4.0385	5.6880	5.6925	3.7618	4.1874	4.3909

TABLE 4-55 The control limit coefficient parameters of EWMA, EEWMA and NEEWMA with PEA_0 under Negative Binomial distribution with p_0 is 0.01

λ_1	λ_2	λ_3	ARL_0								
			$\beta_0 = 5$			$\beta_0 = 10$			$\beta_0 = 50$		
			200	370	500	200	370	500	200	370	500
0.1	0	0	0.0096	0.0113	0.0120	0.0064	0.0076	0.0081	0.0025	0.0030	0.0032
0.1	0.01	0	0.0204	0.0223	0.0231	0.0170	0.0184	0.0190	0.0129	0.0134	0.0137
0.1	0.01	0.0025	0.0235	0.0255	0.0264	0.0202	0.0215	0.0221	0.0159	0.0165	0.0168
0.1	0.025	0	0.0441	0.0465	0.0475	0.0404	0.0421	0.0428	0.0358	0.0366	0.0369
0.1	0.025	0.00625	0.0577	0.0604	0.0616	0.0540	0.0558	0.0566	0.0492	0.0500	0.0504
0.1	0.05	0	0.1277	0.1318	0.1335	0.1234	0.1263	0.1275	0.1180	0.1193	0.1199
0.1	0.05	0.0125	0.2208	0.2269	0.2292	0.2163	0.2207	0.2224	0.2106	0.2128	0.2136
0.3	0	0	0.0127	0.0142	0.0150	0.0085	0.0095	0.0099	0.0034	0.0038	0.0040
0.3	0.01	0	0.0145	0.0161	0.0169	0.0103	0.0113	0.0118	0.0051	0.0055	0.0057
0.3	0.01	0.0025	0.0150	0.0166	0.0174	0.0107	0.0118	0.0123	0.0056	0.0060	0.0062
0.3	0.025	0	0.0177	0.0193	0.0201	0.0133	0.0144	0.0149	0.0080	0.0085	0.0087
0.3	0.025	0.00625	0.0190	0.0207	0.0215	0.0146	0.0157	0.0162	0.0093	0.0098	0.0099
0.3	0.05	0	0.0239	0.0257	0.0265	0.0194	0.0205	0.0210	0.0138	0.0143	0.0145
0.3	0.05	0.0125	0.0274	0.0292	0.0300	0.0228	0.0239	0.0245	0.0171	0.0176	0.0178
0.7	0	0	0.0147	0.0164	0.0172	0.0097	0.0107	0.0112	0.0038	0.0042	0.0044
0.7	0.01	0	0.0152	0.0168	0.0176	0.0101	0.0111	0.0116	0.0042	0.0046	0.0048
0.7	0.01	0.0025	0.0153	0.0169	0.0177	0.0102	0.0112	0.0117	0.0043	0.0047	0.0049
0.7	0.025	0	0.0158	0.0175	0.0183	0.0108	0.0118	0.0123	0.0049	0.0053	0.0054
0.7	0.025	0.00625	0.0161	0.0178	0.0186	0.0110	0.0121	0.0126	0.0052	0.0055	0.0057
0.7	0.05	0	0.0170	0.0187	0.0195	0.0119	0.0130	0.0135	0.0060	0.0064	0.0066
0.7	0.05	0.0125	0.0176	0.0193	0.0201	0.0125	0.0136	0.0141	0.0066	0.0070	0.0072

4.2.3.1. Accuracy of ARL Evaluation Method

This section compares the MCA and the MC methods for calculating the ARL and the SDRL of the EWMA, EEWMA, and NEEWMA control charts under a discretely distributed process. Three discrete distributions are considered, namely the Poisson, the binomial with $p_0 = 0.01$, and the negative binomial, representing the equidispersed, underdispersed, and overdispersed cases respectively. The chart parameters are fixed at $\lambda_1 = 0.1$, $\lambda_2 = 0.01$, and $\lambda_3 = \lambda_2/4$ throughout, the corresponding control limits being taken from TABLES 4-53 to 4-55. The results are as follows.

For the Poisson distribution, comparing the MCA and the MC methods at $ARL_0 = 370$, the values obtained by the two methods are close at every shift level and for all three charts, the in-control average run length being returned as 369.1403 by the MCA against 369.2460 by the simulation for the EWMA chart. The run lengths fall away far more steeply than under any continuous distribution, the EWMA chart requiring 34.0598 at a shift of one unit and 2.4929 at ten units against 34.3084 and 2.4994 for the simulation, and the SDRL values agree to the same degree. The results for $ARL_0 = 200$ and 500 are shown in TABLES A-35 and B-35 respectively.

For the binomial distribution, comparing the MCA and the MC methods at $ARL_0 = 370$, the values obtained by the two methods are close at every shift level and for all three charts, and the in-control average run length is attained in each case. This distribution yields the longest run lengths of the three, the EWMA chart requiring 225.2211 observations at a shift of one unit and 63.6813 at five units against 225.9808 and 63.0421 for the simulation, and the SDRL exceeds the ARL at every shift level, indicating a heavier right tail than in the preceding sections. The results are shown in TABLE 4-57, and those for $ARL_0 = 200$ and 500 in TABLES A-36 and B-36 respectively.

For the negative binomial distribution, comparing the MCA and the MC methods at $ARL_0 = 370$, the values obtained by the two methods are close at every shift level and for all three charts, and the in-control average run length is attained in each case. The run lengths between of the other two distributions, the EWMA chart requiring 37.1202 at a shift of one unit and 5.6333 at five units against 37.2526 and 5.6382 for the simulation, and the agreement does not deteriorate as lagged terms are added, the NEEWMA chart being reproduced as faithfully as the EWMA chart under all three discrete distributions. The results are shown in TABLE 4-58, and those for $ARL_0 = 200$ and 500 in TABLES A-37 and B-37 respectively.

Turning to the overall comparison, TABLE 4-59 summarizes the closeness of the two methods and TABLE 4-60 their computation times. In point of closeness, the binomial distribution separates itself from the other two: its relative error lies between 0.36 and 0.63 and its MPE and MAPE below 0.06 per cent, whereas under the Poisson and negative binomial distributions the relative error lies between 11.05 and 50.63 and the MAPE between 0.85 and 4.22 per cent.

Within these two the NEEWMA chart is approximated most accurately and the EWMA chart least accurately. In point of computation time, the MCA requires between 3.08 and 3.53 seconds per shift level, as in the two preceding sections, whereas the MC simulation requires only between 1.68 and 8.63 seconds because the run lengths generated under a discrete process are short. The approximation therefore retains its advantage under the binomial distribution but not under the other two.

TABLE 4-56 ARL of the EWMA EEWMA and NEEWMA control chart using PEA_0 compared to MC under a Poisson distribution, with $ARL_0 = 370$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MC	MCA	MC	MCA	MC
0	ARL	369.1403	369.2460	369.9239	369.1327	369.9349	369.6371
	SDRL	347.0782	347.0827	343.4435	343.3882	342.2230	342.8219
	CPU Time	3.28	25.61	2.89	25.55	2.91	25.98
1	ARL	34.0598	34.3084	35.6946	35.6621	36.1678	36.2799
	SDRL	20.1197	20.2190	19.3714	19.4767	19.1881	19.0415
	CPU Time	3.43	2.66	3.25	2.77	3.47	2.75
2	ARL	15.0213	15.0776	16.5309	16.5591	16.9550	16.9356
	SDRL	6.0842	6.1501	6.0360	6.0679	6.0281	6.9988
	CPU Time	3.53	1.22	3.36	1.28	3.32	1.34
3	ARL	9.5856	9.5655	10.8053	10.8486	11.1473	11.1511
	SDRL	3.2313	3.1859	3.2752	3.2915	3.2888	3.2768
	CPU Time	3.09	0.82	2.81	0.92	2.86	0.97
4	ARL	7.0059	7.9762	8.0149	8.9891	8.2979	8.2776
	SDRL	2.1533	2.1176	2.2112	2.2051	2.2277	2.1989
	CPU Time	3.14	0.59	3.02	0.72	3.08	0.68
5	ARL	5.4882	5.4750	6.3459	6.3697	6.5868	6.6208
	SDRL	1.6075	1.6051	1.6645	1.6889	1.6803	1.6989
	CPU Time	3.09	0.50	3.07	0.57	3.11	0.58
10	ARL	2.4929	2.4994	2.9790	2.9682	3.1175	3.1298
	SDRL	0.7256	0.7271	0.7672	0.7673	0.7763	0.7839
	CPU Time	2.93	0.26	3.11	0.31	3.13	0.31
15	ARL	1.4853	1.4742	1.8461	1.8521	1.9454	1.9393
	SDRL	0.5362	0.5327	0.5406	0.5395	0.5332	0.5369
	CPU Time	2.97	0.19	3.14	0.23	2.94	0.23
20	ARL	1.0283	1.0255	1.2195	1.2259	1.2951	1.2984
	SDRL	0.3087	0.3157	0.4241	0.4244	0.4620	0.4648
	CPU Time	3.01	0.16	3.19	0.17	2.97	0.17
25	ARL	0.7997	0.7957	0.9730	0.9745	1.0100	1.0118
	SDRL	0.4062	0.4096	0.2544	0.2451	0.2327	0.2286
	CPU Time	3.06	0.14	3.24	0.15	3.03	0.16
50	ARL	0.0016	0.0011	0.0147	0.0134	0.0300	0.0295
	SDRL	0.0401	0.0332	0.1203	0.1150	0.1707	0.1692
	CPU Time	3.14	0.08	3.11	0.08	3.08	0.11
100	ARL	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	SDRL	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	CPU Time	3.44	0.08	3.25	0.09	3.48	0.08

TABLE 4-57 ARL of the EWMA EEWMA and NEEWMA control chart using PEA_0 compared to MC under a Binomial distribution, with $ARL_0 = 370$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MC	MCA	MC	MCA	MC
0	ARL	368.8959	368.8476	369.8534	369.6864	369.3809	369.3593
	SDRL	382.7506	382.9805	383.1681	383.9654	382.7380	382.8649
	CPU Time	3.17	27.34	3.16	25.90	3.08	26.14
1	ARL	225.2211	225.9808	224.4171	224.4438	223.7450	223.2388
	SDRL	235.3577	235.6328	233.9562	233.1951	233.2520	233.0046
	CPU Time	3.44	17.08	3.39	16.38	3.34	16.07
2	ARL	150.8464	150.1903	149.7488	149.0330	149.1548	149.7540
	SDRL	158.4628	158.4401	156.7555	156.2350	156.0936	156.9579
	CPU Time	3.47	11.41	3.39	10.92	3.39	11.01
3	ARL	108.0223	108.0186	107.0191	107.3460	106.5422	106.4252
	SDRL	113.8579	113.7522	112.2584	112.0545	111.6978	111.8961
	CPU Time	3.45	8.16	3.36	8.01	3.44	7.91
4	ARL	81.3427	81.2717	80.5179	80.9818	80.1450	80.8898
	SDRL	85.8808	85.7890	84.4869	84.1515	84.0252	84.5503
	CPU Time	3.51	6.27	3.43	5.98	3.29	5.93
5	ARL	63.6813	63.0421	63.0310	63.0578	62.7407	62.6690
	SDRL	67.2511	67.1323	66.0658	66.5282	65.6875	65.9893
	CPU Time	3.53	5.08	3.38	4.76	3.29	4.75
10	ARL	26.9083	26.6360	26.7538	26.7926	26.6637	26.9205
	SDRL	28.0858	28.5507	27.5714	27.4762	27.4212	27.4220
	CPU Time	3.51	2.13	3.39	2.10	3.39	2.22
15	ARL	15.7247	15.9071	15.7205	15.8219	15.6881	15.7518
	SDRL	16.0951	16.3412	15.8571	15.8830	15.7895	15.7727
	CPU Time	3.48	1.29	3.29	1.38	3.38	1.28
20	ARL	10.7543	10.8584	10.7952	10.7685	10.7824	10.7352
	SDRL	10.7978	10.9287	10.6874	10.6658	10.6549	10.5450
	CPU Time	3.39	0.95	3.42	0.89	3.08	0.93
25	ARL	8.0448	8.1719	8.0965	8.2439	8.0912	8.0170
	SDRL	7.9420	7.0761	7.8971	7.9871	7.8809	7.6986
	CPU Time	3.03	0.73	2.86	0.70	2.89	0.69
50	ARL	3.3654	3.3796	3.3926	3.3365	3.3925	3.3626
	SDRL	3.1674	3.1708	3.1970	3.1283	3.1964	3.1268
	CPU Time	3.43	0.36	2.91	0.33	2.91	0.36
100	ARL	1.4124	1.4248	1.4155	1.4155	1.4155	1.4135
	SDRL	1.4058	1.4075	1.4159	1.4098	1.4159	1.4310
	CPU Time	3.61	0.18	3.01	0.17	3.26	0.19

TABLE 4-58 ARL of the EWMA EEWMA and NEEWMA control chart using PEA_0 compared to MC under a Negative Binomial distribution, with ARL_0 = 370.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MC	MCA	MC	MCA	MC
0	ARL	369.8651	369.1171	369.8813	369.1852	369.9044	369.5127
	SDRL	348.9665	348.9434	344.5799	344.5229	343.3682	343.0754
	CPU Time	3.25	26.80	3.34	26.48	3.29	27.52
1	ARL	37.1202	37.2526	38.3652	38.2039	38.7491	38.5463
	SDRL	23.0078	23.2861	21.8309	21.6507	21.5427	21.5867
	CPU Time	3.54	3.04	3.66	3.07	3.61	3.16
2	ARL	15.8347	15.8094	17.2632	17.3665	17.6710	17.6077
	SDRL	6.6360	6.7553	6.5015	6.5877	6.4722	6.4518
	CPU Time	3.64	1.31	3.68	1.48	3.59	1.50
3	ARL	9.9555	9.9479	11.1485	11.0983	11.4864	11.5035
	SDRL	3.4125	3.4076	3.4300	3.4009	3.4370	3.4950
	CPU Time	3.11	0.87	3.67	0.96	3.62	1.08
4	ARL	7.2224	7.2605	8.2200	8.2166	8.5022	8.4764
	SDRL	2.2343	2.2699	2.2816	2.2863	2.2953	2.2759
	CPU Time	3.16	0.64	3.51	0.74	3.52	0.73
5	ARL	5.6333	5.6382	6.4856	6.4900	6.7268	6.7068
	SDRL	1.6503	1.6484	1.7024	1.7021	1.7169	1.7011
	CPU Time	3.14	0.55	3.50	0.59	3.24	0.64
10	ARL	2.5394	2.5261	3.0275	3.0326	3.1661	3.1691
	SDRL	0.7336	0.7338	0.7713	0.7758	0.7810	0.7913
	CPU Time	3.20	0.27	3.55	0.32	3.25	0.33
15	ARL	1.5175	1.5216	1.8750	1.8765	1.9709	1.9725
	SDRL	0.5373	0.5333	0.5333	0.5292	0.5306	0.5288
	CPU Time	3.25	0.20	3.33	0.25	3.28	0.25
20	ARL	1.0311	1.0337	1.2428	1.2391	1.3216	1.3160
	SDRL	0.3148	0.3187	0.4393	0.4352	0.4738	0.4712
	CPU Time	3.28	0.17	3.56	0.18	3.25	0.18
25	ARL	0.8181	0.8143	0.9793	0.9807	1.0092	1.0130
	SDRL	0.3902	0.3935	0.2413	0.2411	0.2370	0.2511
	CPU Time	3.21	0.14	3.56	0.17	3.24	0.16
50	ARL	0.0011	0.0010	0.0165	0.0156	0.0303	0.0317
	SDRL	0.0324	0.0316	0.1275	0.1239	0.1715	0.1752
	CPU Time	3.09	0.08	3.44	0.12	3.18	0.08
100	ARL	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	SDRL	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	CPU Time	3.03	0.08	3.57	0.09	3.13	0.07

TABLE 4-59 Closeness of the ARL of the MEWMA control chart with PEA_0 compared to MC under a discrete distribution.

Distribution	ARL_0	Charts	RE	RMSE	MAE	MPE	MAPE
Poisson	200	EWMA	39.0214	58.2246	4.8779	2.3072	3.2518
		EEWMA	14.9399	86.0030	5.9083	0.2084	1.2450
		NEEWMA	12.4720	92.9551	6.1847	-0.0219	1.0393
	370	EWMA	41.1675	72.2739	5.4360	2.4801	3.4306
		EEWMA	14.1757	97.4357	6.3917	0.1488	1.1813
		NEEWMA	11.0498	107.8585	6.6485	-0.1230	0.9208
	500	EWMA	22.0455	72.5319	5.4348	0.9028	1.8371
		EEWMA	12.2895	104.3730	6.5593	-0.0008	1.0241
		NEEWMA	11.3060	113.1601	6.8311	-0.0926	0.9422
Binomial	200	EWMA	0.6342	13.9893	2.5587	0.0121	0.0529
		EEWMA	0.5139	12.1487	2.3014	0.0035	0.0428
		NEEWMA	0.5684	12.9350	2.3274	-0.0016	0.0474
	370	EWMA	0.4879	35.4125	4.0308	0.0223	0.0407
		EEWMA	0.3904	31.5083	3.6270	0.0173	0.0325
		NEEWMA	0.3565	30.5410	3.5068	0.0174	0.0297
	500	EWMA	0.3873	38.8827	4.1655	0.0257	0.0323
		EEWMA	0.4261	43.8707	4.3590	0.0258	0.0355
		NEEWMA	0.4097	38.9467	4.0984	0.0254	0.0341
Negative Binomial	200	EWMA	50.6282	54.6572	4.8063	3.2761	4.2190
		EEWMA	14.7224	79.4331	5.8219	0.1882	1.2269
		NEEWMA	13.3311	88.4315	6.1280	0.0559	1.1109
	370	EWMA	37.0405	65.1558	5.2558	2.1456	3.0867
		EEWMA	13.3054	93.0508	6.2908	0.0867	1.1088
		NEEWMA	10.6183	102.8284	6.5924	-0.1500	0.8849
	500	EWMA	33.3715	69.9469	5.4538	1.8420	2.7810
		EEWMA	12.4027	101.0972	6.5335	0.0189	1.0336
		NEEWMA	10.1965	109.4759	6.8115	-0.1768	0.8497

TABLE 4-60 Computation time efficiency of the ARL of the MEWMA control chart with PEA_0 compared to MC under a discrete distribution.

Distribution	ARL_0	Charts	TCA		ACT	
			MCA	MC	MCA	MC
Poisson	200	EWMA	39.86	20.13	3.32	1.68
		EEWMA	37.09	20.75	3.09	1.73
		NEEWMA	39.40	20.94	3.28	1.75
	370	EWMA	38.11	32.31	3.18	2.69
		EEWMA	37.44	32.84	3.12	2.74
		NEEWMA	37.38	33.36	3.12	2.78
	500	EWMA	40.23	40.98	3.35	3.42
		EEWMA	36.95	42.10	3.08	3.51
		NEEWMA	39.01	41.64	3.25	3.47
Binomial	200	EWMA	41.32	51.18	3.44	4.27
		EEWMA	39.30	47.94	3.28	4.00
		NEEWMA	39.14	47.47	3.26	3.96
	370	EWMA	41.02	80.98	3.42	6.75
		EEWMA	38.99	77.52	3.25	6.46
		NEEWMA	38.74	77.48	3.23	6.46
	500	EWMA	41.27	103.50	3.44	8.63
		EEWMA	38.04	98.32	3.17	8.19
		NEEWMA	38.90	97.10	3.24	8.09
Negative Binomial	200	EWMA	40.14	21.42	3.35	1.79
		EEWMA	39.93	22.05	3.33	1.84
		NEEWMA	40.66	22.73	3.39	1.89
	370	EWMA	38.90	34.15	3.24	2.85
		EEWMA	42.37	34.45	3.53	2.87
		NEEWMA	40.20	35.70	3.35	2.98
	500	EWMA	40.86	44.15	3.41	3.68
		EEWMA	41.35	44.18	3.45	3.68
		NEEWMA	39.33	44.47	3.28	3.71

4.2.3.2. Efficiency of Control charts

This section applies the MCA method to compute the ARL and the SDRL and compares the performance of the EWMA, EEWMA, and NEEWMA control charts under a discretely distributed process, namely the Poisson, binomial, and negative binomial distributions, which represent the equidispersed, underdispersed, and overdispersed cases respectively. The effects of the three weighted parameters are examined in turn, and the results are as follows.

For the Poisson distribution, with $ARL_0 = 370$, it was found that the detection performance varied inversely with λ_1 for small shifts and directly with it for moderate and large shifts, reproducing the behaviors observed under the symmetric and asymmetric distributions. When λ_1 was held constant, however, lower values of λ_2 and λ_3 gave better performance for the EEWMA and NEEWMA charts at every shift magnitude, so that the additional smoothing is uniformly unhelpful here and no configuration of λ_2 and λ_3 was found at which the extended charts improved upon the conventional one. The results are shown in TABLE 4-61, and those for $ARL_0 = 200$ and 500 in TABLES A-38 and B-38 respectively.

For the binomial distribution, with $ARL_0 = 370$, the relationships are less monotone. Increasing λ_1 improved the detection of small shifts up to $\lambda_1 = 0.3$, beyond which performance declined again, while lower values of λ_1 gave better detection of moderate and large shifts. Increasing λ_2 improved the detection of small shifts up to $\lambda_2 = 0.025$, beyond which performance likewise deteriorated, and increasing λ_3 improved the detection of small shifts while lower values moderate and large ones. An interior optimum thus exists for both λ_1 and λ_2 when small shifts are the concern, which is not the case under any of the continuous distributions examined, so that the parameters of a chart designed for an underdispersed process must be chosen by search over the range rather than by a monotone rule. The results are shown in TABLE 4-62, and those for $ARL_0 = 200$ and 500 in TABLES A-39 and B-39 respectively.

For the negative binomial distribution, with $ARL_0 = 370$, the pattern returns to that of the Poisson: increasing λ_1 improved the detection of moderate and large shifts while lower values improved the detection of small shifts, and increasing λ_2 and λ_3 reduced detection performance at every shift magnitude. The agreement between the equidispersed and the overdispersed cases suggests that the behaviour of the weighted parameters under a discrete process is governed by the discreteness of the support rather than by the degree of dispersion. The results are shown in TABLE 4-63, and those for $ARL_0 = 200$ and 500 in TABLES A-40 and B-40 respectively.

The same patterns were obtained at all three in-control levels. Taken together, the three distributions show that the second and third weighted parameters are of no benefit under a discrete process except in the underdispersed

case, and that even there the benefit is confined to small shifts and vanishes once the parameters are raised beyond a moderate value.

TABLE 4-61 Effect of control charts parameters on ARL under a Poisson distribution, with PEA_0 and $ARL_0 = 370$.

δ	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
No change	0	369.14	369.95	369.87	369.92	369.86	369.74	369.93	369.95	369.92
Small	1	34.06	39.23	66.63	35.69	39.08	49.87	36.17	41.00	60.95
	2	15.02	11.81	19.49	16.53	19.48	28.10	16.95	21.08	36.51
	3	9.59	5.87	7.93	10.81	13.18	20.05	11.15	14.46	26.72
	4	7.01	3.67	4.00	8.01	9.98	15.67	8.30	11.04	21.21
	5	5.49	2.57	2.31	6.35	8.02	12.88	6.59	8.92	17.61
Medium	10	2.49	0.77	0.31	2.98	3.94	6.76	3.12	4.46	9.51
	15	1.49	0.22	0.04	1.85	2.51	4.50	1.95	2.88	6.45
	20	1.03	0.04	0.00	1.22	1.81	3.31	1.30	2.07	4.82
	25	0.80	0.00	0.00	0.97	1.27	2.59	1.01	1.57	3.82
Large	50	0.00	0.00	0.00	0.01	0.28	1.01	0.03	0.59	1.85
	100	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.89

TABLE 4-62 Effect of control charts parameters on ARL under a Binomial distribution, with PEA_0 and $ARL_0 = 370$.

δ	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
No change	0	368.90	303.51	313.34	369.85	369.12	369.57	369.38	370.00	369.13
Small	1	225.22	211.30	219.24	224.42	221.18	216.00	223.74	220.58	213.67
	2	150.85	156.14	162.78	149.75	146.58	141.90	149.15	145.90	140.51
	3	108.02	120.47	126.16	107.02	104.43	101.14	106.54	103.92	100.77
	4	81.34	96.04	101.00	80.52	78.50	76.46	80.14	78.20	76.84
	5	63.68	78.55	82.94	63.03	61.49	60.40	62.74	61.36	61.27
Medium	10	26.91	36.85	39.56	26.75	26.38	27.23	26.66	26.61	28.80
	15	15.72	21.93	23.82	15.72	15.67	16.82	15.69	15.94	18.33
	20	10.75	14.84	16.24	10.80	10.83	11.95	10.78	11.09	13.34
	25	8.04	10.87	11.95	8.10	8.15	9.17	8.09	8.38	10.46
Large	50	3.37	4.07	4.48	3.39	3.42	3.92	3.39	3.51	5.01
	100	1.41	1.51	1.62	1.42	1.42	1.52	1.42	1.43	2.35

TABLE 4-63 Effect of control charts parameters on ARL under a Negative Binomial distribution, with PEA_0 and $ARL_0 = 370$.

δ	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
No change	0	369.87	369.91	369.97	369.88	369.90	369.95	369.90	369.95	369.97
Small	1	37.12	51.51	96.82	38.37	41.24	51.27	38.75	42.95	61.99
	2	15.83	15.20	32.59	17.26	20.11	28.58	17.67	21.66	36.90
	3	9.96	7.14	13.54	11.15	13.49	20.31	11.49	14.75	26.95
	4	7.22	4.29	6.66	8.22	10.17	15.85	8.50	11.22	21.37
	5	5.63	2.93	3.71	6.49	8.16	13.01	6.73	9.05	17.74
Medium	10	2.54	0.89	0.47	3.03	3.99	6.81	3.17	4.51	9.57
	15	1.52	0.30	0.06	1.88	2.54	4.53	1.97	2.91	6.49
	20	1.03	0.05	0.00	1.24	1.83	3.33	1.32	2.08	4.85
	25	0.82	0.00	0.00	0.98	1.30	2.61	1.01	1.60	3.85
Large	50	0.00	0.00	0.00	0.02	0.32	1.01	0.03	0.64	1.86
	100	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.89

4.2.3.3. Behavior of Detection

This section applies the MCA method to compute the ARL and the SDRL in order to study the effect of the parameter of a discretely distributed process on the performance of the EWMA, EEWMA, and NEEWMA control charts under the Poisson, binomial, and negative binomial distributions. The chart parameters were fixed at $\lambda_1 = 0.1$, $\lambda_2 = 0.01$, and $\lambda_3 = \lambda_2/4$, and the results are as follows.

For the Poisson distribution, with $ARL_0 = 370$, it was found that increasing the process parameter reduced the detection performance of all three charts at every shift magnitude, the EWMA chart requiring 34.06 observations to signal a shift of one unit when the process parameter is five but 54.63 when it is ten and 132.23 when it is fifty. A process generating larger counts therefore conceals a shift of a given size for considerably longer. The comparison among the three charts is uniform across the whole range: the EWMA chart gave the best performance at every level of the process parameter and at every shift magnitude, the EEWMA chart came second, and the NEEWMA chart third, the three requiring 34.06, 49.87, and 60.95 observations respectively at a shift of one unit when the process parameter is five. The gap between them moreover widens as the process parameter rises. The results are shown in TABLE 4-64, and those for $ARL_0 = 200$ and 500 in TABLES A-41 and B-41 respectively.

For the binomial distribution, with $ARL_0 = 370$, increasing the process parameter likewise reduced the detection performance of all three charts, the EWMA chart requiring 225.22 observations to signal a shift of one unit when the process parameter is five but 320.47 when it is fifty. The ordering of the charts,

however, differs from that obtained under the other two distributions: here the extended charts are marginally the faster, the EEWMA and NEEWMA charts requiring 216.00 and 213.67 observations against 225.22 for the EWMA chart at the same setting, and the advantage is retained at every level of the process parameter. The margin is nevertheless small in comparison with the deterioration caused by the process parameter itself. The results are shown in TABLE 4-65, and those for ARL_0 = 200 and 500 in TABLES A-42 and B-42 respectively.

For the negative binomial distribution, with ARL_0 = 370, the results closely follow those obtained under the Poisson distribution. Increasing the process parameter reduced the detection performance of all three charts, the EWMA chart requiring 37.12 observations to signal a shift of one unit when the process parameter is five but 135.82 when it is fifty, and the EWMA chart again gave the best performance at every level of the process parameter, ahead of the EEWMA and NEEWMA charts. The results are shown in TABLE 4-66, and those for ARL_0 = 200 and 500 in TABLES A-43 and B-43 respectively.

The same patterns were obtained at all three in-control levels. The superiority of the conventional EWMA chart under the equidispersed and overdispersed reverses the ordering obtained under the symmetric and asymmetric form and runs counter to the claims of uniform superiority made in the studies that introduced the extended statistics. A plausible explanation lies in the control limits reported in TABLES 4-53 to 4-55: at λ_1 = 0.1 the limits required by the extended charts are several times wider than those of the EWMA chart, so that a shift must displace the monitoring statistic further before a signal is produced. The lagged terms that extend the memory of the statistic also inflate its variance, and where the observations are counts taking few distinct values that inflation is not repaid by any gain in information least so in the underdispersed, which is the one in which the extended charts retain a slight advantage.

TABLE 4-64 Effect of the parameters in process on the ARL of control charts under a Poisson distribution, with PEA_0 and $ARL_0 = 370$.

δ	Charts	EWMA			EEWMA			NEEWMA		
	θ_0	5	10	50	5	10	50	5	10	50
No change	0	369.14	367.95	369.52	369.74	369.99	369.33	369.92	369.97	369.45
Small	1	34.06	54.63	132.23	49.87	72.66	147.38	60.95	86.32	163.35
	2	15.02	24.72	68.86	28.10	42.62	93.28	36.51	54.33	111.74
	3	9.59	16.06	45.46	20.05	31.47	71.64	26.72	41.32	89.56
	4	7.01	11.98	34.34	15.67	25.32	59.94	21.21	33.80	76.81
	5	5.49	9.57	28.00	12.88	21.30	52.41	17.61	28.76	68.23
Medium	10	2.49	4.72	15.68	6.76	12.06	34.63	9.51	16.77	46.72
	15	1.49	3.05	11.30	4.50	8.40	26.82	6.45	11.87	36.74
	20	1.03	2.19	8.90	3.31	6.41	22.13	4.82	9.16	30.61
	25	0.80	1.69	7.35	2.59	5.15	18.92	3.82	7.43	26.35
Large	50	0.00	0.72	3.90	1.01	2.44	11.08	1.85	3.74	15.75
	100	0.00	0.00	1.98	0.02	1.00	6.01	0.89	1.86	8.76

TABLE 4-65 Effect of the parameters in process on the ARL of control charts under a Binomial distribution, with PEA_0 and $ARL_0 = 370$.

δ	Charts	EWMA			EEWMA			NEEWMA		
	α_0	5	10	50	5	10	50	5	10	50
No change	0	368.90	369.87	369.60	369.57	368.02	369.37	369.13	369.68	369.84
Small	1	225.22	263.00	320.47	216.00	253.11	315.68	213.67	252.43	315.66
	2	150.85	194.79	279.56	141.90	184.04	272.43	140.51	183.51	272.53
	3	108.02	149.25	245.29	101.14	139.94	237.27	100.77	140.11	237.80
	4	81.34	117.65	216.41	76.46	110.33	208.40	76.84	111.20	209.51
	5	63.68	95.01	191.93	60.40	89.59	184.51	61.27	91.03	186.24
Medium	10	26.91	42.22	113.02	27.23	42.41	110.91	28.80	44.92	115.19
	15	15.72	24.50	73.44	16.82	26.41	75.67	18.33	28.86	81.12
	20	10.75	16.48	51.51	11.95	18.84	56.24	13.34	21.03	62.03
	25	8.04	12.13	38.32	9.17	14.54	44.33	10.46	16.47	50.07
Large	50	3.37	4.78	14.69	3.92	6.61	21.02	5.01	7.69	25.45
	100	1.41	1.85	5.80	1.52	3.01	10.04	2.35	3.44	12.79

TABLE 4-66 Effect of the parameters in process on the ARL of control charts under a Negative Binomial distribution, with PEA_0 and $ARL_0 = 370$.

δ	Charts	EWMA			EEWMA			NEEWMA		
	β_0	5	10	50	5	10	50	5	10	50
No change	0	369.87	369.80	369.55	369.95	369.94	369.83	369.97	369.96	369.88
Small	1	37.12	58.47	135.82	51.27	74.34	149.30	61.99	87.55	164.73
	2	15.83	25.89	70.90	28.58	43.23	94.30	36.90	54.83	112.51
	3	9.96	16.59	46.59	20.31	31.81	72.25	26.95	41.62	90.05
	4	7.22	12.28	35.02	15.85	25.54	60.35	21.37	34.01	77.17
	5	5.63	9.77	28.44	13.01	21.47	52.72	17.74	28.93	68.52
Medium	10	2.54	4.78	15.81	6.81	12.13	34.77	9.57	16.85	46.87
	15	1.52	3.08	11.36	4.53	8.45	26.91	6.49	11.92	36.86
	20	1.03	2.21	8.94	3.33	6.45	22.20	4.85	9.20	30.70
	25	0.82	1.71	7.38	2.61	5.18	18.98	3.85	7.46	26.43
Large	50	0.00	0.75	3.91	1.01	2.47	11.11	1.86	3.77	15.81
	100	0.00	0.00	1.98	0.02	1.00	6.02	0.89	1.88	8.80

4.3. Real Application

This chapter presents the application of the proposed MCA method to five real-world datasets: industrial sensor anomaly detection, GDP growth rate, unemployment insurance claims, solar energy output, and manufacturing defect counts. These datasets are selected to reflect the range of distributional characteristics examined in Chapters 4 through 7—symmetric, asymmetric, and discrete processes—providing an empirical test of the framework's practical applicability. The ARL and SDRL results obtained from the MCA method for each dataset are compared with those generated by the MC approach and discussed in relation to the corresponding numerical findings reported in the preceding chapters.

4.3.1. Industrial Sensor Anomaly Detection

This research applies the proposed method to an industrial system using the Kaggle Industrial Sensor Anomaly Detection dataset. The dataset simulates high-dimensional sensor data from multiple processing units, including real-time measurements such as pressure, flow rate, water level, and valve status. It contains both normal and abnormal conditions, where anomalies are represented by events such as pressure spikes, flow inconsistencies, and sensor malfunctions. These scenarios reflect potential system faults and cyber-physical attacks and are used to develop and evaluate models for anomaly detection, fault diagnosis, and system monitoring.

This study employs sixteen sensor variables for anomaly detection using a MEWMA control chart. Prior to monitoring, the data is decomposed to capture its underlying structure. Although no significant trend is observed, the data exhibits a seasonal pattern that can be approximated by a sinusoidal function. Therefore, each

sensor signal is fitted with a sinusoidal model to represent its temporal behavior. The residuals, defined as the difference between observed values and model predictions, are then extracted and used as inputs for anomaly detection. Figure 4-1 illustrates the original sensor data, the fitted sinusoidal model, and the resulting residuals.

A MEWMA control chart is used to monitor the residuals from all 16 sensors at the same time, which is how the anomaly detection process works. The MEWMA parameters are weighted, difference and limit parameters are 0.1, 0.7 and 2.8881 for the best detection performance. The monitoring statistics are calculated one by one, and when it goes over the set control limit, it finds anomalies. The results show that the control chart can find the first anomaly at observation 25, which shows that it can find changes in multivariate sensor data. Figure 4-2 shows the control chart that goes with this.

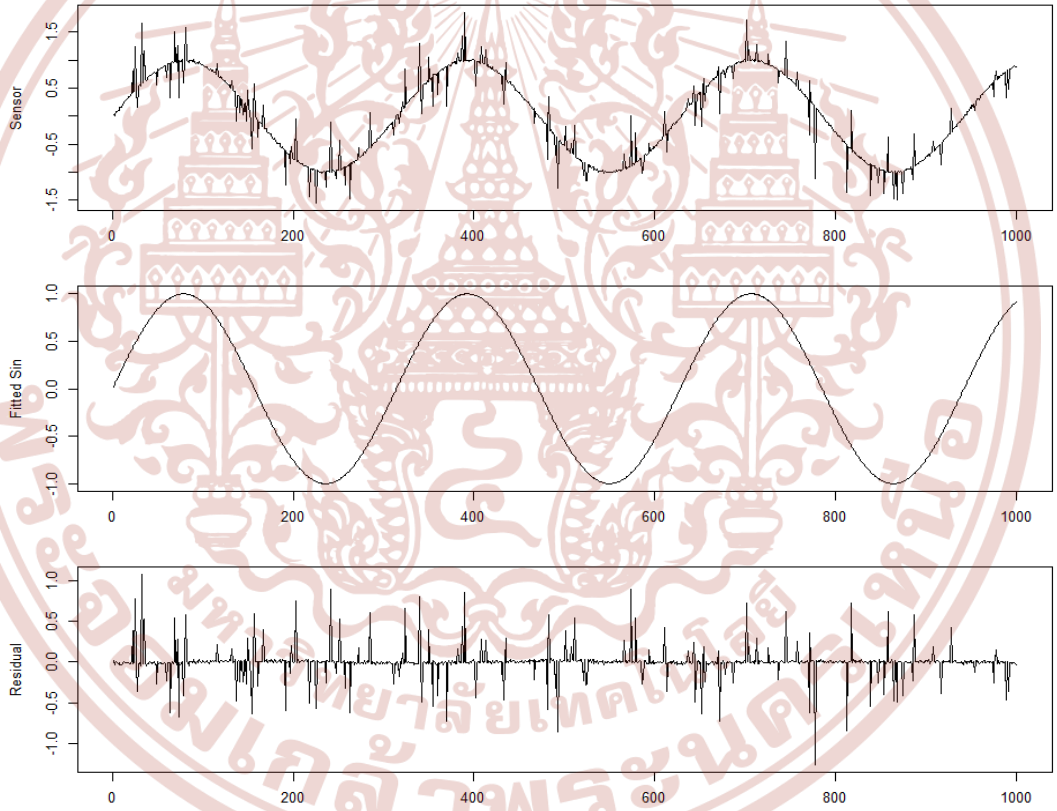


FIGURE 4-1 Decomposition of Sensor Data into Sinusoidal Model and Residual Component

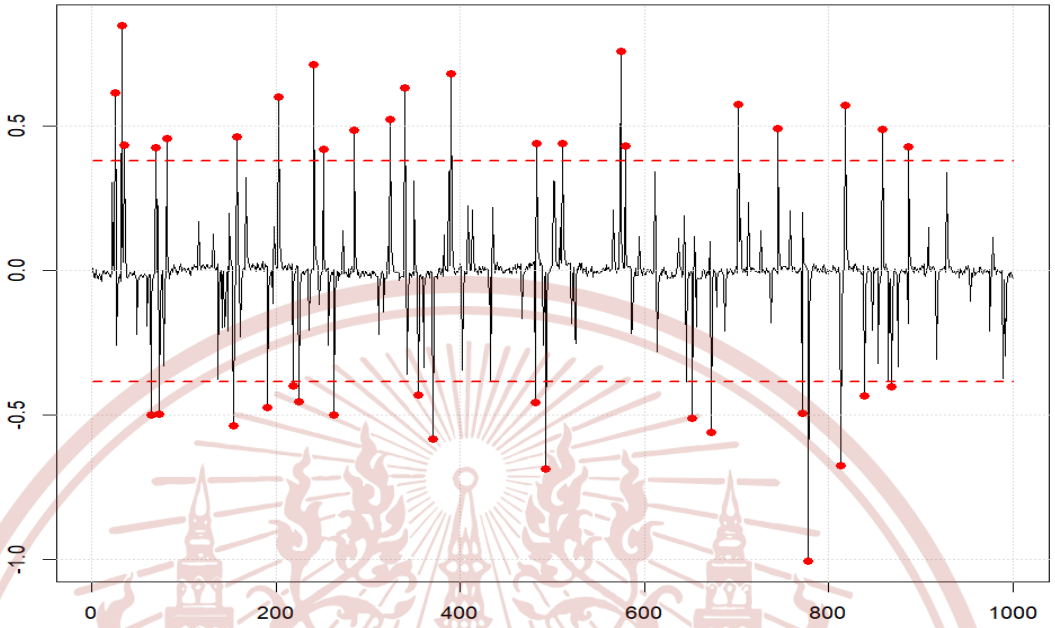


FIGURE 4-2 MEWMA Control Chart Applied to Residual Signals for Anomaly Detection

4.3.2. GDP Growth (Annual %) Rate

This study demonstrates the practical application of the EWMA, EEWMA, and NEEWMA control charts using real-world GDP growth (annual %) data for Australia (AUS), Japan (JPN), Singapore (SGP), and Thailand (THA) in the period 2000–2024, obtained from the World Bank Open Data repository. The data set in Tables xxx. For each country, the data was divided into two subsets:

1) Phase I, the period from 2000 to 2010, comprising the observations available before the period of interest, which was used for statistical analysis and for constructing the control limits of the control charts;
2) Phase II, the period from 2011 to 2024, which was employed to evaluate the effectiveness of the control charts in detecting structural changes in GDP growth.

Given weighted parameters of the control charts are $\lambda_1 = 0.3$, $\lambda_2 = 0.05$ and $\lambda_3 = \lambda_2 / 4$.

For the training set, descriptive statistical analysis and normality testing using the Kolmogorov–Smirnov (KS) test indicates that the GDP growth data for all four countries follow a normal distribution, as all p-values exceed the 0.05 significance level. The corresponding summary statistics and control limits for the EWMA, EEWMA, and NEEWMA control charts are reported in Tables 4-67 to 4-68

TABLE 4-67 Descriptive Statistics of GDP growth (annual %) of Australia, Japan, Singapore, and Thailand of the training data set

Stat	Australia	Japan	Singapore	Thailand
Mean	3.1557	0.7959	6.2077	4.6060
SD	0.7812	2.4496	405218	2.3171
KS test (p-value)	0.2000	0.1250	0.2000	0.2000

TABLE 4-68 Control limit of EWMA, EEWMA, and NEEWMA of GDP growth (annual %) of Australia, Japan, Singapore, and Thailand of the training data set

Control charts		Australia	Japan	Singapore	Thailand
EWMA	LCL	2.6240	-0.902	1.0220	0.2648
	UCL	3.6873	2.4938	11.3934	8.9472
EEWMA	LCL	2.6495	-0.8209	1.2699	0.4724
	UCL	3.6619	2.4126	11.1455	8.7397
NEEWMA	LCL	2.6590	-0.7903	1.3632	0.5505
	UCL	3.6523	2.3821	11.0522	8.6615

For the testing set, the application of the three control charts to GDP growth data of the four countries is detailed as follows:

For Australia, GDP growth remained relatively stable during 2011–2019 within the control range. In 2020, GDP growth slowed significantly and approached or fell below the LCL, reflecting global economic turmoil. The economy gradually recovered during 2021–2023, followed by a slight slowdown in 2024. The NEEWMA control chart was the first to detect anomalies, providing clearer signals of change than the EWMA and EEWMA charts, as shown in Figure 4-3.

For Japan, GDP growth remained relatively low but stable during 2011–2019 within the control range. In 2020, GDP growth slowed rapidly and approached or fell below the lower control limit, indicating a significant economic downturn. Although economic conditions improved during 2021–2023, growth remained relatively weak. The NEEWMA control chart provided the clearest signals of change, while EWMA and EEWMA to detect anomalies. As shown in Figure 4-4.

For Singapore, GDP growth showed a gradual decline during 2011–2018 but remained within the control range. In 2020, GDP growth declined rapidly and approached the lower control limit, indicating a major economic shift. The economy gradually recovered during 2021–2024, but did not return to pre-recession levels. The NEEWMA control chart provides the clearest signal of the shift, while EWMA and EEWMA failed to detect any anomalies, as shown in Figure 4-5.

For Thailand, GDP growth remained moderately stable during 2011–2019, within the controlled range. In 2020, GDP growth declined rapidly and approached or fell below the lower control limit, indicating a major disruption. The

economy gradually recovered during 2021–2024 but remained below pre-recession levels. The NEEWMA control chart provides the clearest signal of the shift, while EWMA and EEWMA failed to detect any anomalies, as shown in Figure 4-6.

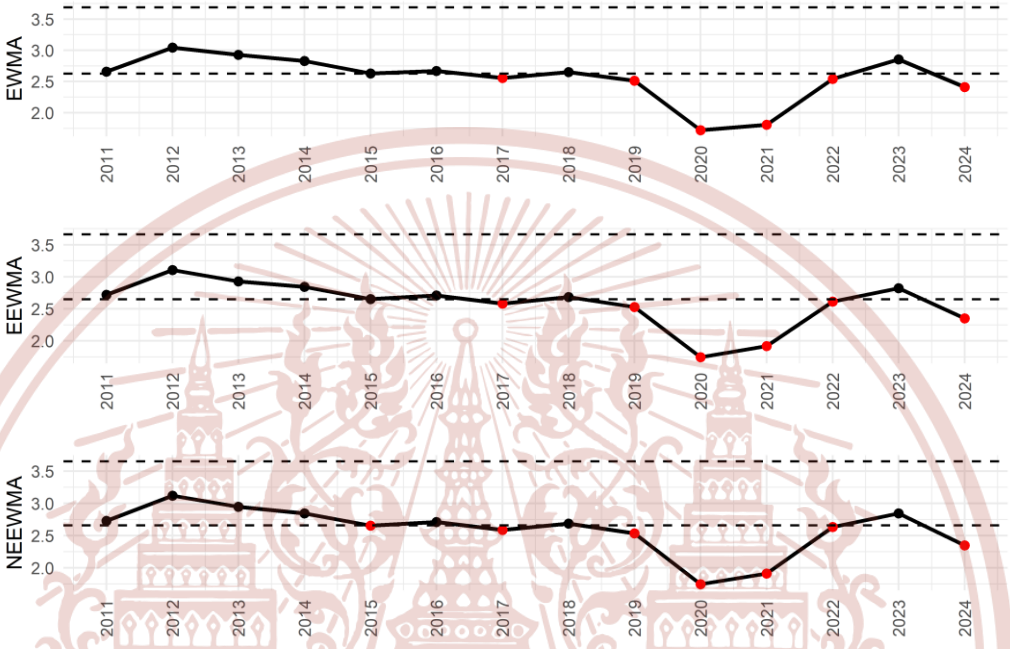


FIGURE 4-3 Performance of EWMA, EEWMA, and NEEWMA control charts to detect changes in Australia's GDP growth

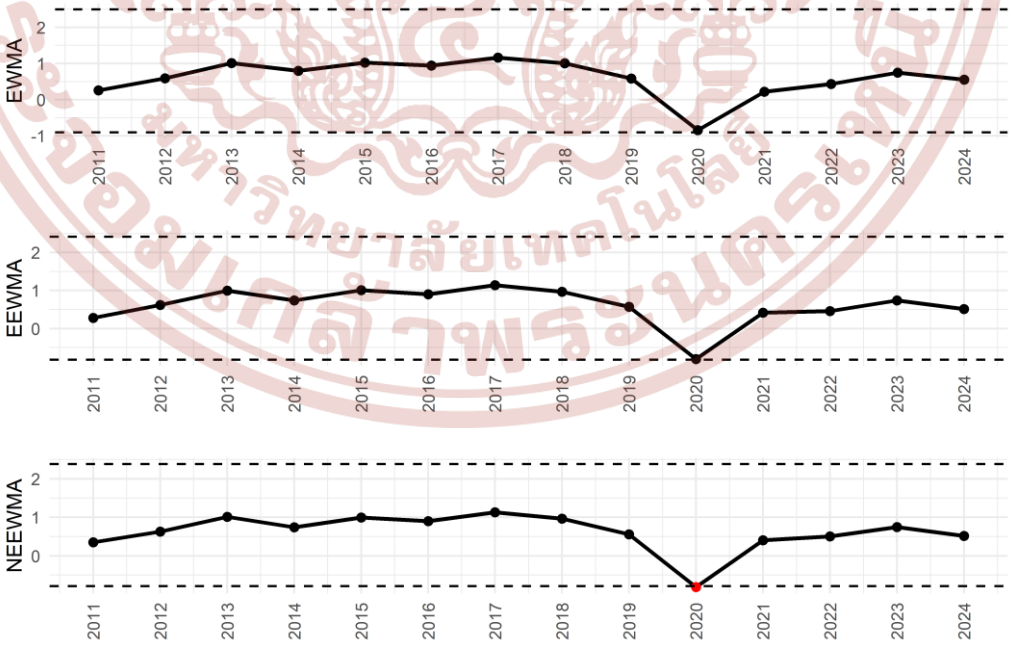


FIGURE 4-4 Performance of EWMA, EEWMA, and NEEWMA control charts to detect changes in Japan's GDP growth

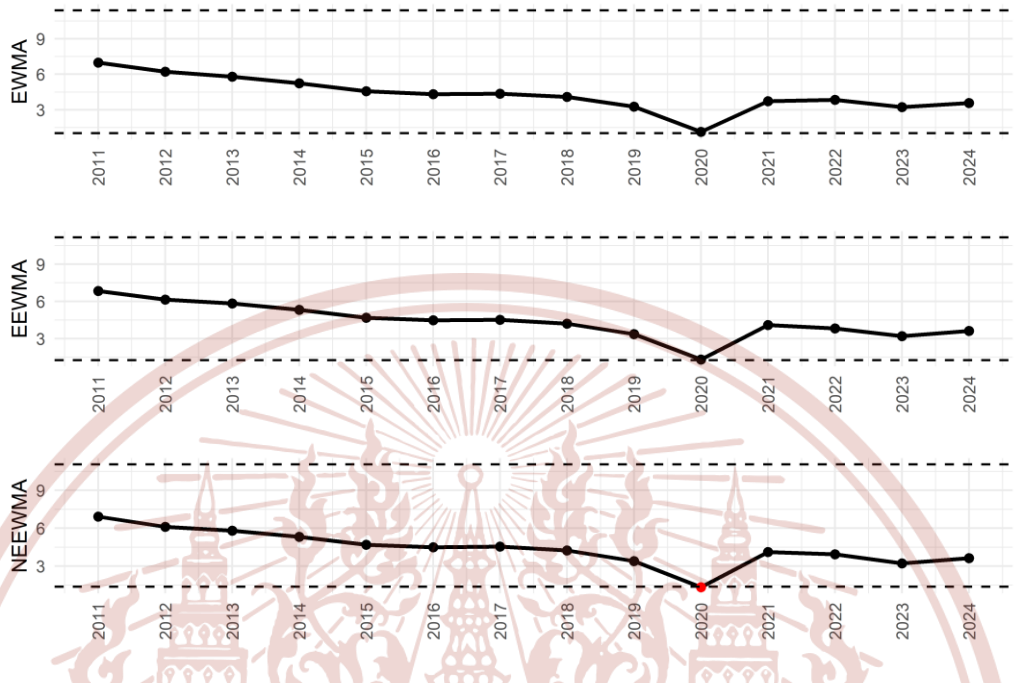


FIGURE 4-5 Performance of EWMA, EEWMA, and NEEWMA control charts to detect changes in Singapore's GDP growth

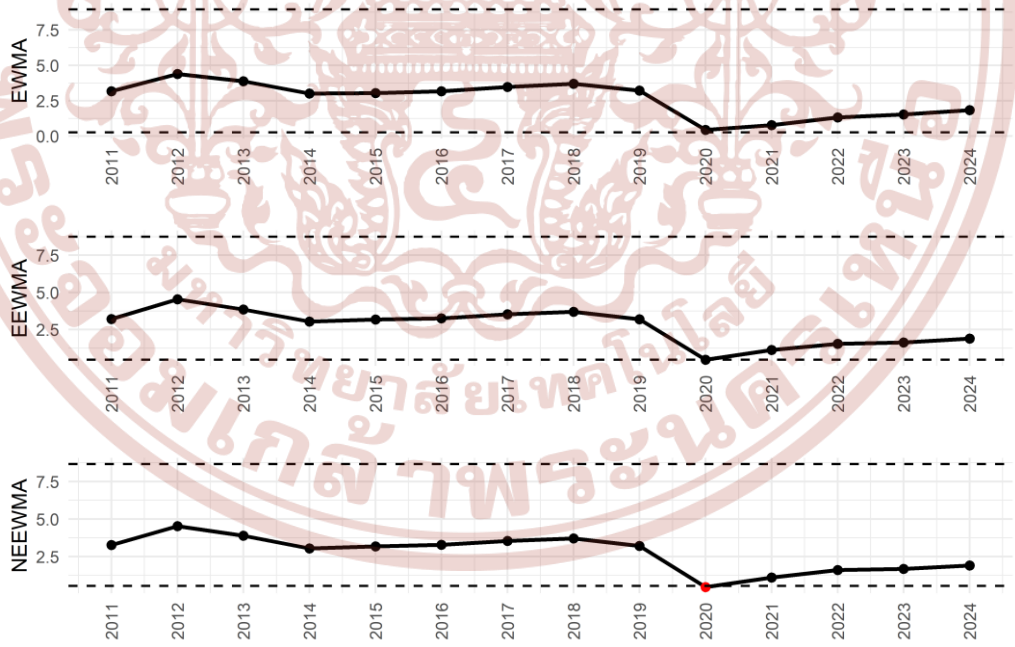


FIGURE 4-6 Performance of EWMA, EEWMA, and NEEWMA control charts to detect changes in Thailand's GDP growth

Notably, the detected change points in 2020 for Japan, Singapore, and Thailand coincide with the onset of the COVID-19 pandemic, which led to severe disruptions in global economic activity. The close agreement between the statistically

detected signals and real-world economic events provides strong empirical evidence supporting the effectiveness and practical relevance of the NEEWMA control chart for identifying significant structural changes in economic time series data.

4.3.3. Unemployment Insurance Claims (UI)

This section discusses the application of control charts to real-world data to demonstrate how they can be used in various fields. The application involves setting the control chart parameters to $\lambda_1 = 0.1$, $\lambda_2 = 0.05$ and $\lambda_3 = \lambda_2 / 4$ using Unemployment Insurance Claims (UI), it refers to the number of individuals applying for unemployment benefits, measured by the Initial Claims (ICSA) and Continued Claims (CCSA) indicators derived from Federal Reserve Economic Data (FRED). These indicators are important because they provide timely and high-frequency insights into labor market conditions, capturing both the inflow of newly unemployed individuals and the persistence of unemployment. In this study, the variables are constructed as rates of change in ICSA and CCSA to better capture short-term fluctuations and dynamic movements in labor market conditions. The dataset covers the period from January 2021 to December 2025, as illustrated by the time-series plots and ACF/PACF of the difference of rate of change in Figure 4-7 to 4-8, and is divided into a training set (2021-2024) and a test set (2025). Before this division, an ARIMA (3,1,0) model was fitted to the data, and the residuals were extracted for structural change detection using the proposed control charts. The model was identified based on the time series behavior and the ACF and PACF plots shown in Figure 4-3 and 4-4. After first-order differences (d=1), the series became approximately stationary. The ACF gradually decayed, while the PACF showed significant spikes up to lag 3 followed by a cutoff, indicating an AR (3) structure. Consequently, the ARIMA (3,1,0) model was selected for analysis.

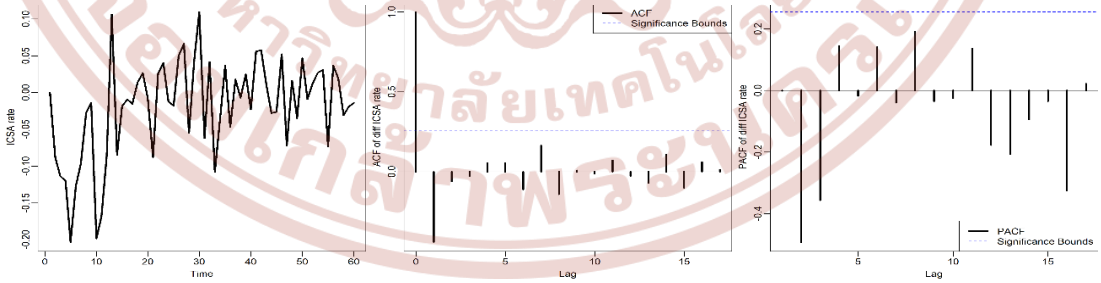


FIGURE 4-7 Time series analysis of no change claims seasonally adjusted (ICSA)

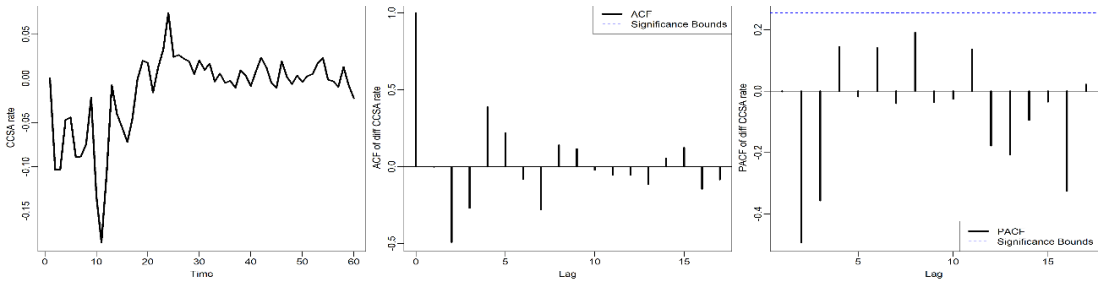


FIGURE 4-8 Time series analysis of cumulative claims seasonally adjusted (CCSA)

First, an ARIMA (3,1,0) model is fitted to extract residuals for change detection. Because control charts require independent observations, the ARIMA model helps remove autocorrelation and isolates the independent components of the series. Residuals that meet the assumptions of independence and normality are then used for further analysis. The results show that residuals from the ARIMA (3,1,0) model exhibit an approximately symmetrical distribution, as shown in Tables 4-69, and demonstrate approximate independence, supported by ACF and PACF plots where most autocorrelation values fall within confidence bounds. The data are then split into a training set (2021-2024) for constructing control charts and a test set (2025) for detecting changes.

For the training set, the construction of EWMA, EEWMA, and NEEWMA control charts is based on the calculation of the mean and variance of the residual (error) values. These statistics are then used to determine the control limit parameters (L), including UCL and LCL. The results of the analysis are presented in the table below.

TABLE 4-69 Distributional tests of ARIMA residuals for ICSA and CCSA.

UI Index	Mean	SD	Kolmogorov-Smirnov Test (p-Value)		
			Normal	Logistic	Laplace
ICSA	0.0005	0.0635	0.7098 *	0.0207	0.4756 *
CCSA	0.0005	0.0259	0.3387 *	0.0054	0.2212 *

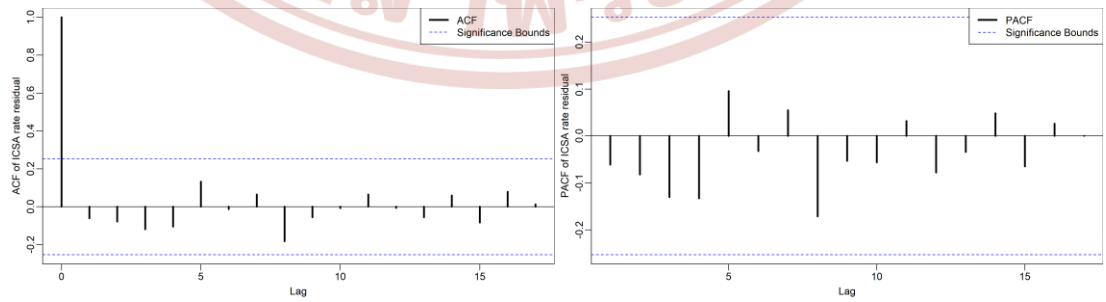


FIGURE 4-9 Autocorrelation analysis of residual ARIMA of ICSA

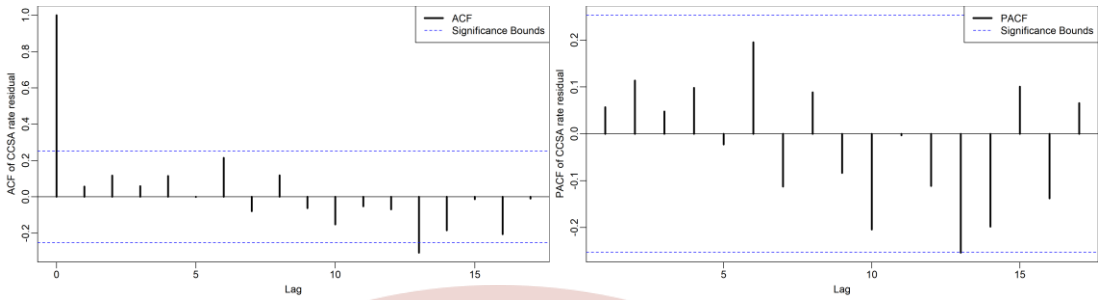


FIGURE 4-10 Autocorrelation analysis of residual ARIMA of CCSA

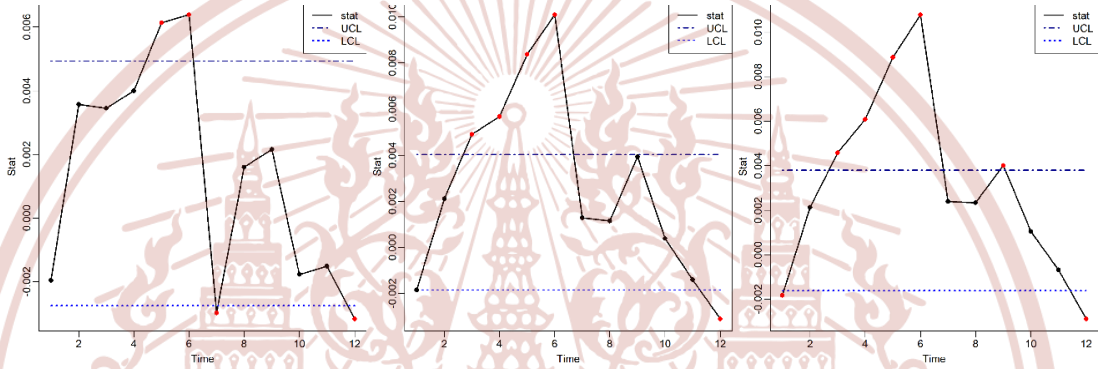


FIGURE 4-11 Detection of control charts for residual ARIMA of ICSA

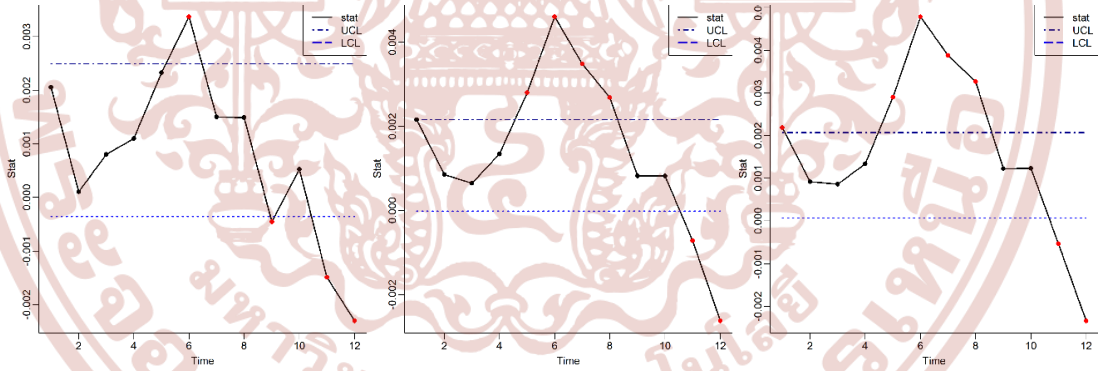


FIGURE 4-12 Detection of control charts for residual ARIMA of CCSA

4.3.4. Solar Energy

This section presents the application of the proposed control chart using the Multi-Region Solar Energy Dataset obtained from Kaggle. The dataset contains hourly solar power generation observations collected from January 1, 2024, to December 31, 2024, at Norway, Brazil, India, and Australia. The data were divided into two subsets: a training set comprising observations from January to March and a testing set comprising observations from April to December. The application procedure consists of two main stages: developing a regression model and constructing control charts based on the model residuals.

The training dataset was used to develop the regression model and set up the control charts. To reflect the natural relationship between solar irradiance and power generation, a regression model without an intercept was chosen, expressed as

$$Y = \beta X + \varepsilon$$

where Y is the AC power output and X is the solar irradiance. This model fits the physical reality of photovoltaic systems, where no electrical power is generated when there is no solar irradiance. Therefore, the regression line was forced to pass through the origin. The estimated regression coefficients and their goodness-of-fit measures are shown in Tables 4-70. For process monitoring, residuals from the regression model were used monitoring statistics. These residuals were calculated as

$$e_i = Y_i - \hat{Y}_i$$

where Y_i and $\hat{Y}_i$ are the observed and predicted AC power outputs, respectively. After fitting the model, the residuals were separated based on their signs. Positive residuals ($e_i \geq 0$) were analyzed directly, while negative residuals ($e_i < 0$) were converted to their absolute values. Analyzing these transformed residuals revealed that they followed a Weibull distribution. The estimated distribution parameters and goodness-of-fit test results are summarized in Tables 4-71. These transformed residuals were then used to build the EWMA, EEWMA, and NEEWMA control charts, with smoothing parameters $\lambda_1 = 0.1$, $\lambda_2 = 0.05$ and $\lambda_3 = \lambda_2 / 4$, respectively. The limit parameter for EWMA, EEWMA, and NEEWMA charts was determined as 2.4948, 14.6419, and 23.9200, respectively. Since the monitoring statistics are non-negative, the lower control limit was fixed at (LCL = 0), while the upper control limit was calculated following the proposed charting procedure.

The testing set was used to evaluate the monitoring performance of the proposed control charts; the regression model developed from the training dataset was applied to the testing dataset. The predicted AC power values were first obtained, and the corresponding residuals were then calculated following the same procedure described in the previous section. These residuals were subsequently transformed and monitored using the EWMA, EEWMA, and NEEWMA control charts. The monitoring results demonstrated that the proposed control charts were effective in detecting potential abnormal conditions in the solar power generation process. For the Norway dataset, an out-of-control signal at first time was detected on date 05/04/2024 at 2 PM by EEWMA. Likewise, abnormal signals were identified in the Brazil, India, and Australia datasets on 03/04/2024 at 3 PM by EWMA and EEWMA, 06/04/2024 at 12 PM by EEWMA, and 16/04/2024 at 10 AM by NEEWMA, respectively. The first detect timestamps are summarized in Tables 4-72 and Figures 4-9 to 4-12. These findings indicate that the proposed monitoring framework can successfully identify unusual variations in photovoltaic power generation systems and provide timely warnings of potential process abnormalities.

TABLE 4-70 Distributional tests of Regression residuals for Solar Energy in 4 Country.

Country	β	RMSE	R^2	Residual	S_e
Norway	45.0150	1.2530	0.9982	0.9711	0.7389
Brazil	64.3131	1.7328	0.9983	1.3155	1.0610
India	54.3332	1.5080	0.9983	1.1175	0.8774
Australia	A57.8044	1.6271	0.9982	1.2693	1.0082

TABLE 4-71 Distributional tests of Regression residuals for Solar Energy in 4 Country.

Country	Distribution	Parameter	KS test	-2LL	AIC	BIC
Norway	Gamma (α, β)	(1.28,0.76)	0.06	785.72	789.72	797.76
	Weibull (α, β)	(1.22,1.03)	0.38	776.60	780.60	788.64
	Gumbel (θ, β)	(0.64,0.58)	0.27	837.78	841.78	849.83
	TP Rayleigh (θ, β)	(-0.44,1.13)	0.02	848.97	841.78	849.83
Brazil	Gamma (α, β)	(1.29,1.02)	0.11	1441.15	1445.15	1453.86
	Weibull (α, β)	(1.20,1.40)	0.33	1436.11	1440.11	1448.81
	Gumbel (θ, β)	(0.84,0.83)	0.01	1556.86	1560.86	1569.56
	TP Rayleigh (θ, β)	(-0.72,1.62)	0.00	1589.89	1560.86	1569.56
India	Gamma (α, β)	(1.30,0.86)	0.14	1145.52	1149.52	1158.04
	Weibull (α, β)	(1.22,1.19)	0.57	1136.81	1140.81	1149.33
	Gumbel (θ, β)	(0.72,0.68)	0.07	1220.11	1224.11	1232.64
	TP Rayleigh (θ, β)	(-0.56,1.34)	0.00	1254.03	1224.11	1232.64
Australia	Gamma (α, β)	(1.25,1.02)	0.28	1461.39	1465.39	1474.17
	Weibull (α, β)	(1.19,1.34)	0.83	1452.04	1456.04	1464.82
	Gumbel (θ, β)	(0.82,0.79)	0.03	1554.21	1558.21	1566.99
	TP Rayleigh (θ, β)	(-0.66,1.54)	0.00	1592.03	1558.21	1566.99

TABLE 4-72 Detection of Control charts for Solar Energy in 4 Country.

Chart	EWMA		EEWMA		NEEWMA	
Timestamp	Date	Time	Date	Time	Date	Time
Norway	29/04/2024	1 PM	05/04/2024	2 PM	06/04/2024	1 PM
Brazil	03/04/2024	3 PM	03/04/2024	3 PM	05/04/2024	8 AM
India	09/04/2024	3 PM	06/04/2024	12 PM	06/04/2024	1 PM
Australia	16/05/2024	12 PM	16/04/2024	12 PM	16/04/2024	4 AM

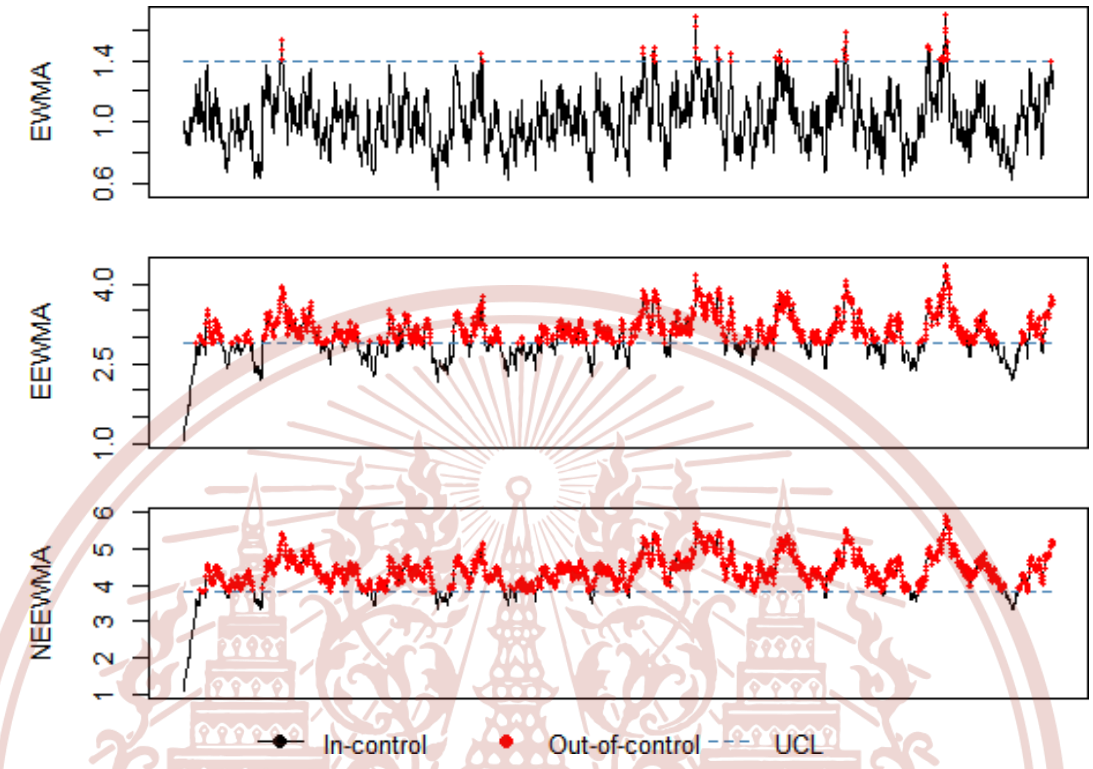


FIGURE 4-13 Detection of control charts for residual regression of solar energy in Norway

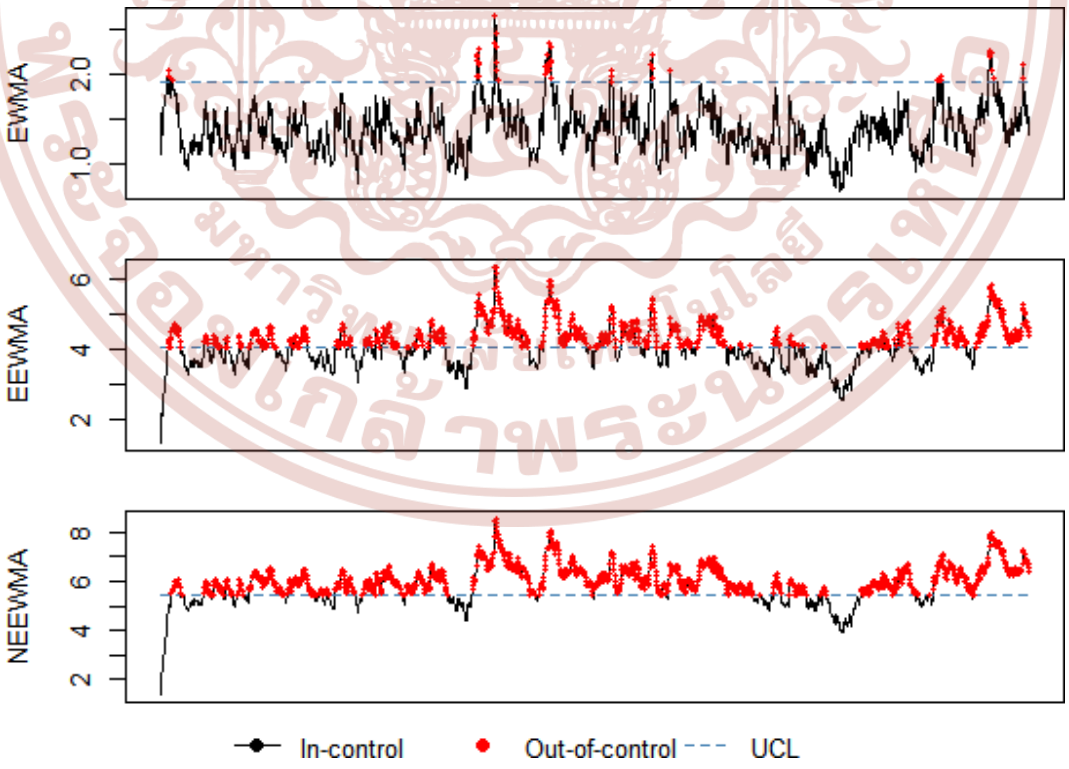


FIGURE 4-14 Detection of control charts for residual regression of solar energy in Brazil

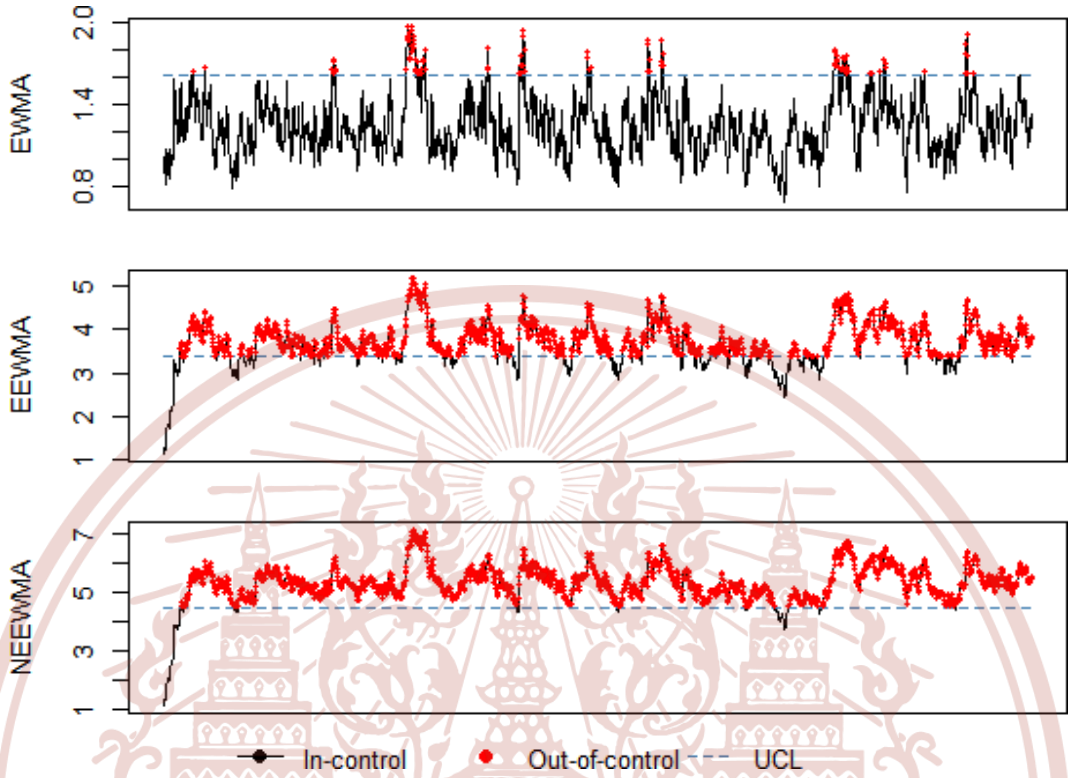


FIGURE 4-15 Detection of control charts for residual regression of solar energy in India

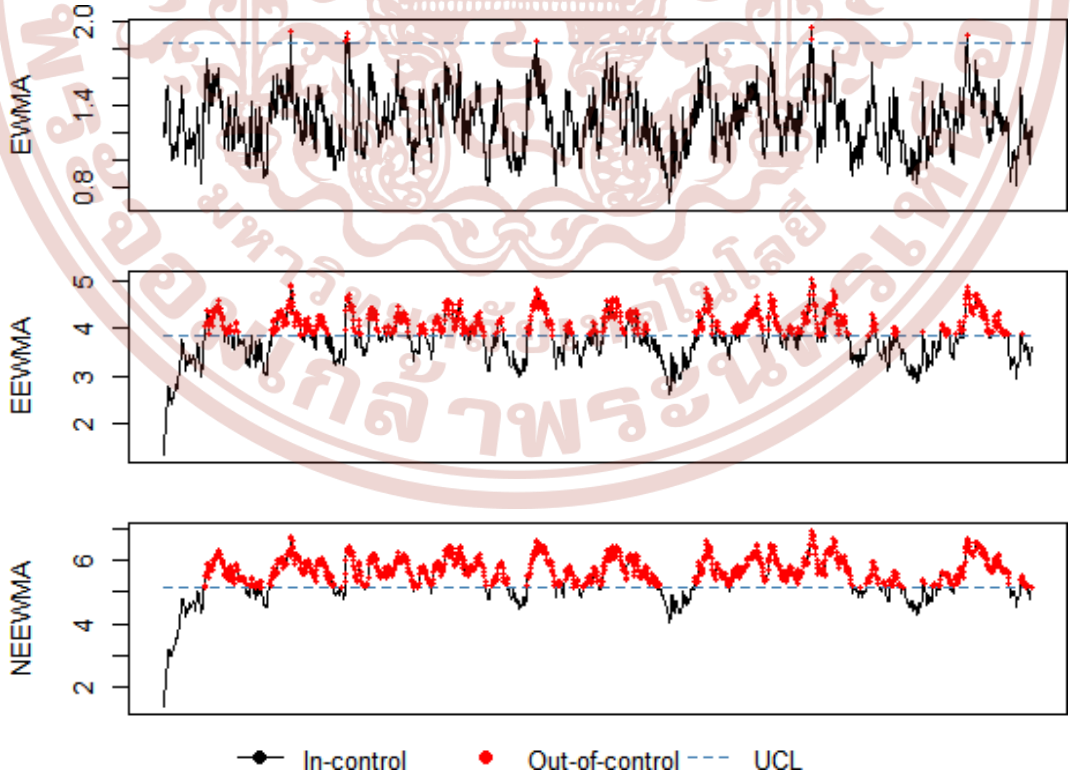


FIGURE 4-16 Detection of control charts for residual regression of solar energy in Australia

CHAPTER 5

Conclusion, Discussion, and Recommendations

This chapter summarizes the overall findings of the study concerning the estimation of the Average Run Length (ARL) of the MEWMA, EEWMA, and NEEWMA control charts by means of the Markov chain approach (MCA) method. The principal results reported in Chapter 4 are drawn together, encompassing the comparative performance of the Joint Distribution Approach (JDA) and the Point Estimator Approach (PEA) relative to Monte Carlo (MC) simulation in terms of both closeness of estimation and computational time efficiency, the behavior of the proposed method under symmetric, asymmetric, and discrete process distributions, and its application to real-world data. These findings are subsequently discussed in relation to the existing body of literature, and their theoretical and practical implications are considered, together with the limitations of the proposed approach. Recommendations for the practical implementation of the method and directions for future research are then offered. The chapter is organized into the following sections:

5.1 Conclusions
5.2 Discussions
5.3 Recommendations

The details pertaining to each section are presented in turn below.

5.1. Conclusions

This study sets out to construct closed-form expressions for the ARL of the MEWMA, EEWMA, and NEEWMA control charts by means of the Markov chain approach, and to establish their accuracy, their computational cost, and the behavior of the charts under a range of process distributions. Four conclusions follow.

First, closed-form expressions for the ARL and the SDRL were derived from the first and second moments of the run-length distribution, together with two approaches to the construction of the transition probability matrix: the Joint Distribution Approach for the MEWMA chart and the Point Estimator Approach, in its in-control and out-of-control variants, for all three charts.

Second, the ARL values obtained from the proposed method agree closely with those obtained by Monte Carlo simulation. Under the standard normal process examined in Section 4.1 the two methods never differ by as much as one run-length unit at any combination of chart parameters, process shift, and in-control ARL, and under the symmetric and asymmetric distributions examined in Section 4.2 the mean percentage error and the mean absolute percentage error remain below one per cent throughout. The agreement is closer still under the binomial distribution but is less close under the Poisson and negative binomial distributions, where the mean absolute percentage error reaches 4.22 per cent for the EWMA chart. The computational

advantage is substantial wherever the run lengths are long: the time required by the approximation is fixed by the number of subintervals alone and is insensitive to the chart parameters and to the in-control ARL, whereas the time required by simulation rises with both, so that the saving exceeds 95 per cent under the symmetric distributions and widens as the in-control ARL increases.

Third, the performance of the EEWMA and NEEWMA charts was established under three symmetric, four asymmetric, and three discrete process distributions. Under the symmetric distributions the location parameter was found to have no effect on chart performance while the scale parameter has an inverse effect, and all three charts detect shifts most readily under the normal distribution, followed by the Laplace and then the logistic. Under the asymmetric distributions the shape parameter of the gamma distribution improves detection for all three charts, whereas under the Weibull distribution an increase in the shape parameter degrades detection so severely that the out-of-control ARL exceeds the in-control value; within the location–scale family the location parameter leaves the EWMA chart entirely unaffected while the extended charts lose sensitivity as it rises. Under the discrete distributions an increase in the process parameter degrades detection for all three charts, and the ordinary EWMA chart performs best in the equidispersed and overdispersed, the extended charts retaining a slight advantage only in the underdispersed.

Fourth, the charts were applied to four real-world datasets drawn from industrial monitoring, macroeconomics, labor statistics, and renewable energy, in each case detecting changes that correspond to identifiable events in the underlying process.

5.2. Discussion

On the accuracy of the approximation and the direction of its error. The discrepancy between the two methods behaves differently across the three families of process distribution, and the difference is informative about its source. Under the symmetric and asymmetric distributions the mean percentage error is negative in almost every parameter combination and equal in magnitude to the mean absolute percentage error, which can occur only if the error carries the same sign at every shift level; the approximation therefore lies uniformly below the simulation rather than scattering about it. Under Poisson and negative binomial distributions, the mean percentage error is positive for most combinations and no longer equals the mean absolute percentage error, so that the error changes sign across shift levels and the approximation exceeds the simulation on balance. The reversal is consistent with the mechanism of the approximation. Partitioning the region between the control limits into subintervals and representing the monitoring statistic within each by a single point is a finer representation when the process is continuous than when its support is itself discrete and sparse relative to the width of the limits, which is the case for the Poisson and negative binomial distributions at the parameter values examined here. The binomial distribution with $p_0 = 0.01$ places its mass over a narrower range and does not encounter the difficulty, and it is accordingly the distribution under which the

closest agreement of the whole study was obtained. That the extended charts are approximated more accurately than the conventional one under the two problematic distributions supports the same explanation, since their monitoring statistics combine several observations and therefore take value on a finer grid than the observations themselves.

On computational efficiency. The cost structures of the two methods differ in kind, not merely in degree. The cost of the Markov chain approach is dominated by the construction and inversion of the transition probability matrix and is therefore fixed by the number of partitions alone: across the whole of Chapter 4 it remained between roughly 3.0 and 3.5 seconds per shift level, irrespective of the distribution, the chart, and the prescribed in-control ARL. The cost of Monte Carlo simulation is proportional to the expected run itself, since every simulated run must be iterated until a signal occurs, and it ranged from under 2 seconds to nearly 200 seconds per shift level over the same experiments. The advantage of the proposed method therefore widens as the in-control ARL increases and as the shift decreases, which is precisely the region in which an EWMA-type chart is designed. The corollary is that the advantage disappears where the run lengths are short: under the Poisson and negative binomial distributions the simulated run lengths are brief enough that simulation is the cheaper of the two methods at $ARL_0 = 200$ and 370. The proposed method is thus to be recommended for its speed where estimation is genuinely difficult, and for its exactness rather than its speed elsewhere.

On the behavior of the control charts. The effect of the first weighted parameter reproduces the classical result of Lucas and Saccucci (1990) for the ordinary EWMA chart: small values accumulate information over a longer history and detect small and moderate shifts more quickly, whereas large values weight the current observation more heavily and respond faster to large shifts. This is held under every distribution examined. The novel finding concerns the second and third weighted parameters, whose effect is not stable across distributional families. Under the symmetric distributions an increase in λ_2 and λ_3 improved the detection of small and moderate shifts at the cost of large ones, the gain exceeding the loss. Under the gamma, Gumbel, and two-parameter Rayleigh distributions the same increase reduced detection at every shift magnitude; under the Weibull distribution it improved detection at every shift magnitude; and under the discrete distributions it was harmful except in the underdispersed case, where an interior optimum appeared. The parameter recommendations given in the papers introducing these charts should therefore not be transferred across distributional families without re-evaluation, and the practitioner who selects λ_2 and λ_3 by a rule calibrated on normal data may obtain a chart that is worse than the conventional one.

On the discrete case. Under the Poisson and negative binomial distributions, the ordinary EWMA chart outperformed both the EEWMA and the NEEWMA chart at every level of the process parameter. This result runs counter to the claims made in the

papers that introduced the extended statistics (Naveed et al., 2018; Javed et al., 2024), which report uniform superiority over the EWMA chart, and it deserves comment rather than omission. The control limits reported in Section 4.2.3 offer an explanation: at $\lambda_1 = 0.1$ the limits required by the extended charts to attain the prescribed in-control ARL are several times wider than those of the EWMA chart, so that a shift must displace the monitoring statistic considerably further before a signal is produced. The lagged terms that extend the memory of the statistics also inflate their variance, and where the observations are counts taking few distinct values that inflation is not repaid by any gain in information. The binomial distribution, in which the variance of the process is smaller than its means, is the case in which the inflation is least costly, and it is there alone that the extended charts retained an advantage — a small one. The finding indicates that the choice among EWMA-type charts should be conditioned on the nature of the quality characteristic, and that the extended charts are best regarded as instruments for continuous processes.

On the real applications. The four applications examined bear out the numerical findings in part. In the GDP growth series of Australia, Japan, Singapore, and Thailand, the NEEWMA chart signaled the 2020 structural break in every country, while the EWMA and EEWMA charts failed to signal at all in three of the four — consistent with the superiority of the extended statistics for small and moderate shifts in approximately normal processes. The industrial sensor application demonstrated that the MEWMA formulation extends naturally to the simultaneous monitoring of sixteen residual series, the chart signaling at observation 25. In the solar energy application, where the transformed regression residuals followed a Weibull distribution, the EEWMA chart signaled earliest in three of the four countries. This last result should be read with care: Section 4.2.2.3 shows that detection under a Weibull process deteriorates for all three charts as the shape parameter departs from unity, so the advantage of the EEWMA chart here is best attributed to the effect of the second weighted parameter reported in Section 4.2.2.2, under which increasing λ_2 and λ_3 improved detection at every shift magnitude — the only distribution of the four for which this was so.

Limitations. Five limitations should be acknowledged. First, the Markov chain approach is an approximation: its accuracy depends on the number of partitions, and $n = 1{,}000$ was adopted throughout without a formal study of the convergence of the ARL as n increases. Second, its accuracy is not uniform across process distributions, being appreciably lower under the Poisson and negative binomial distributions than elsewhere, and no correction for the discreteness of the support was attempted. Third, the Point Estimator Approach substitutes the mean for the lagged observations, and its accuracy may be expected to degrade as the process moves further from the in-control state; the comparison between the in-control and out-of-control variants in Sections 4.1.2 and 4.1.3 addresses this only partially. Fourth, the process parameters were treated as known throughout, whereas in practice they are estimated from a Phase I

sample, and the resulting estimation error is not reflected in the reported ARL values. Fifth, the study is confined to the univariate monitoring of a mean shift with subgroup size one and to processes of independent observations; the industrial sensor application required a sinusoidal model and the unemployment claims application an ARIMA (3,1,0) model to remove structure before monitoring could proceed.

5.3. Recommendations

For practitioners. Where the quality characteristic is continuous and the shift of interest is small, the NEEWMA chart with a small λ_1 is recommended, with λ_2 and λ_3 chosen according to the distributional family: larger values under symmetric processes and under the Weibull distribution, smaller values under the gamma, Gumbel, and two-parameter Rayleigh distributions. Where the quality characteristic is a count, the ordinary EWMA chart is recommended in the **equidispersed** and **overdispersed**, since the extended statistics offer no advantage there; in the **underdispersed** the extended charts may be used, but their parameters should be selected by search over the range rather than by a monotone rule, since the best value of λ_1 and λ_2 lies in the interior. For the MEWMA chart, the difference parameter should be set small, as k = 0.1 dominated the larger values at every shift magnitude. Where the process follows a Weibull distribution whose shape parameter departs appreciably from unity, none of the three charts should be relied upon, as the out-of-control ARL then exceeds the in-control value. Where the process exhibits trend, seasonality, or autocorrelation, the charts should be applied to the residuals of a fitted model rather than to the raw observations, as was done in the sensor, unemployment claims, and solar energy applications.

For future research. Five directions suggest themselves. The convergence of the approximation as the number of partitions increases should be studied formally, so that a value of n may be recommended on theoretical rather than empirical grounds. A variant of the transition probability matrix adapted to a discrete support should be developed, which would address the loss of accuracy observed under the Poisson and negative binomial distributions. The Joint Distribution Approach should be extended to the triradiate case required by the NEEWMA statistic, which would remove the need for the point estimator substitution. The effect of parameter estimation from a Phase I sample on the ARL of the extended charts should be quantified. Finally, the framework developed here applies to any monitoring statistic expressible as a linear combination of finitely many lagged observations, and its extension to the CUSUM, GWMA, and homogeneously weighted moving average families, and to the monitoring of dispersion rather than location, would be a natural continuation.

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Appendix A

Appendix A
Table of Results with $ARL_0 = 200$

TABLE A-1 ARL of the MEWMA control chart using JDA compared to MC under a standard normal process, with $ARL_0 = 200$ and $k = 0.1$

δ	$\lambda_1 = 0.1$			$\lambda_1 = 0.7$		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	4.9402 (5.94)	4.9262±1.5131 (0.22)	0.0140	1.0258 (6.08)	1.0338±1.0482 (0.11)	0.0080
-2.00	8.8987 (5.9)	8.8317±2.2305 (0.36)	0.0670	3.4725 (5.99)	3.4961±1.8337 (0.18)	0.0236
-1.00	25.9417 (5.86)	25.0844±4.4515 (1)	0.8573	21.1885 (5.96)	21.2993±4.6234 (0.84)	0.1108
-0.90	30.5948 (6)	30.6508±4.8643 (1.22)	0.0560	26.8098 (6.02)	26.6919±5.1193 (1.04)	0.1179
-0.70	44.8622 (5.92)	44.2435±6.0911 (1.68)	0.6187	44.3798 (6.08)	44.5703±6.6317 (1.74)	0.1905
-0.50	71.2859 (5.98)	71.2588±8.0034 (2.72)	0.0271	76.1207 (6.03)	76.8763±8.6573 (2.86)	0.7556
-0.30	120.5506 (5.86)	120.867±10.6118 (4.41)	0.3164	129.402 (5.96)	129.198±11.2981 (4.82)	0.2040
-0.10	186.2965 (5.94)	186.6893±13.5066 (6.95)	0.3928	189.0085 (5.91)	189.7362±13.6353 (7)	0.7277
-0.07	193.0514 (5.89)	193.7468±13.5267 (7)	0.6954	194.4927 (5.89)	194.7797±13.8459 (7.05)	0.2870
-0.05	196.4041 (6.05)	196.8276±14.0457 (7.44)	0.4235	197.1752 (5.92)	197.7261±14.0476 (7.23)	0.5509
-0.03	198.7048 (6.06)	198.6471±13.8789 (7.34)	0.0577	199.0012 (5.9)	199.0003±14.1187 (7.29)	0.0009
-0.01	199.8749 (6.05)	199.3673±13.8698 (7.5)	0.5076	199.9258 (5.92)	199.4939±14.1404 (7.26)	0.4319
0.00	200.0215 (6.13)	200.6599±13.9763 (7.45)	0.6384	200.0419 (6)	200.2466±14.1629 (7.36)	0.2047
0.01	199.8733 (6.14)	199.8837±13.9603 (7.53)	0.0104	199.9257 (5.96)	199.2414±14.0635 (7.23)	0.6843
0.03	198.7 (6.06)	198.8863±13.8998 (7.44)	0.1863	199.0008 (5.86)	199.7576±14.1081 (7.11)	0.7568
0.05	196.3962 (6)	196.8783±13.9092 (7.11)	0.4821	197.1744 (5.97)	197.6157±13.8708 (7.08)	0.4413
0.07	193.0406 (5.87)	193.102±13.6279 (7)	0.0614	194.4917 (5.98)	194.7602±13.9664 (7.11)	0.2685
0.10	186.2815 (5.92)	186.5875±13.2033 (6.71)	0.3060	189.007 (6)	189.3256±13.7766 (7.08)	0.3186
0.30	120.5215 (5.89)	120.2526±10.7613 (4.57)	0.2689	129.3987 (5.92)	129.0468±11.3723 (4.83)	0.3519
0.50	71.2573 (6.05)	71.431±7.8593 (2.69)	0.1737	76.1171 (5.97)	76.1933±8.6887 (2.88)	0.0762
0.70	44.8372 (5.91)	44.8309±6.1677 (1.7)	0.0063	44.3765 (5.97)	44.4321±6.6852 (1.67)	0.0556
0.90	30.5734 (5.92)	30.3338±4.8854 (1.18)	0.2396	26.8068 (5.9)	26.9126±5.2032 (1.04)	0.1058
1.00	25.9218 (5.86)	25.1293±4.4923 (1.02)	0.7925	21.1856 (5.94)	21.0605±4.5705 (0.81)	0.1251
2.00	8.8874 (5.89)	8.8794±2.2171 (0.38)	0.0080	3.4707 (5.92)	3.4937±1.8369 (0.18)	0.0230
3.00	4.9324 (5.89)	4.9281±1.4935 (0.23)	0.0043	1.0247 (5.95)	1.0274±1.0515 (0.09)	0.0027

TABLE A-2 ARL of the MEWMA control chart using JDA compared to MC under a standard normal process, with $ARL_0 = 200$ and $k = 0.7$

δ	$\lambda_1 = 0.1$			$\lambda_1 = 0.7$		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	52.137 (6.15)	52.2634±6.7313 (1.99)	0.1264	7.0919 (5.88)	7.9325±2.5808 (0.3)	0.8406
-2.00	87.4583 (6.08)	87.4252±8.8609 (3.34)	0.0331	19.7793 (5.96)	19.6559±4.4146 (0.77)	0.1234
-1.00	150.9237 (6.08)	150.8566±12.0543 (5.64)	0.0671	73.0326 (5.92)	73.6984±8.499 (2.77)	0.6658
-0.90	158.2853 (6.06)	158.3177±12.1764 (5.81)	0.0324	84.0928 (5.92)	84.1037±9.114 (3.16)	0.0109
-0.70	172.479 (5.97)	172.9949±12.922 (6.3)	0.5159	110.9262 (5.83)	110.1682±10.4311 (4.01)	0.7580
-0.50	184.9394 (5.91)	184.8563±13.3506 (6.84)	0.0831	143.0891 (5.85)	143.0166±11.9665 (5.36)	0.0725
-0.30	194.3088 (5.95)	194.2198±13.7778 (7.22)	0.0890	175.4012 (6.03)	175.0289±13.3872 (6.67)	0.3723
-0.10	199.3611 (5.97)	199.7686±13.9469 (7.38)	0.4075	196.9765 (5.95)	196.3889±13.9519 (7.28)	0.5876
-0.07	199.692 (5.91)	199.4731±13.8575 (7.26)	0.2189	198.516 (6.11)	198.94±14.043 (7.26)	0.4240
-0.05	199.8479 (5.92)	199.0618±13.8725 (7.37)	0.7861	199.2481 (5.92)	199.2608±14.2713 (7.57)	0.0127
-0.03	199.9519 (5.92)	199.5427±14.0375 (7.46)	0.4092	199.7389 (5.99)	199.4523±14.0455 (7.33)	0.2866
-0.01	200.0038 (5.94)	200.7709±13.7633 (7.09)	0.7671	199.985 (5.89)	199.735±14.2301 (7.22)	0.2500
0.00	200.0101 (5.98)	200.4174±13.8028 (7.27)	0.4073	200.0158 (5.99)	200.0282±14.0614 (7.4)	0.0124
0.01	200.0034 (6.09)	200.2754±13.839 (7.42)	0.2720	199.985 (6.05)	199.5721±14.0014 (7.4)	0.4129
0.03	199.9509 (6.06)	199.0618±14.1066 (7.6)	0.8891	199.7386 (5.92)	199.5921±14.1973 (7.2)	0.1465
0.05	199.8462 (6.05)	199.3853±14.0528 (7.53)	0.4609	199.2477 (5.86)	199.5313±14.0958 (7.12)	0.2836
0.07	199.6896 (6.12)	199.1401±13.9074 (7.46)	0.5495	198.5154 (5.86)	198.8044±14.1322 (7.22)	0.2890
0.10	199.3578 (5.98)	199.3284±13.9796 (7.36)	0.0294	196.9757 (5.89)	196.2222±14.012 (7.18)	0.7535
0.30	194.299 (5.94)	194.2931±13.8385 (7.14)	0.0059	175.399 (5.89)	175.4623±13.2637 (6.31)	0.0633
0.50	184.9238 (5.87)	184.101±13.3256 (6.78)	0.8228	143.0861 (5.92)	143.0377±12.0016 (5.31)	0.0484
0.70	172.4586 (5.97)	172.1477±12.7867 (6.31)	0.3109	110.9227 (5.94)	110.4285±10.4689 (4.06)	0.4942
0.90	158.2612 (6.02)	158.1955±12.2521 (5.86)	0.0657	84.0892 (5.97)	84.1018±9.2906 (3.19)	0.0126
1.00	150.8981 (5.98)	150.0846±12.0002 (5.59)	0.8135	73.0291 (6)	73.939±8.5808 (2.73)	0.9099
2.00	87.4287 (5.91)	87.4538±8.9471 (3.33)	0.0251	19.7765 (6.05)	19.8059±4.407 (0.8)	0.0294
3.00	52.1108 (5.95)	52.1635±6.6634 (2.03)	0.0527	7.0898 (6.04)	7.0526±2.645 (0.3)	0.0372

TABLE A-3 ARL of the MEWMA control chart using JDA compared to MC under a standard normal process, with $ARL_0 = 200$ and $k = 1.0$

δ	$\lambda_1 = 0.1$			$\lambda_1 = 0.7$		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	80.7456 (5.94)	80.2645±8.6869 (2.98)	0.4811	12.9628 (6.03)	12.9864±3.5723 (0.58)	0.0236
-2.00	120.2832 (5.84)	120.6516±10.7797 (4.46)	0.3684	32.8255 (6.05)	32.0111±5.7001 (1.25)	0.8144
-1.00	171.4546 (5.9)	171.8593±12.9058 (6.39)	0.4047	97.5834 (6.07)	97.007±9.9122 (3.66)	0.5764
-0.90	176.2295 (5.97)	176.3465±13.0177 (6.46)	0.1170	108.809 (5.94)	108.6457±10.4712 (4.06)	0.1633
-0.70	184.9097 (5.99)	184.1807±13.4545 (6.82)	0.7290	133.8132 (6.01)	133.6007±11.44 (4.85)	0.2125
-0.50	192.0089 (5.97)	192.8172±13.6874 (7.08)	0.8083	160.3124 (5.89)	160.6198±12.643 (5.81)	0.3074
-0.30	197.055 (6.12)	197.5548±13.7869 (7.4)	0.4998	183.8354 (5.85)	183.3301±13.548 (6.66)	0.5053
-0.10	199.6788 (6.06)	199.8177±13.9846 (7.53)	0.1389	198.0895 (5.91)	198.0373±14.1275 (7.22)	0.0522
-0.07	199.8482 (6.17)	199.2631±13.8985 (7.5)	0.5851	199.0666 (5.82)	199.6219±14.057 (7.21)	0.5553
-0.05	199.928 (6.08)	199.3376±14.0093 (7.55)	0.5904	199.5294 (5.97)	199.6832±14.2006 (7.27)	0.1538
-0.03	199.9811 (6.03)	199.8896±13.8615 (7.35)	0.0915	199.839 (6)	199.9501±14.2481 (7.42)	0.1111
-0.01	200.0076 (5.94)	200.1258±13.9465 (7.38)	0.1182	199.9942 (5.95)	199.1311±14.1346 (7.41)	0.8631
0.00	200.0108 (5.9)	200.855±13.94 (7.44)	0.8442	200.0135 (6.03)	200.1236±14.2073 (7.45)	0.1101
0.01	200.0073 (6.03)	200.7987±13.9639 (7.38)	0.7914	199.9941 (5.92)	199.821±14.2087 (7.36)	0.1731
0.03	199.9804 (5.91)	199.9281±13.9239 (7.33)	0.0523	199.8389 (6)	199.085±14.2002 (7.47)	0.7539
0.05	199.9268 (5.97)	199.0014±13.7476 (7.22)	0.9254	199.5291 (6.03)	199.0122±14.0927 (7.28)	0.5169
0.07	199.8465 (5.87)	199.1507±13.9622 (7.27)	0.6958	199.0661 (5.87)	199.0446±14.044 (7.22)	0.0215
0.10	199.6764 (6)	199.0418±14.0275 (7.41)	0.6346	198.0888 (5.97)	198.6908±14.198 (7.27)	0.6020
0.30	197.0479 (5.94)	197.8309±13.9051 (7.22)	0.7830	183.8336 (5.89)	183.253±13.5333 (6.74)	0.5806
0.50	191.9973 (6.1)	191.1397±13.6141 (7.17)	0.8576	160.3097 (5.95)	160.7262±12.6221 (5.91)	0.4165
0.70	184.8941 (6.09)	184.9014±13.3737 (6.83)	0.0073	133.81 (5.94)	133.6535±11.4772 (4.96)	0.1565
0.90	176.2103 (6.03)	176.4871±13.1744 (6.69)	0.2768	108.8056 (5.99)	108.0009±10.4931 (4.04)	0.8047
1.00	171.4339 (6.14)	171.2123±12.9303 (6.58)	0.2216	97.5798 (6.05)	97.5698±9.8892 (3.67)	0.0100
2.00	120.2541 (6.17)	120.7466±10.7738 (4.64)	0.4925	32.8223 (5.96)	32.6984±5.6448 (1.23)	0.1239
3.00	80.7164 (6.09)	80.2006±8.531 (3.06)	0.5158	12.9603 (5.92)	12.0356±3.5561 (0.52)	0.9247

TABLE A-4 ARL of the MEWMA control chart using PEA_1 compared to MC under a standard normal process, with $ARL_0 = 200$ and $k = 0.1$

δ	$\lambda_1 = 0.1$			$\lambda_1 = 0.7$		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	4.2148 (6.64)	4.2381±1.3617 (0.59)	0.0233	0.9997 (6.75)	0.9978±1.0416 (0.22)	0.0019
-2.00	7.549 (6.69)	7.5991±1.9919 (0.97)	0.0501	3.3966 (6.77)	3.4506±1.8187 (0.5)	0.0540
-1.00	21.7624 (6.61)	21.9071±3.9624 (2.44)	0.1447	20.8111 (6.76)	20.5984±4.5142 (2.31)	0.2127
-0.90	25.6885 (6.62)	25.6169±4.3849 (2.77)	0.0716	26.3581 (6.75)	26.1879±5.0516 (2.88)	0.1702
-0.70	37.9472 (6.62)	37.4931±5.536 (3.96)	0.4541	43.7437 (6.67)	43.0453±6.6358 (4.73)	0.6984
-0.50	61.6512 (6.64)	61.4345±7.3642 (6.28)	0.2167	75.3006 (6.77)	75.305±8.6991 (7.75)	0.0044
-0.30	109.7402 (6.65)	109.2396±10.1118 (10.91)	0.5006	128.6374 (6.66)	128.4519±11.34 (12.65)	0.1855
-0.10	183.1377 (6.64)	183.8266±13.2919 (17.75)	0.6889	188.806 (6.77)	188.4797±13.8142 (18.4)	0.3263
-0.07	191.3641 (6.56)	191.3782±13.6448 (18.58)	0.0141	194.3679 (6.76)	194.4695±13.7205 (18.83)	0.1016
-0.05	195.499 (6.57)	195.3257±13.676 (18.8)	0.1733	197.0899 (6.78)	197.6006±13.937 (18.86)	0.5107
-0.03	198.3568 (6.63)	198.6253±13.9433 (19.31)	0.2685	198.9435 (6.69)	198.9261±14.0627 (19.23)	0.0174
-0.01	199.8167 (6.8)	199.8721±13.8315 (19.12)	0.0554	199.8823 (6.72)	199.3285±14.0419 (18.94)	0.5538
0.00	200.0001 (6.64)	200.1667±13.8763 (19.47)	0.1666	200.0001 (6.72)	200.8086±14.1463 (19.59)	0.8085
0.01	199.8149 (6.66)	199.0097±13.9304 (19.42)	0.8052	199.8821 (6.83)	199.4038±14.2851 (19.72)	0.4783
0.03	198.3515 (6.59)	198.511±13.8866 (18.77)	0.1595	198.9431 (6.72)	198.5163±14.1644 (19.33)	0.4268
0.05	195.4903 (6.62)	195.9654±13.8054 (19.08)	0.4751	197.0891 (6.71)	197.9751±14.3017 (19.55)	0.8860
0.07	191.3521 (6.66)	191.5399±13.6162 (18.64)	0.1878	194.3668 (6.8)	194.7271±13.9701 (18.72)	0.3603
0.10	183.1212 (6.63)	183.8763±13.3927 (18.16)	0.7551	188.8045 (6.78)	188.9401±13.7535 (18.47)	0.1356
0.30	109.7106 (6.61)	109.3026±10.1665 (10.96)	0.4080	128.6342 (6.69)	128.1087±11.375 (12.83)	0.5255
0.50	61.6236 (6.6)	61.2955±7.2672 (6.28)	0.3281	75.2971 (6.7)	75.8053±8.6452 (7.75)	0.5082
0.70	37.9238 (6.56)	37.0127±5.6088 (4.03)	0.9111	43.7404 (6.72)	43.3962±6.5793 (4.65)	0.3442
0.90	25.6687 (6.66)	25.7686±4.416 (2.81)	0.0999	26.3551 (6.72)	26.3715±5.0585 (2.93)	0.0164
1.00	21.744 (6.58)	21.5212±3.9486 (2.34)	0.2228	20.8083 (6.7)	20.7643±4.4617 (2.3)	0.0440
2.00	7.5388 (6.66)	7.5154±1.9744 (0.92)	0.0234	3.3949 (6.65)	3.3956±1.8033 (0.49)	0.0007
3.00	4.2078 (6.72)	4.2188±1.3626 (0.59)	0.0110	0.9986 (6.81)	0.9947±1.0367 (0.22)	0.0039

TABLE A-5 ARL of the MEWMA control chart using PEA_1 compared to MC under a standard normal process, with $ARL_0 = 200$ and $k = 0.7$

δ	$\lambda_1 = 0.1$			$\lambda_1 = 0.7$		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	34.1405 (6.58)	34.9276±5.2674 (3.59)	0.7871	5.2304 (6.7)	5.316±2.2759 (0.69)	0.0856
-2.00	61.6512 (6.67)	61.4165±7.3639 (6.3)	0.2347	15.146 (6.61)	15.1626±3.8335 (1.73)	0.0166
-1.00	127.1654 (6.67)	127.5439±11.1009 (12.53)	0.3785	61.8558 (6.66)	61.9747±7.886 (6.48)	0.1189
-0.90	136.5685 (6.66)	136.3965±11.28 (13.5)	0.1720	72.389 (6.72)	72.455±8.6038 (7.55)	0.0660
-0.70	156.0674 (6.66)	156.6699±12.1959 (15.42)	0.6025	99.0739 (6.75)	99.0773±10.0695 (10.23)	0.0034
-0.50	174.8549 (6.64)	174.9319±12.8896 (16.83)	0.0770	133.228 (6.69)	133.4216±11.5664 (13.22)	0.1936
-0.30	190.1496 (6.61)	190.061±13.5514 (18.35)	0.0886	170.0796 (6.72)	170.9661±13.2356 (17)	0.8865
-0.10	198.8565 (6.72)	198.8495±13.9799 (19.44)	0.0070	196.2148 (6.81)	196.6669±13.9658 (18.72)	0.4521
-0.07	199.4386 (6.56)	199.6899±13.9748 (19.39)	0.2513	198.129 (6.75)	198.838±14.2203 (19.44)	0.7090
-0.05	199.7136 (6.61)	199.701±13.9009 (19.08)	0.0126	199.0417 (6.65)	199.6857±14.274 (19.39)	0.6440
-0.03	199.8971 (6.65)	199.5269±13.9756 (19.52)	0.3702	199.6544 (6.77)	199.1234±14.0966 (19.16)	0.5310
-0.01	199.9889 (6.64)	199.7723±14.0303 (19.5)	0.2166	199.962 (6.7)	199.1066±13.96 (19.11)	0.8554
0.00	200.0002 (6.58)	200.3271±13.9625 (19.28)	0.3269	200.0005 (6.62)	200.6861±14.2632 (19.69)	0.6856
0.01	199.9884 (6.64)	199.5201±14.0961 (19.33)	0.4683	199.9619 (6.69)	199.2016±14.1763 (19.5)	0.7603
0.03	199.8958 (6.59)	199.2836±13.9594 (19.44)	0.6122	199.6541 (6.7)	199.8597±14.0296 (19.36)	0.2056
0.05	199.7113 (6.64)	199.1823±14.0347 (19.56)	0.5290	199.0413 (6.66)	199.9097±14.0335 (19.12)	0.8684
0.07	199.4354 (6.61)	199.7844±13.7852 (19.16)	0.3490	198.1284 (6.78)	198.0196±14.0336 (19.07)	0.1088
0.10	198.852 (6.54)	198.6375±14.0413 (19.22)	0.2145	196.2139 (6.74)	196.897±14.2025 (19.22)	0.6831
0.30	190.1368 (6.6)	190.7333±13.4664 (18.15)	0.5965	170.0772 (6.7)	170.6252±13.076 (16.86)	0.5480
0.50	174.8352 (6.66)	174.7593±12.9755 (17.08)	0.0759	133.2248 (6.73)	133.5674±11.6059 (13.25)	0.3426
0.70	156.0429 (6.62)	156.9493±12.1822 (15.02)	0.9064	99.0704 (6.68)	99.0096±9.9164 (10.11)	0.0608
0.90	136.5408 (6.57)	136.2944±11.4378 (13.66)	0.2464	72.3855 (6.75)	72.049±8.5201 (7.56)	0.3365
1.00	127.1368 (6.64)	127.1859±11.1243 (12.67)	0.0491	61.8523 (6.72)	61.5126±7.8026 (6.56)	0.3397
2.00	61.6236 (6.63)	61.8513±7.437 (6.31)	0.2277	15.1434 (6.67)	15.274±3.8747 (1.74)	0.1306
3.00	34.1181 (6.69)	34.7517±5.1892 (3.58)	0.6336	5.2285 (6.68)	5.1632±2.2312 (0.69)	0.0653

TABLE A-6 ARL of the MEWMA control chart using PEA_1 compared to MC under a standard normal process, with $ARL_0 = 200$ and $k = 1.0$

δ	$\lambda_1 = 0.1$			$\lambda_1 = 0.7$		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	54.7132 (6.65)	54.8414±6.9661 (5.64)	0.1282	8.8387 (6.63)	8.8316±2.9162 (1.07)	0.0071
-2.00	90.7841 (6.69)	90.6294±9.2951 (9.27)	0.1547	23.881 (6.72)	23.196±4.859 (2.63)	0.6850
-1.00	153.3986 (6.65)	153.671±12.1508 (14.94)	0.2724	81.6675 (6.76)	81.0707±8.9629 (8.21)	0.5968
-0.90	160.4865 (6.64)	160.2331±12.3975 (15.81)	0.2534	92.9341 (6.78)	92.5058±9.5681 (9.4)	0.4283
-0.70	174.053 (6.66)	174.4407±12.9104 (17.08)	0.3877	119.4235 (6.69)	119.5949±10.7914 (11.58)	0.1714
-0.50	185.856 (6.65)	185.6133±13.3519 (18.1)	0.2427	149.7402 (6.78)	149.6089±12.2557 (14.8)	0.1313
-0.30	194.6662 (6.62)	194.2659±13.7977 (18.82)	0.4003	178.7706 (6.78)	178.5454±13.379 (17.5)	0.2252
-0.10	199.3941 (6.62)	199.367±13.7774 (19.16)	0.0271	197.4186 (6.67)	197.5034±14.0454 (19.28)	0.0848
-0.07	199.7031 (6.58)	199.9565±13.9394 (19.63)	0.2534	198.7277 (6.71)	198.378±14.0571 (19.07)	0.3497
-0.05	199.8487 (6.61)	199.8386±13.8699 (19.12)	0.0101	199.3492 (6.72)	199.096±14.2163 (19.54)	0.2532
-0.03	199.9458 (6.6)	199.6943±13.8093 (19)	0.2515	199.7656 (6.78)	199.8908±13.9627 (18.94)	0.1252
-0.01	199.9943 (6.61)	199.5576±13.8508 (19.15)	0.4367	199.9743 (6.7)	199.0071±14.182 (19.52)	0.9672
0.00	200.0002 (6.6)	200.301±13.9498 (19.28)	0.3008	200.0004 (6.78)	200.8197±14.2801 (19.91)	0.8193
0.01	199.9939 (6.64)	199.899±13.7957 (19.03)	0.0949	199.9742 (6.79)	199.1209±14.057 (19.4)	0.8533
0.03	199.9449 (6.64)	199.3872±13.9138 (19.28)	0.5577	199.7653 (6.76)	199.9785±13.9702 (19.16)	0.2132
0.05	199.8471 (6.61)	199.7518±13.8544 (19.12)	0.0953	199.3489 (6.73)	199.8381±14.1233 (19.13)	0.4892
0.07	199.7008 (6.6)	199.484±13.855 (19.11)	0.2168	198.7272 (6.76)	198.2319±14.0294 (19.1)	0.4953
0.10	199.3908 (6.67)	199.9834±13.8487 (19.16)	0.5926	197.4178 (6.78)	197.1468±14.0863 (19.31)	0.2710
0.30	194.6567 (6.67)	194.3528±13.85 (19.09)	0.3039	178.7685 (6.74)	178.6035±13.4454 (17.67)	0.1650
0.50	185.8408 (6.63)	185.5998±13.6735 (18.51)	0.2410	149.7372 (6.77)	149.3755±12.2202 (14.71)	0.3617
0.70	174.0331 (6.66)	174.6939±12.7234 (16.7)	0.6608	119.4201 (6.77)	119.0045±10.8973 (11.84)	0.4156
0.90	160.4629 (6.64)	160.8059±12.4264 (15.41)	0.3430	92.9305 (6.78)	92.3837±9.5615 (9.41)	0.5468
1.00	153.3736 (6.66)	153.5541±12.1252 (14.97)	0.1805	81.664 (6.75)	81.8681±9.2355 (8.39)	0.2041
2.00	90.7545 (6.66)	90.6841±9.1845 (9.21)	0.0704	23.8781 (6.75)	23.0639±4.8987 (2.61)	0.8142
3.00	54.6865 (6.61)	54.2731±6.8461 (5.64)	0.4134	8.8365 (6.74)	8.8514±2.9402 (1.06)	0.0149

TABLE A-7 ARL of the EEWMA and NEEWMA control chart using PEA_0 compared to MC under a normal process, with $ARL_0 = 200$ and $\lambda_1 = 0.1$

δ	*EEWMA (0.01)*			*NEEWMA (0.01,0.0025)*		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	1.6045 (6.81)	1.6021±0.7867 (0.2)	0.0024	1.6232 (6.67)	1.6201±0.7889 (0.19)	0.0031
-2.00	2.892 (6.81)	2.8951±1.0603 (0.35)	0.0031	2.9173 (6.69)	2.9096±1.0617 (0.33)	0.0077
-1.00	7.6462 (6.72)	7.6552±1.9709 (0.7)	0.009	7.6745 (6.62)	7.6583±1.9616 (0.67)	0.0162
-0.90	12.8791 (6.73)	12.9512±2.7975 (1.1)	0.0721	12.8845 (6.65)	12.8637±2.7872 (1)	0.0208
-0.70	21.4863 (6.81)	21.6027±3.8954 (1.7)	0.1164	21.4248 (6.69)	21.316±3.8412 (1.53)	0.1088
-0.50	46.5496 (6.72)	46.4656±6.2561 (3.42)	0.084	46.2557 (6.73)	46.9545±6.1795 (3.29)	0.6988
-0.30	144.2877 (6.75)	144.1993±11.767 (9.72)	0.0884	143.77 (6.67)	143.9621±11.6573 (9.79)	0.1921
-0.10	168.0964 (6.72)	168.2432±12.6759 (11.16)	0.1468	167.7403 (6.66)	167.4561±12.7948 (11.47)	0.2842
-0.07	182.3211 (6.76)	182.9245±13.3253 (12.17)	0.6034	182.1039 (6.76)	182.5634±13.4309 (12.41)	0.4595
-0.05	193.2497 (6.91)	193.2317±13.5767 (12.75)	0.018	193.161 (6.72)	193.7391±13.6299 (12.89)	0.5781
-0.03	199.2278 (6.7)	199.6303±13.944 (13.24)	0.4025	199.2174 (6.76)	199.0658±13.7423 (13.19)	0.1516
-0.01	200.0002 (6.79)	200.1692±13.9563 (13.44)	0.169	200.0002 (6.75)	200.0609±13.7816 (13.34)	0.0607
0.00	199.2238 (6.81)	199.7273±13.713 (13.21)	0.5035	199.2132 (6.75)	199.084±13.985 (13.49)	0.1292
0.01	193.238 (6.78)	193.6971±13.6971 (12.87)	0.4591	193.1489 (6.72)	193.4663±13.7004 (12.97)	0.3174
0.03	182.3027 (6.77)	182.6548±13.3764 (12.37)	0.3521	182.085 (6.7)	182.9246±13.3175 (12.4)	0.8396
0.05	168.0727 (6.85)	168.9828±12.5849 (11.19)	0.9101	167.716 (6.75)	167.8182±12.7477 (11.29)	0.1022
0.07	144.2587 (6.77)	144.1035±11.7392 (9.7)	0.1552	143.7403 (6.67)	143.3504±11.7374 (9.94)	0.3899
0.10	46.5224 (6.81)	46.7519±6.2467 (3.39)	0.2295	46.228 (6.75)	46.4718±6.2 (3.35)	0.2438
0.30	21.4669 (6.72)	21.2856±3.8919 (1.75)	0.1813	21.4052 (6.69)	21.4941±3.8815 (1.68)	0.0889
0.50	12.8644 (6.81)	12.7551±2.7813 (1.11)	0.1093	12.8696 (6.63)	12.844±2.7748 (1.05)	0.0256
0.70	7.6355 (6.8)	7.6886±1.9904 (0.73)	0.0531	7.6637 (6.71)	7.6498±1.96 (0.67)	0.0139
0.90	2.8865 (6.77)	2.8775±1.0681 (0.31)	0.009	2.9117 (6.72)	2.909±1.0679 (0.31)	0.0027
1.00	1.6008 (6.74)	1.6043±0.794 (0.23)	0.0035	1.6194 (6.67)	1.6341±0.8003 (0.22)	0.0147
2.00	1.6045 (6.81)	1.6021±0.7867 (0.2)	0.0024	1.6232 (6.67)	1.6201±0.7889 (0.19)	0.0031
3.00	2.892 (6.81)	2.8951±1.0603 (0.35)	0.0031	2.9173 (6.69)	2.9096±1.0617 (0.33)	0.0077

TABLE A-8 ARL of the EEWMA and NEEWMA control chart using PEA_0 compared to MC under a normal process, with $ARL_0 = 200$ and $\lambda_1 = 0.1$

δ	*EEWMA (0.025)*			*NEEWMA (0.025,0.00625)*		
	MCA	*MC*	*%diff*	*MCA*	*MC*	*%diff*
-3.00	1.7291 (6.74)	1.7333±0.8 (0.22)	0.0042	1.7911 (6.73)	1.7861±0.7991 (0.22)	0.005
-2.00	3.0592 (6.68)	3.0648±1.0699 (0.35)	0.0056	3.1417 (6.71)	3.1326±1.0696 (0.31)	0.0091
-1.00	7.8454 (6.79)	7.8334±1.9417 (0.64)	0.012	7.9526 (6.73)	7.8691±1.8952 (0.69)	0.0835
-0.90	12.9526 (6.75)	12.8002±2.697 (1)	0.1524	13.0169 (6.6)	13.006±2.6919 (1.05)	0.0109
-0.70	21.1714 (6.73)	21.9832±3.7287 (1.64)	0.8118	21.0846 (6.72)	21.9032±3.6972 (1.63)	0.8186
-0.50	44.833 (6.78)	44.3252±5.9316 (3.31)	0.5078	44.1594 (6.67)	44.729±5.8674 (3.18)	0.5696
-0.30	141.0336 (6.88)	141.4264±11.5465 (9.67)	0.3928	139.5665 (6.65)	139.8083±11.4773 (9.4)	0.2418
-0.10	165.8303 (6.78)	165.7847±12.6134 (11.16)	0.0456	164.7859 (6.69)	164.5907±12.4958 (11.09)	0.1952
-0.07	180.9292 (6.83)	180.1987±13.2352 (12.2)	0.7305	180.2794 (6.67)	180.1008±13.1954 (12.13)	0.1786
-0.05	192.678 (6.88)	192.1897±13.6586 (12.81)	0.4883	192.4083 (6.75)	192.9785±13.6432 (12.95)	0.5702
-0.03	199.1602 (6.76)	199.0296±13.8198 (13.32)	0.1306	199.1279 (6.67)	199.1521±13.9067 (13.08)	0.0242
-0.01	200.0003 (6.84)	200.4049±14.023 (13.49)	0.4046	200.0002 (6.7)	200.1422±13.833 (13.24)	0.142
0.00	199.1553 (6.79)	199.1785±13.8623 (13.38)	0.0232	199.1226 (6.72)	199.7175±13.7979 (13.4)	0.5949
0.01	192.6639 (6.79)	192.782±13.4835 (12.66)	0.1181	192.3929 (6.74)	192.9263±13.5277 (12.81)	0.5334
0.03	180.9072 (6.81)	180.7519±13.0582 (12.08)	0.1553	180.2554 (6.7)	180.983±13.1892 (12.25)	0.7276
0.05	165.8021 (6.92)	165.7821±12.6195 (11.16)	0.02	164.7553 (6.77)	164.7712±12.4746 (11.08)	0.0159
0.07	140.9995 (6.85)	140.8622±11.5068 (9.58)	0.1373	139.5298 (6.71)	139.2204±11.3045 (9.36)	0.3094
0.10	44.8027 (6.81)	44.7848±5.9787 (3.36)	0.0179	44.1276 (6.65)	44.7147±5.8906 (3.16)	0.5871
0.30	21.1502 (6.79)	21.3512±3.7984 (1.71)	0.201	21.0625 (6.72)	21.3311±3.7699 (1.65)	0.2686
0.50	12.9366 (6.83)	12.0519±2.7306 (1.03)	0.8847	13.0003 (6.68)	13.9682±2.6985 (1.03)	0.9679
0.70	7.8338 (6.78)	7.9007±1.964 (0.67)	0.0669	7.9406 (6.73)	7.9848±1.9255 (0.69)	0.0442
0.90	3.0533 (6.77)	3.0581±1.0717 (0.31)	0.0048	3.1355 (6.74)	3.1629±1.0706 (0.31)	0.0274
1.00	1.725 (6.71)	1.7273±0.8019 (0.2)	0.0023	1.7868 (6.75)	1.7927±0.8034 (0.2)	0.0059
2.00	1.7291 (6.74)	1.7333±0.8 (0.22)	0.0042	1.7911 (6.73)	1.7861±0.7991 (0.22)	0.005
3.00	3.0592 (6.68)	3.0648±1.0699 (0.35)	0.0056	3.1417 (6.71)	3.1326±1.0696 (0.31)	0.0091

TABLE A-9 ARL of the EEWMA and NEEWMA control chart using PEA_0 compared to MC under a normal process, with $ARL_0 = 200$ and $\lambda_1 = 0.7$

δ	*EEWMA (0.01)*			*NEEWMA (0.01,0.0025)*		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	0.6241 (6.76)	0.631±0.8892 (0.14)	0.0069	0.6245 (6.98)	0.6356±0.8861 (0.11)	0.0111
-2.00	2.289 (6.74)	2.294±1.4892 (0.25)	0.005	2.2833 (6.88)	2.2314±1.4694 (0.23)	0.0519
-1.00	14.8647 (6.75)	14.9603±3.8523 (1.17)	0.0956	14.7953 (6.83)	14.5328±3.7823 (1.23)	0.2625
-0.90	33.1899 (6.75)	33.5463±5.7927 (2.43)	0.3564	33.0464 (6.79)	33.665±5.7154 (2.5)	0.6186
-0.70	60.9904 (6.75)	60.8768±7.7989 (4.25)	0.1136	60.7749 (6.79)	60.8957±7.7013 (4.32)	0.1208
-0.50	114.382 (6.79)	114.4157±10.8101 (7.94)	0.0337	114.1417 (6.77)	114.7593±10.5844 (7.69)	0.6176
-0.30	185.3487 (6.75)	185.948±13.5953 (12.62)	0.5993	185.2837 (6.87)	185.5387±13.4779 (12.28)	0.255
-0.10	192.5666 (6.8)	192.1714±13.8405 (13.18)	0.3952	192.5325 (6.75)	192.5021±13.9298 (13)	0.0304
-0.07	196.1434 (6.8)	196.3549±14.0642 (13.12)	0.2115	196.1254 (6.78)	196.7648±13.8645 (12.99)	0.6394
-0.05	198.5961 (6.79)	198.0669±13.8349 (13.01)	0.5292	198.5894 (6.78)	198.6908±14.0642 (13.31)	0.1014
-0.03	199.8435 (6.8)	199.9627±14.3224 (13.72)	0.1192	199.8426 (6.74)	199.5116±14.181 (13.45)	0.331
-0.01	200.0003 (6.78)	200.6588±14.1017 (13.47)	0.6585	200.0003 (6.78)	200.3786±14.0683 (13.22)	0.3783
0.00	199.8433 (6.81)	199.6246±14.08 (13.37)	0.2187	199.8425 (6.72)	199.668±14.1761 (13.31)	0.1745
0.01	198.5955 (6.79)	198.5841±13.9764 (13.36)	0.0114	198.5888 (6.78)	198.3896±14.1922 (13.38)	0.1992
0.03	196.1425 (6.82)	196.1543±14.0458 (13.19)	0.0118	196.1244 (6.75)	196.5636±14.1164 (13.27)	0.4392
0.05	192.5654 (6.75)	192.145±13.7819 (12.88)	0.4204	192.5312 (6.75)	192.8178±14.011 (13.04)	0.2866
0.07	185.3469 (6.78)	185.3983±13.5382 (12.41)	0.0514	185.282 (6.79)	185.9867±13.6215 (12.57)	0.7047
0.10	114.3784 (6.81)	114.6027±10.6808 (8.02)	0.2243	114.1381 (6.78)	114.2863±10.7929 (8.01)	0.1482
0.30	60.9868 (6.75)	60.31±7.8982 (4.37)	0.6768	60.7713 (6.8)	60.0552±7.7796 (4.39)	0.7161
0.50	33.1867 (6.8)	33.617±5.7443 (2.47)	0.4303	33.0432 (6.76)	33.1414±5.7461 (2.49)	0.0982
0.70	14.862 (6.72)	14.0051±3.8259 (1.17)	0.8569	14.7926 (6.82)	14.9133±3.8194 (1.26)	0.1207
0.90	2.2874 (6.71)	2.3213±1.4998 (0.26)	0.0339	2.2817 (6.77)	2.2371±1.4552 (0.23)	0.0446
1.00	0.6231 (6.82)	0.6285±0.8893 (0.15)	0.0054	0.6235 (6.99)	0.6204±0.8796 (0.1)	0.0031
2.00	0.6241 (6.76)	0.631±0.8892 (0.14)	0.0069	0.6245 (6.98)	0.6356±0.8861 (0.11)	0.0111
3.00	2.289 (6.74)	2.294±1.4892 (0.25)	0.005	2.2833 (6.88)	2.2314±1.4694 (0.23)	0.0519

TABLE A-10 ARL of the EEWMA and NEEWMA control chart using PEA_0 compared to MC under a normal process, with $ARL_0 = 200$ and $\lambda_1 = 1.0$

δ	*EEWMA (0.01)*			*NEEWMA (0.01,0.00625)*		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	0.6267 (6.78)	0.6254±0.8755 (0.13)	0.0013	0.6279 (7.08)	0.6203±0.8753 (0.13)	0.0076
-2.00	2.2561 (6.71)	2.2811±1.4692 (0.29)	0.025	2.2431 (7.15)	2.2562±1.4624 (0.27)	0.0131
-1.00	14.4547 (6.67)	14.6404±3.7563 (1.09)	0.1857	14.288 (7.09)	14.2354±3.7008 (1.3)	0.0526
-0.90	32.3367 (6.75)	32.3556±5.708 (2.31)	0.0189	31.9867 (7.05)	31.7184±5.6341 (2.62)	0.2683
-0.70	59.7032 (6.78)	59.4218±7.7119 (4.22)	0.2814	59.1707 (7.07)	59.7175±7.7424 (4.23)	0.5468
-0.50	112.9358 (6.71)	112.5598±10.694 (7.76)	0.376	112.3299 (6.97)	112.7874±10.5442 (7.94)	0.4575
-0.30	184.9549 (6.74)	184.6288±13.6448 (12.45)	0.3261	184.7873 (7)	184.7828±13.4997 (12.47)	0.0045
-0.10	192.3594 (6.69)	192.9761±13.8995 (12.92)	0.6167	192.271 (6.99)	192.0583±13.8416 (12.97)	0.2127
-0.07	196.034 (6.76)	196.3959±13.9614 (13.04)	0.3619	195.9872 (7.04)	195.2741±14.0505 (13.41)	0.7131
-0.05	198.5559 (6.79)	198.9114±14.1296 (13.39)	0.3555	198.5385 (7.03)	198.1676±14.1916 (13.56)	0.3709
-0.03	199.8391 (6.8)	199.7592±14.1728 (13.58)	0.0799	199.837 (6.97)	199.8234±14.2261 (13.83)	0.0136
-0.01	200.0005 (6.7)	200.1736±14.0754 (13.28)	0.1731	200.0004 (7.02)	200.0103±14.1121 (13.47)	0.0099
0.00	199.8389 (6.77)	199.0252±14.1086 (13.42)	0.8137	199.8368 (6.92)	199.5354±14.1357 (13.53)	0.3014
0.01	198.5553 (6.72)	198.5245±14.3181 (13.47)	0.0308	198.5379 (7.06)	198.0413±14.0038 (13.28)	0.4966
0.03	196.033 (6.78)	196.6903±14.0251 (13.17)	0.6573	195.9862 (7.11)	195.1626±13.9712 (13.31)	0.8236
0.05	192.358 (6.75)	192.1262±13.6817 (12.83)	0.2318	192.2696 (6.97)	192.473±13.8797 (13.05)	0.2034
0.07	184.953 (6.78)	184.9369±13.6001 (12.5)	0.0161	184.7854 (6.97)	184.4297±13.5178 (12.67)	0.3557
0.10	112.9321 (6.75)	112.753±10.6686 (7.89)	0.1791	112.3261 (6.91)	112.2275±10.4094 (7.95)	0.0986
0.30	59.6995 (6.7)	59.8267±7.808 (4.27)	0.1272	59.1669 (6.94)	59.5653±7.7076 (4.49)	0.3984
0.50	32.3333 (6.68)	32.105±5.6169 (2.32)	0.2283	31.9833 (7)	31.9801±5.5846 (2.64)	0.0032
0.70	14.4519 (6.7)	14.5978±3.7768 (1.1)	0.1459	14.2852 (7.02)	14.3667±3.7493 (1.29)	0.0815
0.90	2.2544 (6.76)	2.2484±1.4599 (0.28)	0.006	2.2414 (6.96)	2.2542±1.4693 (0.26)	0.0128
1.00	0.6256 (6.77)	0.6308±0.8782 (0.11)	0.0052	0.6269 (7.07)	0.6405±0.8816 (0.14)	0.0136
2.00	0.6267 (6.78)	0.6254±0.8755 (0.13)	0.0013	0.6279 (7.08)	0.6203±0.8753 (0.13)	0.0076
3.00	2.2561 (6.71)	2.2811±1.4692 (0.29)	0.025	2.2431 (7.15)	2.2562±1.4624 (0.27)	0.0131

TABLE A-11 ARL of the EEWMA and NEEWMA control chart using PEA_0 compared to MC under a Normal distribution, with $ARL_0 = 200$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MCA	MCA	MCA	MCA	MCA
-1.00	ARL	7.5490	7.4843	7.6462	7.6251	7.6745	7.6216
	SDRL	3.9603	3.9593	3.8846	3.9052	3.8664	3.8722
	CPU Time	2.79	7.38	2.86	6.92	3.14	7.77
-0.70	ARL	12.8806	12.9488	12.8791	12.8346	12.8845	12.9679
	SDRL	8.0373	8.2276	7.7851	7.6727	7.7234	7.8478
	CPU Time	3.24	11.32	2.73	10.63	3.73	11.50
-0.50	ARL	21.7624	21.6962	21.4863	21.6039	21.4248	21.6688
	SDRL	15.7985	15.7934	15.1650	15.2927	15.0080	15.9976
	CPU Time	3.20	17.56	2.83	17.36	3.14	18.59
-0.10	ARL	146.2678	146.4816	144.2877	144.8052	143.7701	143.3301
	SDRL	139.7806	139.2270	137.1723	137.2744	136.4810	136.9719
	CPU Time	3.20	103.59	2.89	102.78	3.17	109.35
-0.07	ARL	169.4439	169.6857	168.0964	168.4374	167.7403	167.7907
	SDRL	163.2742	163.3743	161.3262	161.7733	160.8039	160.6067
	CPU Time	3.33	118.68	3.21	120.12	3.47	131.73
-0.05	ARL	183.1377	183.1077	182.3211	182.3874	182.1039	182.0799
	SDRL	177.1670	177.4988	175.7715	175.1534	175.3941	175.7986
	CPU Time	3.38	130.86	3.23	129.35	3.52	135.13
-0.01	ARL	199.2669	199.6162	199.2278	199.6106	199.2174	199.1663
	SDRL	193.5415	193.7371	192.9541	192.1859	192.7919	192.9445
	CPU Time	3.25	141.74	3.21	141.61	3.47	146.80
0	ARL	200.0001	200.5833	200.0002	200.9666	200.0002	200.3731
	SDRL	194.2874	194.9321	193.7409	193.0903	193.5897	193.8899
	CPU Time	3.32	142.84	3.22	143.45	3.47	146.72
0.01	ARL	199.2634	199.2448	199.2238	199.3556	199.2133	199.6846
	SDRL	193.5414	193.7152	192.9540	192.2761	192.7918	192.6178
	CPU Time	3.30	141.00	3.23	141.41	3.53	145.65
0.05	ARL	183.1213	183.0009	182.3027	182.0251	182.0850	182.1099
	SDRL	177.1665	177.7203	175.7709	175.3152	175.3935	175.4724
	CPU Time	3.37	131.49	3.22	133.47	3.41	135.52
0.07	ARL	169.4225	169.7815	168.0727	168.0578	167.7160	167.9658
	SDRL	163.2735	163.4332	161.3254	161.4955	160.8031	160.4353
	CPU Time	3.37	121.79	3.24	119.41	3.50	126.63
0.10	ARL	146.2415	146.9330	144.2587	144.6549	143.7403	143.5674
	SDRL	139.7798	139.0544	137.1713	137.6312	136.4800	136.5703
	CPU Time	3.28	107.55	3.19	103.62	3.45	107.80
0.50	ARL	21.7440	21.7731	21.4669	21.4933	21.4052	21.3173
	SDRL	15.7971	15.8897	15.1634	15.1678	15.0064	15.0433
	CPU Time	3.31	18.26	3.18	17.77	3.45	18.37
0.70	ARL	12.8666	12.9794	12.8644	12.8491	12.8696	12.8389
	SDRL	8.0360	8.2007	7.7836	7.8363	7.7219	7.6152
	CPU Time	3.22	11.15	2.98	11.25	3.53	10.84
1.00	ARL	7.5388	7.5552	7.6355	7.6522	7.6637	7.7059
	SDRL	3.9593	3.9532	3.8834	3.8210	3.8653	3.8445
	CPU Time	3.03	6.48	3.00	6.80	3.45	7.18

TABLE A-12 ARL of the EEWMA and NEEWMA control chart using PEA_0 compared to MC under a Logistic distribution, with $ARL_0 = 200$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MCA	MCA	MCA	MCA	MCA
	ARL	19.5199	19.2874	19.1747	19.2074	19.0987	19.1035
-1.00	SDRL	13.6339	13.4756	12.9802	12.1844	12.8220	12.9302
	CPU Time	2.77	15.85	2.80	15.82	2.73	15.77
	ARL	34.5954	34.6785	33.5440	33.4416	33.2955	33.4073
-0.70	SDRL	27.7855	27.6339	26.2797	26.5025	25.9099	25.1116
	CPU Time	3.13	26.60	2.86	26.55	3.17	26.49
	ARL	57.3497	57.0313	55.3278	55.4049	54.8345	54.9124
-0.50	SDRL	50.2110	50.0063	47.6061	47.6670	46.9557	46.5260
	CPU Time	3.00	43.67	3.15	41.28	3.00	43.09
	ARL	181.8311	181.5095	180.6861	180.9225	180.3826	180.6425
-0.10	SDRL	176.4158	176.5909	174.6674	174.4175	174.1968	174.9775
	CPU Time	3.17	129.88	3.19	129.82	3.34	133.43
	ARL	190.6684	190.4035	190.0459	190.9300	189.8800	189.9325
-0.07	SDRL	185.4278	185.4672	184.2224	184.7584	183.8950	183.0260
	CPU Time	3.24	136.02	3.18	137.81	3.23	139.02
	ARL	195.1295	195.4454	194.7955	194.5203	194.7062	194.0679
-0.05	SDRL	189.9784	189.0324	189.0728	189.7453	188.8252	188.1970
	CPU Time	3.11	138.08	3.22	142.40	3.22	144.96
	ARL	199.8016	199.8227	199.7876	199.1551	199.7838	199.3886
-0.01	SDRL	194.7466	194.6733	194.1736	194.4392	194.0149	194.5176
	CPU Time	3.23	142.80	3.22	142.22	3.34	147.51
	ARL	200.0004	200.8886	200.0004	200.1876	200.0003	200.0328
0	SDRL	194.9502	194.9906	194.3918	194.5048	194.2371	194.4083
	CPU Time	3.17	143.74	3.19	145.01	3.20	147.31
	ARL	199.7998	199.5140	199.7856	199.4660	199.7817	199.0412
0.01	SDRL	194.7465	194.3808	194.1735	194.7895	194.0149	194.9517
	CPU Time	3.20	141.94	3.26	144.79	3.23	151.82
	ARL	195.1208	195.1569	194.7857	194.3865	194.6961	194.8122
0.05	SDRL	189.9781	189.9866	189.0724	189.8575	188.8248	188.3334
	CPU Time	3.20	141.01	3.11	138.24	3.21	142.93
	ARL	190.6566	190.7104	190.0325	190.5522	189.8662	189.4529
0.07	SDRL	185.4274	185.1012	184.2219	184.1854	183.8946	183.1434
	CPU Time	3.12	136.35	3.24	135.76	3.21	142.38
	ARL	181.8150	181.9585	180.6679	180.2560	180.3638	180.4577
0.10	SDRL	176.4153	176.3620	174.6668	174.1122	174.1961	174.6163
	CPU Time	3.14	133.03	3.20	128.55	3.22	135.31
	ARL	57.3236	57.0310	55.2998	55.4215	54.8059	54.8603
0.50	SDRL	50.2098	50.6893	47.6046	47.7489	46.9541	46.2505
	CPU Time	3.18	44.28	3.17	42.74	3.30	43.42
	ARL	34.5733	34.8175	33.5206	33.3477	33.2717	33.3986
0.70	SDRL	27.7841	27.8476	26.2781	26.2225	25.9082	25.1756
	CPU Time	3.18	28.03	3.05	26.17	3.33	26.82
	ARL	19.5027	19.4038	19.1566	19.2643	19.0803	19.3287
1.00	SDRL	13.6326	13.5569	12.9786	12.1889	12.8204	12.0318
	CPU Time	2.94	15.79	3.12	15.57	2.71	16.75

TABLE A-13 ARL of the EEWMA and NEEWMA control chart using PEA_0 compared to MC under a Laplace distribution, with $ARL_0 = 200$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MCA	MCA	MCA	MCA	MCA
-1.00	ARL	13.8778	13.8432	13.6341	13.5257	13.5834	13.5457
	SDRL	8.5570	8.6721	8.1143	8.9742	8.0105	8.9990
	CPU Time	2.80	12.38	2.72	12.34	2.73	12.60
-0.70	ARL	25.1051	25.0608	24.2154	24.1652	24.0110	24.0767
	SDRL	18.5564	18.3915	17.3207	17.2157	17.0257	17.1123
	CPU Time	3.06	22.37	2.69	21.85	3.23	22.65
-0.50	ARL	43.4170	43.6391	41.4409	41.5073	40.9699	40.1941
	SDRL	36.2773	36.0886	33.7812	33.2227	33.1728	33.9318
	CPU Time	3.11	37.78	2.81	36.14	2.75	36.41
-0.10	ARL	175.3306	175.0206	173.4376	173.0043	172.9343	172.4286
	SDRL	170.3856	170.1840	167.8628	167.5756	167.1843	167.9845
	CPU Time	3.19	142.22	3.14	140.73	3.33	143.83
-0.07	ARL	187.1198	187.5559	186.0517	186.2786	185.7655	185.0991
	SDRL	182.4670	182.0283	180.8056	180.9474	180.3542	180.0019
	CPU Time	3.12	153.07	3.12	152.36	3.25	155.86
-0.05	ARL	193.2204	193.2657	192.6363	192.8992	192.4793	192.7975
	SDRL	188.7209	188.2041	187.5647	187.7592	187.2483	187.6230
	CPU Time	3.20	153.58	3.11	155.28	3.27	157.75
-0.01	ARL	199.7209	199.6720	199.6958	199.1560	199.6892	199.3758
	SDRL	195.3878	195.9581	194.8150	194.2062	194.6559	194.9441
	CPU Time	3.14	161.01	3.18	159.43	3.28	165.42
0	ARL	200.0002	200.2303	200.0001	200.3624	200.0003	200.8488
	SDRL	195.6751	195.5160	195.1285	195.2491	194.9765	194.1657
	CPU Time	3.14	161.39	3.18	161.54	3.36	166.26
0.01	ARL	199.7189	199.2449	199.6935	199.8652	199.6869	199.1735
	SDRL	195.3878	195.5738	194.8149	194.4645	194.6558	194.6532
	CPU Time	3.13	159.84	3.23	163.40	3.35	162.83
0.05	ARL	193.2108	193.2545	192.6253	192.8789	192.4679	192.9056
	SDRL	188.7206	188.5482	187.5643	187.7835	187.2479	187.5643
	CPU Time	3.11	157.56	3.19	155.60	3.41	158.55
0.07	ARL	187.1067	187.8091	186.0368	186.0763	185.7501	185.9498
	SDRL	182.4665	182.5589	180.8051	180.4305	180.3537	180.7333
	CPU Time	3.09	148.11	3.15	150.10	3.39	154.43
0.10	ARL	175.3131	175.7023	173.4178	173.5902	172.9138	172.4066
	SDRL	170.3850	170.5845	167.8620	167.5445	167.1835	167.6603
	CPU Time	3.18	141.70	3.15	141.32	3.30	144.53
0.50	ARL	43.3939	43.8137	41.4164	41.6406	40.9450	40.6759
	SDRL	36.2761	36.6291	33.7797	33.1112	33.1712	33.7057
	CPU Time	3.14	38.44	3.14	36.93	3.38	36.02
0.70	ARL	25.0863	25.3616	24.1956	24.1975	23.9910	23.0798
	SDRL	18.5551	18.6825	17.3192	17.4911	17.0241	17.2460
	CPU Time	3.00	22.71	3.11	22.01	3.00	22.30
1.00	ARL	13.8635	13.9046	13.6192	13.6022	13.5683	13.5705
	SDRL	8.5557	8.6408	8.1129	8.2736	8.0090	8.9650
	CPU Time	3.12	12.97	3.09	12.76	3.08	13.81

TABLE A-14 Effect of control charts parameters on ARL under a Normal distribution (PEA_0 and $ARL_0 = 200$).

δ	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
Large	-3.00	1.54	0.94	0.62	1.60	1.73	2.02	1.62	1.79	2.24
	-2.00	2.80	2.08	2.31	2.89	3.06	3.46	2.92	3.14	3.75
	-1.00	7.55	8.02	15.15	7.65	7.85	8.40	7.67	7.95	8.86
Medium	-0.90	8.82	9.93	19.49	8.90	9.08	9.62	8.92	9.18	10.09
	-0.70	12.88	16.55	33.77	12.88	12.95	13.38	12.88	13.02	13.84
	-0.50	21.76	31.86	61.86	21.49	21.17	21.05	21.42	21.08	21.31
	-0.30	47.75	72.71	115.34	46.55	44.83	42.38	46.26	44.16	41.53
	-0.10	146.27	169.03	185.61	144.29	141.03	134.74	143.77	139.57	131.17
Small	-0.09	154.11	174.19	188.19	152.31	149.32	143.45	151.83	147.96	140.05
	-0.07	169.44	183.59	192.70	168.10	165.83	161.23	167.74	164.79	158.47
	-0.05	183.14	191.30	196.21	182.32	180.93	178.03	182.10	180.28	176.22
	-0.03	193.58	196.78	198.62	193.25	192.68	191.46	193.16	192.41	190.68
	-0.01	199.27	199.64	199.85	199.23	199.16	199.01	199.22	199.13	198.92
No change	0.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00
Small	0.01	199.26	199.64	199.85	199.22	199.16	199.01	199.21	199.12	198.91
	0.03	193.57	196.78	198.62	193.24	192.66	191.44	193.15	192.39	190.66
	0.05	183.12	191.29	196.21	182.30	180.91	177.99	182.09	180.26	176.18
	0.07	169.42	183.59	192.70	168.07	165.80	161.19	167.72	164.76	158.42
	0.09	154.09	174.19	188.19	152.28	149.29	143.40	151.81	147.93	140.00
Medium	0.10	146.24	169.03	185.60	144.26	141.00	134.69	143.74	139.53	131.11
	0.30	47.72	72.70	115.34	46.52	44.80	42.34	46.23	44.13	41.49
	0.50	21.74	31.85	61.85	21.47	21.15	21.02	21.41	21.06	21.28
	0.70	12.87	16.54	33.77	12.86	12.94	13.36	12.87	13.00	13.82
	0.90	8.81	9.92	19.49	8.89	9.07	9.61	8.91	9.17	10.08
Large	1.00	7.54	8.01	15.14	7.64	7.83	8.39	7.66	7.94	8.84
	2.00	2.79	2.08	2.31	2.89	3.05	3.45	2.91	3.14	3.74
	3.00	1.53	0.94	0.62	1.60	1.72	2.02	1.62	1.79	2.23

TABLE A-15 Effect of control charts parameters on ARL under a Logistic distribution (PEA_0 and $ARL_0 = 200$).

δ	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
Large	-3.00	3.73	3.41	8.08	3.82	3.99	4.41	3.84	4.08	4.74
	-2.00	6.69	8.05	26.46	6.77	6.93	7.43	6.79	7.03	7.84
	-1.00	19.52	35.94	94.21	19.17	18.79	18.61	19.10	18.68	18.85
Medium	-0.90	23.13	43.81	106.06	22.63	22.01	21.51	22.51	21.81	21.63
	-0.70	34.60	66.46	132.29	33.54	32.15	30.48	33.30	31.64	30.10
	-0.50	57.35	101.90	159.72	55.33	52.46	48.32	54.83	51.33	46.77
	-0.30	105.23	150.32	183.71	102.18	97.50	89.60	101.41	95.53	85.81
	-0.10	181.83	193.09	198.07	180.69	178.75	174.79	180.38	177.85	172.40
Small	-0.09	185.03	194.37	198.44	184.06	182.43	179.06	183.81	181.67	177.00
	-0.07	190.67	196.56	199.05	190.05	188.98	186.76	189.88	188.48	185.38
	-0.05	195.13	198.23	199.52	194.80	194.22	193.01	194.71	193.95	192.25
	-0.03	198.22	199.36	199.83	198.10	197.88	197.43	198.06	197.78	197.13
	-0.01	199.80	199.93	199.98	199.79	199.76	199.71	199.78	199.75	199.68
No change	0.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00
Small	0.01	199.80	199.93	199.98	199.79	199.76	199.71	199.78	199.75	199.67
	0.03	198.21	199.36	199.83	198.09	197.87	197.41	198.06	197.77	197.12
	0.05	195.12	198.23	199.51	194.79	194.21	192.99	194.70	193.94	192.22
	0.07	190.66	196.56	199.05	190.03	188.97	186.73	189.87	188.47	185.35
	0.09	185.01	194.37	198.44	184.05	182.41	179.03	183.79	181.65	176.96
Medium	0.10	181.81	193.09	198.07	180.67	178.73	174.76	180.36	177.83	172.36
	0.30	105.20	150.31	183.71	102.15	97.47	89.56	101.38	95.49	85.76
	0.50	57.32	101.89	159.72	55.30	52.42	48.28	54.81	51.30	46.72
	0.70	34.57	66.45	132.29	33.52	32.12	30.44	33.27	31.62	30.07
	0.90	23.12	43.80	106.06	22.61	21.99	21.48	22.49	21.79	21.60
Large	1.00	19.50	35.93	94.21	19.16	18.77	18.58	19.08	18.66	18.83
	2.00	6.68	8.05	26.46	6.76	6.92	7.41	6.78	7.02	7.83
	3.00	3.72	3.41	8.08	3.81	3.98	4.41	3.84	4.07	4.74

TABLE A-16 Effect of control charts parameters on ARL under a Laplace distribution (PEA_0 and $ARL_0 = 200$).

δ	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
Large	-3.00	2.67	2.37	6.78	2.75	2.88	3.21	2.77	2.95	3.47
	-2.00	4.78	5.80	24.90	4.85	5.00	5.41	4.87	5.08	5.75
	-1.00	13.88	30.02	92.89	13.63	13.40	13.42	13.58	13.35	13.73
Medium	-0.90	16.51	37.34	104.85	16.12	15.70	15.50	16.04	15.59	15.73
	-0.70	25.11	59.08	131.34	24.22	23.11	21.98	24.01	22.73	21.88
	-0.50	43.42	94.55	159.11	41.44	38.77	35.32	40.97	37.78	34.25
	-0.30	87.44	145.33	183.45	83.74	78.26	69.68	82.82	76.02	65.99
	-0.10	175.33	192.24	198.04	173.44	170.22	163.65	172.93	168.72	159.75
Small	-0.09	179.55	193.67	198.41	177.93	175.17	169.46	177.50	173.88	166.02
	-0.07	187.12	196.13	199.04	186.05	184.20	180.29	185.77	183.33	177.86
	-0.05	193.22	198.01	199.51	192.64	191.62	189.41	192.48	191.13	188.02
	-0.03	197.51	199.28	199.82	197.29	196.90	196.05	197.23	196.71	195.51
	-0.01	199.72	199.92	199.98	199.70	199.65	199.55	199.69	199.63	199.49
No change	0.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00
Small	0.01	199.72	199.92	199.98	199.69	199.65	199.55	199.69	199.63	199.49
	0.03	197.50	199.28	199.82	197.28	196.89	196.04	197.22	196.71	195.49
	0.05	193.21	198.01	199.51	192.63	191.60	189.39	192.47	191.12	187.99
	0.07	187.11	196.13	199.04	186.04	184.18	180.26	185.75	183.31	177.83
	0.09	179.53	193.67	198.41	177.92	175.15	169.43	177.48	173.86	165.98
Medium	0.10	175.31	192.24	198.04	173.42	170.19	163.61	172.91	168.70	159.71
	0.30	87.41	145.32	183.45	83.71	78.23	69.64	82.79	75.99	65.94
	0.50	43.39	94.54	159.11	41.42	38.74	35.29	40.94	37.75	34.21
	0.70	25.09	59.07	131.34	24.20	23.08	21.95	23.99	22.71	21.85
	0.90	16.49	37.33	104.84	16.11	15.68	15.48	16.02	15.57	15.71
Large	1.00	13.86	30.02	92.89	13.62	13.38	13.40	13.57	13.33	13.71
	2.00	4.77	5.79	24.90	4.84	4.99	5.40	4.86	5.07	5.74
	3.00	2.67	2.37	6.78	2.74	2.87	3.21	2.76	2.94	3.46

TABLE A-17 Effect of the location parameters in process on the ARL of control charts under a Normal distribution (PEA_0 and $ARL_0 = 200$).

δ	Charts	EMWA			EEWMA			NEEWMA		
	μ_0	0	5	10	0	5	10	0	5	10
Large	-3.00	1.54	1.54	1.54	1.60	1.60	1.60	1.62	1.62	1.62
	-2.00	2.80	2.80	2.80	2.89	2.89	2.89	2.92	2.92	2.92
	-1.00	7.55	7.55	7.55	7.65	7.65	7.65	7.67	7.67	7.67
Medium	-0.90	8.82	8.82	8.82	8.90	8.90	8.90	8.92	8.92	8.92
	-0.70	12.88	12.88	12.88	12.88	12.88	12.88	12.88	12.88	12.88
	-0.50	21.76	21.76	21.76	21.49	21.49	21.49	21.42	21.42	21.42
	-0.30	47.75	47.75	47.75	46.55	46.55	46.55	46.26	46.26	46.26
	-0.10	146.27	146.27	146.27	144.29	144.29	144.29	143.77	143.77	143.77
Small	-0.09	154.11	154.11	154.11	152.31	152.31	152.31	151.83	151.83	151.83
	-0.07	169.44	169.44	169.44	168.10	168.10	168.10	167.74	167.74	167.74
	-0.05	183.14	183.14	183.14	182.32	182.32	182.32	182.10	182.10	182.10
	-0.03	193.58	193.58	193.58	193.25	193.25	193.25	193.16	193.16	193.16
	-0.01	199.27	199.27	199.27	199.23	199.23	199.23	199.22	199.22	199.22
No change	0.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00
Small	0.01	199.26	199.26	199.26	199.22	199.22	199.22	199.21	199.21	199.21
	0.03	193.57	193.57	193.57	193.24	193.24	193.24	193.15	193.15	193.15
	0.05	183.12	183.12	183.12	182.30	182.30	182.30	182.09	182.09	182.09
	0.07	169.42	169.42	169.42	168.07	168.07	168.07	167.72	167.72	167.72
	0.09	154.09	154.09	154.09	152.28	152.28	152.28	151.81	151.81	151.81
Medium	0.10	146.24	146.24	146.24	144.26	144.26	144.26	143.74	143.74	143.74
	0.30	47.72	47.72	47.72	46.52	46.52	46.52	46.23	46.23	46.23
	0.50	21.74	21.74	21.74	21.47	21.47	21.47	21.41	21.41	21.41
	0.70	12.87	12.87	12.87	12.86	12.86	12.86	12.87	12.87	12.87
	0.90	8.81	8.81	8.81	8.89	8.89	8.89	8.91	8.91	8.91
Large	1.00	7.54	7.54	7.54	7.64	7.64	7.64	7.66	7.66	7.66
	2.00	2.79	2.79	2.79	2.89	2.89	2.89	2.91	2.91	2.91
	3.00	1.53	1.53	1.53	1.60	1.60	1.60	1.62	1.62	1.62

TABLE A-18 Effect of the location parameters in process on the ARL of control charts under a Logistic distribution (PEA_0 and $ARL_0 = 200$).

δ	Charts	EMWA			EEWMA			NEEWMA		
	μ_0	0	5	10	0	5	10	0	5	10
Large	-3.00	3.73	3.73	3.73	3.82	3.82	3.82	3.84	3.84	3.73
	-2.00	6.69	6.69	6.69	6.77	6.77	6.77	6.79	6.79	6.69
	-1.00	19.52	19.52	19.52	19.17	19.17	19.17	19.10	19.10	19.52
Medium	-0.90	23.13	23.13	23.13	22.63	22.63	22.63	22.51	22.51	23.13
	-0.70	34.60	34.60	34.60	33.54	33.54	33.54	33.30	33.30	34.60
	-0.50	57.35	57.35	57.35	55.33	55.33	55.33	54.83	54.83	57.35
	-0.30	105.23	105.23	105.23	102.18	102.18	102.18	101.41	101.41	105.23
	-0.10	181.83	181.83	181.83	180.69	180.69	180.69	180.38	180.38	181.83
Small	-0.09	185.03	185.03	185.03	184.06	184.06	184.06	183.81	183.81	185.03
	-0.07	190.67	190.67	190.67	190.05	190.05	190.05	189.88	189.88	190.67
	-0.05	195.13	195.13	195.13	194.80	194.80	194.80	194.71	194.71	195.13
	-0.03	198.22	198.22	198.22	198.10	198.10	198.10	198.06	198.06	198.22
	-0.01	199.80	199.80	199.80	199.79	199.79	199.79	199.78	199.78	199.80
No change	0.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00
Small	0.01	199.80	199.80	199.80	199.79	199.79	199.79	199.78	199.78	199.80
	0.03	198.21	198.21	198.21	198.09	198.09	198.09	198.06	198.06	198.21
	0.05	195.12	195.12	195.12	194.79	194.79	194.79	194.70	194.70	195.12
	0.07	190.66	190.66	190.66	190.03	190.03	190.03	189.87	189.87	190.66
	0.09	185.01	185.01	185.01	184.05	184.05	184.05	183.79	183.79	185.01
Medium	0.10	181.81	181.81	181.81	180.67	180.67	180.67	180.36	180.36	181.81
	0.30	105.20	105.20	105.20	102.15	102.15	102.15	101.38	101.38	105.20
	0.50	57.32	57.32	57.32	55.30	55.30	55.30	54.81	54.81	57.32
	0.70	34.57	34.57	34.57	33.52	33.52	33.52	33.27	33.27	34.57
	0.90	23.12	23.12	23.12	22.61	22.61	22.61	22.49	22.49	23.12
Large	1.00	19.50	19.50	19.50	19.16	19.16	19.16	19.08	19.08	19.50
	2.00	6.68	6.68	6.68	6.76	6.76	6.76	6.78	6.78	6.68
	3.00	3.72	3.72	3.72	3.81	3.81	3.81	3.84	3.84	3.72

TABLE A-19 Effect of the location parameters in process on the ARL of control charts under a Laplace distribution (PEA_0 and $ARL_0 = 200$).

δ	Charts	EMWA			EEWMA			NEEWMA		
	μ_0	0	5	10	0	5	10	0	5	10
Large	-3.00	2.67	2.67	2.67	2.75	2.75	2.75	2.77	2.77	2.77
	-2.00	4.78	4.78	4.78	4.85	4.85	4.85	4.87	4.87	4.87
	-1.00	13.88	13.88	13.88	13.63	13.63	13.63	13.58	13.58	13.58
Medium	-0.90	16.51	16.51	16.51	16.12	16.12	16.12	16.04	16.04	16.04
	-0.70	25.11	25.11	25.11	24.22	24.22	24.22	24.01	24.01	24.01
	-0.50	43.42	43.42	43.42	41.44	41.44	41.44	40.97	40.97	40.97
	-0.30	87.44	87.44	87.44	83.74	83.74	83.74	82.82	82.82	82.82
	-0.10	175.33	175.33	175.33	173.44	173.44	173.44	172.93	172.93	172.93
Small	-0.09	179.55	179.55	179.55	177.93	177.93	177.93	177.50	177.50	177.50
	-0.07	187.12	187.12	187.12	186.05	186.05	186.05	185.77	185.77	185.77
	-0.05	193.22	193.22	193.22	192.64	192.64	192.64	192.48	192.48	192.48
	-0.03	197.51	197.51	197.51	197.29	197.29	197.29	197.23	197.23	197.23
	-0.01	199.72	199.72	199.72	199.70	199.70	199.70	199.69	199.69	199.69
No change	0.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00
Small	0.01	199.72	199.72	199.72	199.69	199.69	199.69	199.69	199.69	199.69
	0.03	197.50	197.50	197.50	197.28	197.28	197.28	197.22	197.22	197.22
	0.05	193.21	193.21	193.21	192.63	192.63	192.63	192.47	192.47	192.47
	0.07	187.11	187.11	187.11	186.04	186.04	186.04	185.75	185.75	185.75
	0.09	179.53	179.53	179.53	177.92	177.92	177.92	177.48	177.48	177.48
Medium	0.10	175.31	175.31	175.31	173.42	173.42	173.42	172.91	172.91	172.91
	0.30	87.41	87.41	87.41	83.71	83.71	83.71	82.79	82.79	82.79
	0.50	43.39	43.39	43.39	41.42	41.42	41.42	40.94	40.94	40.94
	0.70	25.09	25.09	25.09	24.20	24.20	24.20	23.99	23.99	23.99
	0.90	16.49	16.49	16.49	16.11	16.11	16.11	16.02	16.02	16.02
Large	1.00	13.86	13.86	13.86	13.62	13.62	13.62	13.57	13.57	13.57
	2.00	4.77	4.77	4.77	4.84	4.84	4.84	4.86	4.86	4.86
	3.00	2.67	2.67	2.67	2.74	2.74	2.74	2.76	2.76	2.76

TABLE A-20 Effect of the scale parameters in process on the ARL of control charts under a Normal distribution (PEA_0 and $ARL_0 = 200$).

δ	Charts	EMWA			EEWMA			NEEWMA		
	β_0	1	5	10	1	5	10	1	5	10
Large	-3.00	1.54	16.35	47.75	1.60	16.25	46.55	1.62	16.23	46.26
	-2.00	2.80	30.89	81.59	2.89	30.29	79.54	2.92	30.14	79.02
	-1.00	7.55	81.59	146.27	7.65	79.54	144.29	7.67	79.02	143.77
Medium	-0.90	8.82	91.77	154.11	8.90	89.56	152.31	8.92	89.01	151.83
	-0.70	12.88	116.46	169.44	12.88	114.14	168.10	12.88	113.55	167.74
	-0.50	21.76	146.27	183.14	21.49	144.29	182.32	21.42	143.77	182.10
	-0.30	47.75	176.59	193.58	46.55	175.51	193.25	46.26	175.22	193.16
	-0.10	146.27	197.10	199.27	144.29	196.94	199.23	143.77	196.90	199.22
Small	-0.09	154.11	197.64	199.41	152.31	197.52	199.37	151.83	197.48	199.37
	-0.07	169.44	198.57	199.64	168.10	198.49	199.62	167.74	198.47	199.62
	-0.05	183.14	199.27	199.82	182.32	199.23	199.81	182.10	199.22	199.80
	-0.03	193.58	199.74	199.93	193.25	199.72	199.93	193.16	199.72	199.93
	-0.01	199.27	199.97	199.99	199.23	199.97	199.99	199.22	199.97	199.99
No change	0.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00
Small	0.01	199.26	199.97	199.99	199.22	199.97	199.99	199.21	199.97	199.99
	0.03	193.57	199.73	199.93	193.24	199.72	199.93	193.15	199.72	199.93
	0.05	183.12	199.26	199.81	182.30	199.22	199.81	182.09	199.21	199.80
	0.07	169.42	198.56	199.64	168.07	198.49	199.62	167.72	198.46	199.61
	0.09	154.09	197.64	199.40	152.28	197.51	199.37	151.81	197.48	199.36
Medium	0.10	146.24	197.09	199.26	144.26	196.94	199.22	143.74	196.89	199.21
	0.30	47.72	176.57	193.57	46.52	175.49	193.24	46.23	175.20	193.15
	0.50	21.74	146.24	183.12	21.47	144.26	182.30	21.41	143.74	182.09
	0.70	12.87	116.43	169.42	12.86	114.11	168.07	12.87	113.51	167.72
	0.90	8.81	91.74	154.09	8.89	89.53	152.28	8.91	88.97	151.81
Large	1.00	7.54	81.56	146.24	7.64	79.50	144.26	7.66	78.98	143.74
	2.00	2.79	30.87	81.56	2.89	30.26	79.50	2.91	30.12	78.98
	3.00	1.53	16.33	47.72	1.60	16.23	46.52	1.62	16.22	46.23

TABLE A-21 Effect of the scale parameters in process on the ARL of control charts under a Logistic distribution (PEA_0 and $ARL_0 = 200$).

δ	Charts	EMWA			EEWMA			NEEWMA		
	β_0	1	5	10	1	5	10	1	5	10
Large	-3.00	3.73	43.92	105.23	3.82	42.44	102.18	3.84	42.09	101.41
	-2.00	6.69	76.96	142.84	6.77	74.33	140.16	6.79	73.68	139.46
	-1.00	19.52	142.84	181.83	19.17	140.16	180.69	19.10	139.46	180.38
Medium	-0.90	23.13	151.05	185.03	22.63	148.59	184.06	22.51	147.95	183.81
	-0.70	34.60	167.23	190.67	33.54	165.37	190.05	33.30	164.87	189.88
	-0.50	57.35	181.83	195.13	55.33	180.69	194.80	54.83	180.38	194.71
	-0.30	105.23	193.06	198.22	102.18	192.59	198.10	101.41	192.46	198.06
	-0.10	181.83	199.21	199.80	180.69	199.15	199.79	180.38	199.14	199.78
Small	-0.09	185.03	199.36	199.84	184.06	199.31	199.83	183.81	199.30	199.83
	-0.07	190.67	199.61	199.90	190.05	199.58	199.90	189.88	199.58	199.89
	-0.05	195.13	199.80	199.95	194.80	199.79	199.95	194.71	199.78	199.95
	-0.03	198.22	199.93	199.98	198.10	199.92	199.98	198.06	199.92	199.98
	-0.01	199.80	199.99	200.00	199.79	199.99	200.00	199.78	199.99	200.00
No change	0.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00
Small	0.01	199.80	199.99	200.00	199.79	199.99	200.00	199.78	199.99	200.00
	0.03	198.21	199.93	199.98	198.09	199.92	199.98	198.06	199.92	199.98
	0.05	195.12	199.80	199.95	194.79	199.79	199.95	194.70	199.78	199.95
	0.07	190.66	199.61	199.90	190.03	199.58	199.89	189.87	199.57	199.89
	0.09	185.01	199.35	199.84	184.05	199.31	199.83	183.79	199.30	199.82
Medium	0.10	181.81	199.20	199.80	180.67	199.15	199.79	180.36	199.13	199.78
	0.30	105.20	193.05	198.21	102.15	192.58	198.09	101.38	192.45	198.06
	0.50	57.32	181.81	195.12	55.30	180.67	194.79	54.81	180.36	194.70
	0.70	34.57	167.21	190.66	33.52	165.34	190.03	33.27	164.85	189.87
	0.90	23.12	151.03	185.01	22.61	148.56	184.05	22.49	147.92	183.79
Large	1.00	19.50	142.82	181.81	19.16	140.13	180.67	19.08	139.43	180.36
	2.00	6.68	76.93	142.82	6.76	74.30	140.13	6.78	73.64	139.43
	3.00	3.72	43.90	105.20	3.81	42.42	102.15	3.84	42.06	101.38

TABLE A-22 Effect of the scale parameters in process on the ARL of control charts under a Laplace distribution (PEA_0 and $ARL_0 = 200$).

δ	Charts	EMWA			EEWMA			NEEWMA		
	β_0	1	5	10	1	5	10	1	5	10
Large	-3.00	2.67	32.41	87.44	2.75	31.07	83.74	2.77	30.76	82.82
	-2.00	4.78	60.53	127.63	4.85	57.71	123.83	4.87	57.02	122.86
	-1.00	13.88	127.63	175.33	13.63	123.83	173.44	13.58	122.86	172.93
Medium	-0.90	16.51	137.12	179.55	16.12	133.53	177.93	16.04	132.60	177.50
	-0.70	25.11	156.68	187.12	24.22	153.77	186.05	24.01	153.01	185.77
	-0.50	43.42	175.33	193.22	41.44	173.44	192.64	40.97	172.93	192.48
	-0.30	87.44	190.38	197.51	83.74	189.56	197.29	82.82	189.34	197.23
	-0.10	175.33	198.89	199.72	173.44	198.79	199.70	172.93	198.76	199.69
Small	-0.09	179.55	199.10	199.77	177.93	199.02	199.75	177.50	198.99	199.75
	-0.07	187.12	199.45	199.86	186.05	199.40	199.85	185.77	199.39	199.85
	-0.05	193.22	199.72	199.93	192.64	199.70	199.92	192.48	199.69	199.92
	-0.03	197.51	199.90	199.98	197.29	199.89	199.97	197.23	199.89	199.97
	-0.01	199.72	199.99	200.00	199.70	199.99	200.00	199.69	199.99	200.00
No change	0.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00
Small	0.01	199.72	199.99	200.00	199.69	199.99	200.00	199.69	199.99	200.00
	0.03	197.50	199.90	199.97	197.28	199.89	199.97	197.22	199.89	199.97
	0.05	193.21	199.72	199.93	192.63	199.69	199.92	192.47	199.69	199.92
	0.07	187.11	199.45	199.86	186.04	199.40	199.85	185.75	199.39	199.85
	0.09	179.53	199.09	199.77	177.92	199.01	199.75	177.48	198.99	199.75
Medium	0.10	175.31	198.88	199.72	173.42	198.78	199.69	172.91	198.75	199.69
	0.30	87.41	190.37	197.50	83.71	189.55	197.28	82.79	189.33	197.22
	0.50	43.39	175.31	193.21	41.42	173.42	192.63	40.94	172.91	192.47
	0.70	25.09	156.65	187.11	24.20	153.75	186.04	23.99	152.98	185.75
	0.90	16.49	137.10	179.53	16.11	133.50	177.92	16.02	132.57	177.48
Large	1.00	13.86	127.60	175.31	13.62	123.80	173.42	13.57	122.83	172.91
	2.00	4.77	60.51	127.60	4.84	57.68	123.80	4.86	56.99	122.83
	3.00	2.67	32.39	87.41	2.74	31.05	83.71	2.76	30.74	82.79

TABLE A-23 ARL of the EWMA EEWMA and NEEWMA control chart using PEA_0 compared to MC under a Gamma distribution, with $ARL_0 = 200$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MC	MCA	MC	MCA	MC
0.00	ARL	200.0000	200.0176	200.0000	200.9144	200.0000	200.9141
	SDRL	198.6787	198.2380	191.5255	191.7925	189.7251	189.0882
	CPU Time	2.87	15.00	3.31	14.68	2.83	14.81
0.01	ARL	183.7241	183.3324	183.1038	183.8396	183.1580	183.4476
	SDRL	182.2853	182.7790	174.6495	174.0106	172.9488	172.5004
	CPU Time	3.41	13.75	3.31	13.69	3.34	13.72
0.03	ARL	155.9694	155.1326	154.9245	154.8704	155.0852	155.6479
	SDRL	154.2960	154.9730	146.5356	146.1489	145.0274	145.4751
	CPU Time	3.42	11.80	3.36	11.66	3.36	11.42
0.05	ARL	133.5315	133.4429	132.6205	132.9386	132.8786	132.0055
	SDRL	131.6397	131.5314	124.3248	124.9952	122.9940	122.1775
	CPU Time	3.40	10.21	3.28	10.06	3.35	10.12
0.07	ARL	115.3045	115.5517	114.7463	114.5264	115.0894	115.9417
	SDRL	113.2201	113.8008	106.5663	106.9571	105.3949	105.3699
	CPU Time	3.46	8.94	3.33	8.61	3.28	8.91
0.09	ARL	100.4016	100.8354	100.2553	100.2158	100.6704	100.0545
	SDRL	98.1535	98.1809	92.2086	92.1866	91.1786	91.9421
	CPU Time	3.31	7.83	3.28	7.64	3.28	7.78
0.10	ARL	93.9728	93.1027	94.0301	94.3607	94.4764	94.6757
	SDRL	91.6536	91.8554	86.0554	86.5448	85.0896	85.5092
	CPU Time	3.33	7.25	3.28	7.25	3.36	7.27
0.30	ARL	34.8674	34.4618	36.8400	36.7003	37.4988	37.6480
	SDRL	32.1668	32.5469	30.5439	30.3170	30.2634	30.2926
	CPU Time	3.30	2.89	3.39	2.94	3.32	3.10
0.50	ARL	19.1290	19.3120	21.1465	21.3616	21.7491	21.2415
	SDRL	16.8005	16.9543	16.2932	16.5178	16.2144	16.8113
	CPU Time	3.33	1.65	3.28	1.77	3.26	1.82
0.70	ARL	12.7248	12.6804	14.5169	14.7307	15.0388	15.0981
	SDRL	10.8257	10.7837	10.7103	10.8886	10.7056	10.6441
	CPU Time	3.28	1.21	3.26	1.28	3.29	1.30
0.90	ARL	9.4122	9.2733	10.9774	10.0028	11.4299	11.5627
	SDRL	7.8743	7.6380	7.9234	7.8819	7.9504	7.0106
	CPU Time	3.45	0.81	3.30	1.00	3.34	1.00
1.00	ARL	8.3078	8.3658	9.7731	9.8250	10.1962	10.1473
	SDRL	6.9218	6.9866	7.0163	7.9852	7.0520	7.0395
	CPU Time	3.08	0.72	2.89	0.84	2.88	0.89
2.00	ARL	3.7629	3.7394	4.6289	4.6429	4.8798	4.8864
	SDRL	3.2375	3.2491	3.4337	3.4629	3.4878	3.4209
	CPU Time	3.11	0.37	2.86	0.47	2.83	0.48
3.00	ARL	2.4185	2.4469	3.0262	3.9957	3.2031	3.2512
	SDRL	2.2330	2.2492	2.4178	2.4136	2.4678	2.4688
	CPU Time	3.10	0.26	2.86	0.33	2.81	0.35

TABLE A-24 ARL of the EWMA EEWMA and NEEWMA control chart using PEA_0 compared to MC under a Weibull distribution, with $ARL_0 = 200$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MC	MCA	MC	MCA	MC
0.00	ARL	200.0000	200.9020	200.0000	200.9982	200.0000	200.5253
	SDRL	198.6787	198.6592	191.5255	191.7418	189.7251	189.2371
	CPU Time	2.87	14.58	2.77	14.42	3.22	14.77
0.01	ARL	183.7241	183.8553	183.1038	183.7010	183.1580	183.5659
	SDRL	182.2853	182.3282	174.6495	174.8201	172.9488	172.2405
	CPU Time	3.31	13.64	3.25	13.31	3.23	13.66
0.03	ARL	155.9694	155.2087	154.9245	154.7776	155.0852	155.7206
	SDRL	154.2960	154.4528	146.5356	146.9307	145.0274	145.7491
	CPU Time	3.28	11.44	3.16	11.42	3.25	11.70
0.05	ARL	133.5315	133.8337	132.6205	132.4911	132.8786	132.0675
	SDRL	131.6397	131.6185	124.3248	124.7529	122.9940	122.5831
	CPU Time	3.25	9.95	3.23	9.75	3.26	9.91
0.07	ARL	115.3045	115.1573	114.7463	114.6226	115.0894	115.3966
	SDRL	113.2201	113.4991	106.5663	106.5038	105.3949	105.3286
	CPU Time	3.29	8.40	3.25	8.53	3.13	8.56
0.09	ARL	100.4016	100.2587	100.2553	100.2435	100.6704	100.6012
	SDRL	98.1535	98.0803	92.2086	92.6440	91.1786	91.9223
	CPU Time	3.17	7.47	3.25	7.50	3.17	7.58
0.10	ARL	93.9728	93.8835	94.0301	94.7481	94.4764	94.4855
	SDRL	91.6536	91.7545	86.0554	86.6254	85.0896	85.5050
	CPU Time	3.30	7.03	3.27	7.11	3.28	7.06
0.30	ARL	34.8674	34.9126	36.8400	36.7798	37.4988	37.5993
	SDRL	32.1668	32.9532	30.5439	30.4003	30.2634	30.4689
	CPU Time	2.97	2.70	3.30	2.93	2.96	2.95
0.50	ARL	19.1290	19.9241	21.1465	21.2145	21.7491	21.5154
	SDRL	16.8005	16.7166	16.2932	16.5103	16.2144	16.0798
	CPU Time	3.23	1.52	3.25	1.71	3.22	1.73
0.70	ARL	12.7248	12.8173	14.5169	14.5443	15.0388	15.9460
	SDRL	10.8257	10.9605	10.7103	10.7085	10.7056	10.3788
	CPU Time	3.01	1.13	2.89	1.25	3.31	1.30
0.90	ARL	9.4122	9.3097	10.9774	10.9918	11.4299	11.3950
	SDRL	7.8743	7.9417	7.9234	7.9184	7.9504	7.9436
	CPU Time	3.01	0.83	2.94	0.92	2.86	0.94
1.00	ARL	8.3078	8.4281	9.7731	9.6643	10.1962	10.2083
	SDRL	6.9218	6.9923	7.0163	7.8098	7.0520	7.1453
	CPU Time	3.25	0.77	2.90	0.86	2.83	0.87
2.00	ARL	3.7629	3.7065	4.6289	4.5795	4.8798	4.8852
	SDRL	3.2375	3.2261	3.4337	3.3983	3.4878	3.4380
	CPU Time	2.86	0.37	2.89	0.44	3.16	0.48
3.00	ARL	2.4185	2.4431	3.0262	3.0631	3.2031	3.1839
	SDRL	2.2330	2.2396	2.4178	2.4422	2.4678	2.4525
	CPU Time	3.17	0.29	3.22	0.33	2.88	0.32

TABLE A-25 ARL of the EWMA EEWMA and NEEWMA control chart using PEA_0 compared to MC under a Gumbel distribution, with $ARL_0 = 200$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MC	MCA	MC	MCA	MC
0.00	ARL	200.0000	200.3965	200.0000	200.1806	200.0000	200.5102
	SDRL	196.1607	196.3626	194.1660	194.3797	193.1480	193.1136
	CPU Time	2.83	16.01	3.17	16.33	3.49	16.20
0.01	ARL	188.1161	188.5098	187.5422	187.5193	187.6248	187.5913
	SDRL	184.3347	184.7122	181.7571	181.0682	180.8420	180.4000
	CPU Time	3.41	15.00	3.39	15.42	3.49	15.11
0.03	ARL	167.1517	167.4897	165.8877	165.9223	166.1129	166.6628
	SDRL	163.4771	163.9906	160.2073	160.5786	159.4737	159.6289
	CPU Time	3.40	13.57	3.37	13.50	3.49	13.39
0.05	ARL	149.3611	149.0566	147.8183	147.5452	148.1599	148.6158
	SDRL	145.7838	145.9481	142.2497	142.0739	141.6697	141.2141
	CPU Time	3.43	12.16	3.49	11.94	3.48	12.20
0.07	ARL	134.1828	134.6584	132.6103	132.9359	133.0468	133.4944
	SDRL	130.6962	130.1222	127.1591	127.0116	126.7092	126.5211
	CPU Time	3.48	10.91	3.40	11.06	3.36	11.08
0.09	ARL	121.1643	121.8595	119.7079	119.7528	120.2212	120.6437
	SDRL	117.7633	117.0270	114.3780	114.5031	114.0385	114.6135
	CPU Time	3.36	9.90	3.49	10.08	3.31	9.93
0.10	ARL	115.3476	115.8231	113.9824	113.7083	114.5284	114.4846
	SDRL	111.9880	111.1284	108.7142	108.3901	108.4236	108.7862
	CPU Time	3.45	9.54	3.50	9.53	3.35	9.61
0.30	ARL	52.9657	52.4744	53.6566	53.5579	54.4328	54.1481
	SDRL	50.3376	50.1977	49.6092	49.9344	49.8012	49.5124
	CPU Time	3.42	4.53	3.44	4.68	3.44	4.77
0.50	ARL	31.5738	31.2301	32.9857	32.2865	33.7267	33.7641
	SDRL	29.5457	29.0383	29.9237	29.2324	30.2384	30.2928
	CPU Time	3.39	2.73	3.41	2.92	3.50	3.03
0.70	ARL	21.7333	21.7812	23.2804	23.0395	23.9514	23.7406
	SDRL	20.1727	20.1880	20.9437	20.8954	21.2920	21.3373
	CPU Time	3.41	1.95	3.42	2.11	3.17	2.17
0.90	ARL	16.2963	16.2468	17.7998	17.7530	18.4040	18.2208
	SDRL	15.0920	15.0585	15.9937	15.9994	16.3462	16.2342
	CPU Time	3.25	1.49	3.41	1.64	3.19	1.70
1.00	ARL	14.4240	14.4349	15.8829	15.0927	16.4570	16.2095
	SDRL	13.3648	13.2719	14.2880	14.3962	14.6377	14.2619
	CPU Time	3.20	1.33	3.30	1.56	3.14	1.58
2.00	ARL	6.3788	6.3775	7.3681	7.5183	7.7396	7.7301
	SDRL	6.1150	6.2329	6.8814	6.0739	7.1602	7.2112
	CPU Time	3.23	0.63	3.36	0.73	3.14	0.75
3.00	ARL	3.9028	3.8923	4.6038	4.5744	4.8680	4.8509
	SDRL	3.9383	3.9231	4.5135	4.5293	4.7269	4.6910
	CPU Time	3.30	0.40	3.44	0.52	3.16	0.51

TABLE A-26 ARL of the EWMA EEWMA and NEEWMA control chart using PEA_0 compared to MC under a TP Reyleigh distribution, with $ARL_0 = 200$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MC	MCA	MC	MCA	MC
0.00	ARL	200.0000	200.0848	200.0000	200.6430	200.0000	200.5692
	SDRL	194.7152	194.0155	184.5122	184.5336	182.0178	182.0396
	CPU Time	2.83	15.43	2.89	15.69	2.95	15.86
0.01	ARL	181.9065	181.8728	168.8054	168.6314	169.0920	169.9677
	SDRL	176.8359	176.8662	153.6480	153.6228	151.5317	151.4215
	CPU Time	3.41	14.30	3.41	13.29	3.28	13.16
0.03	ARL	144.9066	144.0569	124.3503	124.0143	125.0517	125.2837
	SDRL	140.0134	140.1797	109.8849	109.4330	108.3525	108.8688
	CPU Time	3.39	11.48	3.41	9.95	3.34	10.25
0.05	ARL	113.1625	113.6492	95.2683	95.5510	96.2287	96.0632
	SDRL	108.2849	108.1714	81.5141	81.7420	80.3931	80.0217
	CPU Time	3.45	8.95	3.32	7.76	3.35	7.81
0.07	ARL	88.6524	88.9153	75.4611	75.6637	76.5768	76.7888
	SDRL	83.7633	83.0119	62.4162	62.5192	61.5874	61.3693
	CPU Time	3.33	7.21	3.13	6.23	3.30	6.20
0.09	ARL	70.4380	70.8176	61.4788	61.2043	62.6817	62.8971
	SDRL	65.5662	65.1295	49.1258	49.0282	48.5074	48.2881
	CPU Time	3.37	5.89	3.13	5.15	3.31	5.30
0.10	ARL	63.2079	63.7276	56.0012	56.1442	57.2301	57.1927
	SDRL	58.3606	58.2109	43.9844	43.8734	43.4488	43.0935
	CPU Time	3.28	5.21	3.36	4.80	3.06	5.00
0.30	ARL	15.6778	15.5789	17.9316	17.0335	18.9862	18.8936
	SDRL	12.2757	12.1516	10.8029	10.9844	10.7972	10.6806
	CPU Time	3.37	1.45	3.33	1.59	3.14	1.69
0.50	ARL	7.9376	7.8629	10.2320	10.3344	11.0366	11.0710
	SDRL	5.7068	5.6666	5.5124	5.6519	5.5681	5.6032
	CPU Time	3.03	0.77	3.42	0.95	2.94	1.02
0.70	ARL	5.1412	5.0836	7.0784	7.0422	7.7206	7.7139
	SDRL	3.6099	3.5840	3.6817	3.5944	3.7470	3.7362
	CPU Time	2.88	0.54	2.89	0.69	2.90	0.85
0.90	ARL	3.7338	3.7110	5.3705	5.3980	5.9031	5.9277
	SDRL	2.6473	2.6309	2.7931	2.7995	2.8578	2.8829
	CPU Time	3.09	0.39	2.87	0.53	3.18	0.61
1.00	ARL	3.2658	3.2476	4.7803	4.7890	5.2706	5.2543
	SDRL	2.3446	2.3364	2.5041	2.5311	2.5674	2.5500
	CPU Time	2.97	0.39	2.94	0.50	3.32	0.51
2.00	ARL	1.3185	1.3455	2.1693	2.1781	2.4433	2.4486
	SDRL	1.1896	1.1885	1.3437	1.3353	1.3890	1.4079
	CPU Time	2.92	0.22	2.89	0.28	3.03	0.31
3.00	ARL	0.7391	0.7433	1.3107	1.3133	1.5017	1.4752
	SDRL	0.8555	0.8550	1.0085	1.0181	1.0443	1.0271
	CPU Time	2.89	0.13	2.99	0.17	2.85	0.21

TABLE A-27 Effect of control charts parameters on ARL under a Gamma distribution (PEA_0 and $ARL_0 = 200$).

	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
δ	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
No change	0.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00
	0.01	183.72	186.66	189.02	183.10	183.45	184.63	183.16	183.64	186.12
	0.03	155.97	163.38	169.40	154.92	155.96	159.26	155.09	156.50	163.18
Small	0.05	133.53	143.87	152.47	132.62	134.28	139.36	132.88	135.15	145.12
	0.07	115.30	127.43	137.77	114.75	116.96	123.49	115.09	118.10	130.63
	0.09	100.40	113.48	124.97	100.26	102.94	110.64	100.67	104.31	118.81
	0.10	93.97	107.29	119.18	94.03	96.92	105.11	94.48	98.39	113.69
	0.30	34.87	43.45	53.92	36.84	41.13	52.34	37.50	43.28	62.82
Medium	0.50	19.13	23.44	30.19	21.15	25.11	35.41	21.75	27.10	44.84
	0.70	12.72	14.98	19.37	14.52	17.98	27.07	15.04	19.74	35.37
	0.90	9.41	10.65	13.62	10.98	14.00	22.03	11.43	15.55	29.40
	1.00	8.31	9.23	11.72	9.77	12.61	20.19	10.20	14.07	27.15
Large	2.00	3.76	3.74	4.37	4.63	6.34	11.12	4.88	7.25	15.61
	3.00	2.42	2.29	2.53	3.03	4.25	7.73	3.20	4.90	11.03

TABLE A-28 Effect of control charts parameters on ARL under a Weibull distribution XXXXX

	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
δ	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
No change	0.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00
	0.01	183.72	186.66	189.02	183.10	183.45	184.63	183.16	183.64	186.12
	0.03	155.97	163.38	169.40	154.92	155.96	159.26	155.09	156.50	163.18
Small	0.05	133.53	143.87	152.47	132.62	134.28	139.36	132.88	135.15	145.12
	0.07	115.30	127.43	137.77	114.75	116.96	123.49	115.09	118.10	130.63
	0.09	100.40	113.48	124.97	100.26	102.94	110.64	100.67	104.31	118.81
	0.10	93.97	107.29	119.18	94.03	96.92	105.11	94.48	98.39	113.69
	0.30	34.87	43.45	53.92	36.84	41.13	52.34	37.50	43.28	62.82
Medium	0.50	19.13	23.44	30.19	21.15	25.11	35.41	21.75	27.10	44.84
	0.70	12.72	14.98	19.37	14.52	17.98	27.07	15.04	19.74	35.37
	0.90	9.41	10.65	13.62	10.98	14.00	22.03	11.43	15.55	29.40
	1.00	8.31	9.23	11.72	9.77	12.61	20.19	10.20	14.07	27.15
Large	2.00	3.76	3.74	4.37	4.63	6.34	11.12	4.88	7.25	15.61
	3.00	2.42	2.29	2.53	3.03	4.25	7.73	3.20	4.90	11.03

TABLE A-29 Effect of control charts parameters on ARL under a Gumbel distribution (PEA_0 and $ARL_0 = 200$).

δ	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
No change	0.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00
Small	0.01	188.12	188.17	189.36	187.54	188.02	189.22	187.62	188.25	190.33
	0.03	167.15	167.27	170.30	165.89	167.19	170.40	166.11	167.80	173.37
	0.05	149.36	149.49	153.80	147.82	149.79	154.60	148.16	150.70	159.02
	0.07	134.18	134.28	139.44	132.61	135.12	141.21	133.05	136.28	146.76
	0.09	121.16	121.18	126.89	119.71	122.65	129.75	120.22	124.01	136.20
Medium	0.10	115.35	115.31	121.21	113.98	117.11	124.64	114.53	118.55	131.46
	0.30	52.97	51.49	56.45	53.66	58.08	68.40	54.43	60.08	77.60
	0.50	31.57	29.33	32.41	32.99	37.24	46.95	33.73	39.12	55.62
	0.70	21.73	19.29	21.23	23.28	27.17	35.89	23.95	28.87	43.71
	0.90	16.30	13.88	15.14	17.80	21.35	29.17	18.40	22.88	36.20
Large	1.00	14.42	12.07	13.07	15.88	19.29	26.71	16.46	20.75	33.39
	2.00	6.38	4.73	4.60	7.37	9.75	14.77	7.74	10.80	19.17
	3.00	3.90	2.70	2.42	4.60	6.40	10.38	4.87	7.25	13.67

TABLE A-30 Effect of control charts parameters on ARL under a TP Reyleigh distribution (PEA_0 and $ARL_0 = 200$).

δ	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
No change	0.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00
Small	0.01	181.91	174.56	177.79	168.81	170.41	174.18	169.09	171.11	178.00
	0.03	144.91	134.47	142.04	124.35	128.32	137.46	125.05	130.08	146.28
	0.05	113.16	105.29	115.05	95.27	100.77	113.20	96.23	103.20	124.82
	0.07	88.65	83.83	94.37	75.46	81.90	96.32	76.58	84.75	109.49
	0.09	70.44	67.83	78.31	61.48	68.47	84.04	62.68	71.58	98.04
Medium	0.10	63.21	61.37	71.63	56.00	63.17	79.09	57.23	66.36	93.33
	0.30	15.68	15.03	18.54	17.93	24.38	39.07	18.99	27.35	51.87
	0.50	7.94	6.93	8.01	10.23	15.30	27.32	11.04	17.70	37.91
	0.70	5.14	4.18	4.50	7.08	11.20	21.31	7.72	13.20	30.33
	0.90	3.73	2.88	2.93	5.37	8.84	17.56	5.90	10.55	25.42
Large	1.00	3.27	2.47	2.45	4.78	7.99	16.15	5.27	9.58	23.54
	2.00	1.32	0.88	0.77	2.17	4.01	9.00	2.44	4.96	13.66
	3.00	0.74	0.47	0.39	1.31	2.61	6.21	1.50	3.28	9.62

TABLE A-31 Effect of the shape parameters in process on the ARL of control charts under a Gamma distribution (PEA_0 and $ARL_0 = 200$).

δ	Chart	EWMA			EEWMA			NEEWMA		
	α_0	1	5	10	1	5	10	1	5	10
No change	0.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00
Small	0.01	183.72	179.03	175.94	183.10	166.98	156.56	183.16	167.25	157.01
	0.03	155.97	137.63	123.52	154.92	120.69	102.61	155.09	121.37	103.65
	0.05	133.53	104.16	84.52	132.62	91.04	72.53	132.88	92.00	73.88
	0.07	115.30	79.67	59.59	114.75	71.24	54.46	115.09	72.35	55.95
	0.09	100.40	62.20	43.84	100.26	57.48	42.87	100.67	58.70	44.42
Medium	0.10	93.97	55.44	38.20	94.03	52.16	38.59	94.48	53.40	40.14
	0.30	34.87	13.20	8.20	36.84	16.29	12.12	37.50	17.36	13.26
	0.50	19.13	6.63	4.11	21.15	9.28	7.05	21.75	10.08	7.89
	0.70	12.72	4.27	2.61	14.52	6.40	4.90	15.04	7.05	5.56
	0.90	9.41	3.08	1.85	10.98	4.85	3.71	11.43	5.38	4.26
Large	1.00	8.31	2.68	1.59	9.77	4.31	3.30	10.20	4.80	3.79
	2.00	3.76	1.02	0.49	4.63	1.92	1.43	4.88	2.19	1.70
	3.00	2.42	0.53	0.17	3.03	1.14	0.80	3.20	1.33	1.00

TABLE A-32 Effect of the shape parameters in process on the ARL of control charts under a Weibull distribution (PEA_0 and $ARL_0 = 200$).

δ	Chart	EWMA			EEWMA			NEEWMA		
	α_0	1	5	10	1	5	10	1	5	10
No change	0.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00
Small	0.01	183.72	256.09	303.39	183.10	257.73	306.51	183.16	258.20	307.46
	0.03	155.97	455.02	911.67	154.92	468.71	981.29	155.09	472.67	>307.46
	0.05	133.53	894.44	>911.67	132.62	957.17	>981.29	132.88	975.70	>307.46
	0.07	115.30	>894.44	>911.67	114.75	>957.17	>981.29	115.09	>975.70	>307.46
	0.09	100.40	>894.44	>911.67	100.26	>957.17	>981.29	100.67	>975.70	>307.46
Medium	0.10	93.97	>894.44	>911.67	94.03	>957.17	>981.29	94.48	>975.70	>307.46
	0.30	34.87	>894.44	>911.67	36.84	>957.17	>981.29	37.50	>975.70	>307.46
	0.50	19.13	>894.44	>911.67	21.15	>957.17	>981.29	21.75	>975.70	>307.46
	0.70	12.72	>894.44	>911.67	14.52	>957.17	>981.29	15.04	>975.70	>307.46
	0.90	9.41	>894.44	>911.67	10.98	>957.17	>981.29	11.43	>975.70	>307.46
Large	1.00	8.31	>894.44	>911.67	9.77	>957.17	>981.29	10.20	>975.70	>307.46
	2.00	3.76	>894.44	>911.67	4.63	>957.17	>981.29	4.88	>975.70	>307.46
	3.00	2.42	>894.44	>911.67	3.03	>957.17	>981.29	3.20	>975.70	>307.46

TABLE A-33 Effect of the location parameters in process on the ARL of control charts under a Gumbel distribution (PEA_0 and $ARL_0 = 200$).

δ	Chart	EWMA			EEWMA			NEEWMA		
	θ_0	1	5	10	1	5	10	1	5	10
No change	0.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00
Small	0.01	188.12	188.12	188.12	187.54	188.75	189.24	187.62	188.96	189.48
	0.03	167.15	167.15	167.15	165.89	169.09	170.42	166.11	169.68	171.08
	0.05	149.36	149.36	149.36	147.82	152.58	154.58	148.16	153.47	155.57
	0.07	134.18	134.18	134.18	132.61	138.60	141.14	133.05	139.72	142.40
	0.09	121.16	121.16	121.16	119.71	126.66	129.63	120.22	127.98	131.12
Medium	0.10	115.35	115.35	115.35	113.98	121.34	124.50	114.53	122.74	126.08
	0.30	52.97	52.97	52.97	53.66	63.74	68.44	54.43	65.81	70.85
	0.50	31.57	31.57	31.57	32.99	42.74	47.58	33.73	44.83	50.08
	0.70	21.73	21.73	21.73	23.28	32.31	37.03	23.95	34.30	39.47
	0.90	16.30	16.30	16.30	17.80	26.14	30.65	18.40	28.01	32.99
Large	1.00	14.42	14.42	14.42	15.88	23.91	28.31	16.46	25.72	30.61
	2.00	6.38	6.38	6.38	7.37	13.30	16.77	7.74	14.65	18.64
	3.00	3.90	3.90	3.90	4.60	9.46	12.30	4.87	10.52	13.86

TABLE A-34 Effect of the location parameters in process on the ARL of control charts under a TP Reyleigh distribution XXXXX

δ	Chart	EWMA			EEWMA			NEEWMA		
	θ_0	1	5	10	1	5	10	1	5	10
No change	0.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00
Small	0.01	181.91	181.91	181.91	168.81	171.66	172.85	169.09	172.17	173.41
	0.03	144.91	144.91	144.91	124.35	130.88	133.64	125.05	132.10	134.99
	0.05	113.16	113.16	113.16	95.27	103.82	107.50	96.23	105.50	109.35
	0.07	88.65	88.65	88.65	75.46	85.12	89.36	76.58	87.08	91.52
	0.09	70.44	70.44	70.44	61.48	71.72	76.29	62.68	73.86	78.66
Medium	0.10	63.21	63.21	63.21	56.00	66.40	71.09	57.23	68.61	73.53
	0.30	15.68	15.68	15.68	17.93	27.22	32.07	18.99	29.47	34.68
	0.50	7.94	7.94	7.94	10.23	17.86	22.26	11.04	19.83	24.64
	0.70	5.14	5.14	5.14	7.08	13.51	17.50	7.72	15.26	19.69
	0.90	3.73	3.73	3.73	5.37	10.93	14.58	5.90	12.49	16.59
Large	1.00	3.27	3.27	3.27	4.78	9.99	13.48	5.27	11.47	15.42
	2.00	1.32	1.32	1.32	2.17	5.36	7.82	2.44	6.35	9.23
	3.00	0.74	0.74	0.74	1.31	3.62	5.51	1.50	4.36	6.63

TABLE A-35 ARL of the EWMA EEWMA and NEEWMA control chart using PEA_0 compared to MC under a Poisson distribution, with $ARL_0 = 200$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MC	MCA	MC	MCA	MC
0	ARL	199.9770	199.2251	199.9989	199.9113	199.9943	199.7593
	SDRL	179.3467	179.1226	175.3261	175.7531	174.1898	174.6942
	CPU Time	2.95	13.93	2.84	14.32	2.81	14.10
1	ARL	27.8391	27.9158	29.5713	29.5669	30.0657	30.2076
	SDRL	15.1783	15.3632	14.7186	14.7993	14.6040	14.6993
	CPU Time	3.47	2.25	3.34	2.23	3.32	2.36
2	ARL	13.4649	13.5259	14.9284	14.9690	15.3400	15.3423
	SDRL	5.2513	5.3304	5.2417	5.2770	5.2415	5.2712
	CPU Time	3.42	1.17	3.16	1.17	3.36	1.25
3	ARL	8.8386	8.8575	10.0165	10.9878	10.3472	10.3482
	SDRL	2.9375	2.9024	2.9898	2.9919	3.0046	3.0280
	CPU Time	3.42	0.77	3.14	0.85	3.24	0.87
4	ARL	6.5418	6.5403	7.5179	7.5307	7.7922	7.8142
	SDRL	2.0060	2.9994	2.0662	2.0547	2.0828	2.1020
	CPU Time	2.93	0.57	2.90	0.63	3.33	0.69
5	ARL	5.1601	5.1378	5.9917	5.0062	6.2255	6.2193
	SDRL	1.5183	1.5132	1.5759	1.5936	1.5916	1.5834
	CPU Time	2.90	0.50	2.93	0.54	3.33	0.61
10	ARL	2.3701	2.3691	2.8442	2.8343	2.9792	2.9898
	SDRL	0.7050	0.7059	0.7453	0.7503	0.7552	0.7537
	CPU Time	3.17	0.27	2.92	0.29	3.36	0.31
15	ARL	1.4053	1.4125	1.7626	1.7611	1.8596	1.8546
	SDRL	0.5181	0.5186	0.5421	0.5399	0.5356	0.5377
	CPU Time	3.31	0.19	2.99	0.25	3.22	0.22
20	ARL	0.9935	0.9852	1.1685	1.1646	1.2323	1.2288
	SDRL	0.3051	0.3000	0.3903	0.3890	0.4291	0.4267
	CPU Time	3.34	0.16	3.34	0.17	3.21	0.21
25	ARL	0.7459	0.7401	0.9477	0.9481	0.9887	0.9931
	SDRL	0.4386	0.4425	0.2733	0.2732	0.2307	0.2317
	CPU Time	3.36	0.16	3.10	0.14	3.54	0.16
50	ARL	0.0010	0.0009	0.0099	0.0102	0.0213	0.0203
	SDRL	0.0309	0.0300	0.0991	0.1005	0.1442	0.1410
	CPU Time	3.76	0.08	3.11	0.08	3.41	0.07
100	ARL	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	SDRL	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	CPU Time	3.83	0.08	3.32	0.08	3.27	0.09

TABLE A-36 ARL of the EWMA EEWMA and NEEWMA control chart using PEA_0 compared to MC under a Binomial distribution, with $ARL_0 = 200$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MC	MCA	MC	MCA	MC
0	ARL	199.8689	199.3604	199.4673	199.9030	199.9184	199.9765
	SDRL	208.3996	208.0157	208.4969	208.3311	209.0525	209.8318
	CPU Time	3.01	14.55	2.88	14.45	3.00	14.35
1	ARL	130.6847	130.4573	129.1587	129.7580	129.2461	129.7777
	SDRL	136.8519	136.8124	135.4668	135.0903	135.6066	135.8198
	CPU Time	3.42	9.70	3.38	9.44	3.34	9.48
2	ARL	92.5763	92.2282	90.9982	90.8591	90.9619	90.4014
	SDRL	97.1280	97.9345	95.5088	95.7668	95.4929	95.5769
	CPU Time	3.44	7.44	3.25	6.78	3.40	6.68
3	ARL	69.4594	69.5266	68.0805	68.1837	68.0054	68.8392
	SDRL	72.8626	72.3155	71.3513	71.6202	71.2769	71.4014
	CPU Time	3.70	6.21	3.39	5.26	3.38	5.08
4	ARL	54.4017	54.7426	53.2537	53.6425	53.1715	53.5595
	SDRL	56.9640	56.2254	55.6408	55.5267	55.5473	55.2689
	CPU Time	3.51	4.91	3.33	4.00	3.40	4.05
5	ARL	44.0423	44.2690	43.0996	43.0987	43.0222	43.2749
	SDRL	45.9746	45.2623	44.8404	44.0090	44.7444	44.4956
	CPU Time	3.76	3.68	3.38	3.28	3.42	3.30
10	ARL	20.8313	20.1042	20.4602	20.3517	20.4238	20.1793
	SDRL	21.1974	21.6041	20.6723	20.0390	20.6121	20.2528
	CPU Time	3.46	1.71	3.36	1.67	3.36	1.59
15	ARL	12.9787	12.0830	12.8060	12.8404	12.7905	12.6502
	SDRL	12.8192	12.8670	12.5460	12.5547	12.5134	12.5196
	CPU Time	3.64	1.06	3.26	1.14	3.13	1.03
20	ARL	9.2538	9.2290	9.1625	9.0060	9.1557	9.2032
	SDRL	8.8929	8.7033	8.7352	8.5483	8.7176	8.7517
	CPU Time	3.17	0.80	3.32	0.78	3.15	0.78
25	ARL	7.1308	7.2677	7.0782	7.1299	7.0751	7.1315
	SDRL	6.6940	6.7561	6.5956	6.6437	6.5861	6.6649
	CPU Time	3.41	0.62	3.36	0.65	3.15	0.62
50	ARL	3.2178	3.1926	3.2118	3.1726	3.2117	3.2300
	SDRL	2.8581	2.7995	2.8412	2.7661	2.8407	2.8170
	CPU Time	3.42	0.33	3.11	0.32	3.16	0.32
100	ARL	1.4044	1.4075	1.4042	1.3775	1.4042	1.4200
	SDRL	1.3735	1.3700	1.3727	1.3775	1.3727	1.3945
	CPU Time	3.38	0.17	3.28	0.17	3.25	0.19

TABLE A-37 ARL of the EWMA EEWMA and NEEWMA control chart using PEA_0 compared to MC under a Negative Binomial distribution, with ARL_0 = 200.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MC	MCA	MC	MCA	MC
0	ARL	199.9557	199.1323	199.9478	199.0719	199.9513	199.4818
	SDRL	180.3129	180.3930	176.2607	176.8560	175.1335	175.9216
	CPU Time	3.08	14.89	3.36	14.96	3.29	15.19
1	ARL	29.5087	29.2585	31.0409	31.1122	31.4886	31.3977
	SDRL	16.7480	16.5453	16.0587	16.1134	15.8898	15.7468
	CPU Time	3.64	2.42	3.59	2.53	3.58	2.64
2	ARL	13.9798	13.9387	15.3962	15.3502	15.7978	15.9073
	SDRL	5.5931	5.5394	5.5310	5.5050	5.5188	5.6433
	CPU Time	3.19	1.22	3.65	1.32	3.64	1.44
3	ARL	9.0857	9.0747	10.2475	10.2838	10.5757	10.5624
	SDRL	3.0551	3.0707	3.0904	3.0816	3.1015	3.1438
	CPU Time	3.40	0.80	3.52	0.95	3.47	0.97
4	ARL	6.6895	6.6740	7.6591	7.6686	7.9331	7.9486
	SDRL	2.0587	2.0250	2.1120	2.1174	2.1271	2.1128
	CPU Time	3.48	0.64	3.25	0.67	3.53	0.78
5	ARL	5.2601	5.2454	6.0889	6.0661	6.3233	6.3249
	SDRL	1.5456	1.5334	1.6001	1.5970	1.6151	1.6107
	CPU Time	3.69	0.52	3.16	0.56	3.70	0.61
10	ARL	2.4022	2.4148	2.8787	2.8868	3.0141	3.0151
	SDRL	0.7088	0.7123	0.7471	0.7467	0.7569	0.7540
	CPU Time	3.34	0.27	3.14	0.32	3.38	0.31
15	ARL	1.4306	1.4339	1.7860	1.7856	1.8821	1.8869
	SDRL	0.5224	0.5230	0.5330	0.5354	0.5270	0.5276
	CPU Time	3.32	0.19	3.15	0.25	3.25	0.22
20	ARL	0.9932	0.9940	1.1816	1.1884	1.2521	1.2558
	SDRL	0.3055	0.2993	0.4029	0.4093	0.4432	0.4441
	CPU Time	3.33	0.17	3.35	0.17	3.20	0.19
25	ARL	0.7658	0.7619	0.9541	0.9530	0.9853	0.9832
	SDRL	0.4255	0.4278	0.2575	0.2623	0.2348	0.2296
	CPU Time	3.30	0.14	3.33	0.17	3.17	0.20
50	ARL	0.0005	0.0005	0.0100	0.0105	0.0193	0.0172
	SDRL	0.0230	0.0224	0.0997	0.1019	0.1376	0.1300
	CPU Time	3.22	0.08	3.22	0.07	3.22	0.08
100	ARL	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	SDRL	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	CPU Time	3.15	0.08	3.21	0.08	3.23	0.10

TABLE A-38 Effect of control charts parameters on ARL under a Poisson distribution (PEA_0 and $ARL_0 = 200$).

δ	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
No change	0	199.98	200.00	199.99	200.00	199.88	199.61	199.99	199.98	199.94
Small	1	27.84	27.33	41.08	29.57	33.05	43.49	30.07	34.96	53.70
	2	13.46	9.48	13.26	14.93	17.78	26.00	15.34	19.31	33.86
	3	8.84	5.03	5.75	10.02	12.31	18.90	10.35	13.54	25.21
	4	6.54	3.24	3.01	7.52	9.43	14.91	7.79	10.44	20.18
	5	5.16	2.31	1.77	5.99	7.62	12.32	6.23	8.49	16.85
Medium	10	2.37	0.74	0.21	2.84	3.78	6.53	2.98	4.29	9.19
	15	1.41	0.22	0.02	1.76	2.41	4.36	1.86	2.78	6.25
	20	0.99	0.04	0.00	1.17	1.73	3.21	1.23	2.01	4.68
	25	0.75	0.00	0.00	0.95	1.22	2.50	0.99	1.49	3.71
Large	50	0.00	0.00	0.00	0.01	0.23	1.01	0.02	0.54	1.79
	100	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.82

TABLE A-39 Effect of control charts parameters on ARL under a Binomial distribution (PEA_0 and $ARL_0 = 200$).

δ	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
No change	0	199.87	186.47	186.64	199.47	199.89	199.88	199.92	199.97	199.99
Small	1	130.68	131.49	131.66	129.16	128.56	127.67	129.25	128.31	128.11
	2	92.58	98.21	98.37	91.00	90.22	89.67	90.96	89.96	90.76
	3	69.46	76.49	76.64	68.08	67.36	67.27	68.01	67.16	68.84
	4	54.40	61.48	61.63	53.25	52.64	52.93	53.17	52.51	54.79
	5	44.04	50.66	50.80	43.10	42.60	43.16	43.02	42.52	45.17
Medium	10	20.83	24.42	24.54	20.46	20.31	21.25	20.42	20.34	23.32
	15	12.98	14.78	14.86	12.81	12.76	13.60	12.79	12.81	15.46
	20	9.25	10.09	10.15	9.16	9.16	9.83	9.16	9.20	11.48
	25	7.13	7.42	7.47	7.08	7.09	7.61	7.08	7.12	9.08
Large	50	3.22	2.70	2.71	3.21	3.22	3.35	3.21	3.22	4.14
	100	1.40	0.86	0.86	1.40	1.40	1.41	1.40	1.40	1.62

TABLE A-40 Effect of control charts parameters on ARL under a Negative Binomial distribution (PEA_0 and $ARL_0 = 200$).

δ	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
No change	0	199.96	199.93	199.94	199.95	199.96	199.97	199.95	199.96	199.98
Small	1	29.51	33.77	56.41	31.04	34.24	44.25	31.49	36.03	54.23
	2	13.98	11.50	20.44	15.40	18.18	26.29	15.80	19.68	34.10
	3	9.09	5.86	9.07	10.25	12.52	19.07	10.58	13.73	25.35
	4	6.69	3.67	4.70	7.66	9.55	15.02	7.93	10.57	20.28
	5	5.26	2.56	2.72	6.09	7.71	12.40	6.32	8.58	16.93
Medium	10	2.40	0.79	0.35	2.88	3.82	6.56	3.01	4.32	9.23
	15	1.43	0.24	0.04	1.79	2.44	4.38	1.88	2.80	6.28
	20	0.99	0.03	0.00	1.18	1.75	3.23	1.25	2.01	4.71
	25	0.77	0.00	0.00	0.95	1.23	2.52	0.99	1.52	3.73
Large	50	0.00	0.00	0.00	0.01	0.26	1.01	0.02	0.57	1.80
	100	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.84

TABLE A-41 Effect of the parameters in process on the ARL of control charts under a Poisson distribution (PEA_0 and $ARL_0 = 200$).

δ	Charts	EWMA			EEWMA			NEEWMA		
	θ_0	5	10	50	5	10	50	5	10	50
No change	0	199.98	199.95	199.15	199.61	199.97	199.65	199.94	199.96	199.92
Small	1	27.84	42.37	88.72	43.49	61.05	108.79	53.70	73.59	123.46
	2	13.46	21.62	53.38	26.00	38.85	78.56	33.86	49.71	95.21
	3	8.84	14.64	38.29	18.90	29.44	63.89	25.21	38.69	80.22
	4	6.54	11.13	30.31	14.91	23.98	55.01	20.18	32.01	70.56
	5	5.16	8.99	25.39	12.32	20.32	48.90	16.85	27.43	63.62
Medium	10	2.37	4.52	14.92	6.53	11.67	33.31	9.19	16.22	44.83
	15	1.41	2.93	10.89	4.36	8.17	26.05	6.25	11.54	35.60
	20	0.99	2.12	8.63	3.21	6.25	21.59	4.68	8.92	29.80
	25	0.75	1.62	7.16	2.50	5.03	18.51	3.71	7.25	25.73
Large	50	0.00	0.68	3.83	1.01	2.36	10.91	1.79	3.65	15.49
	100	0.00	0.00	1.97	0.01	1.00	5.96	0.82	1.80	8.63

TABLE A-42 Effect of the parameters in process on the ARL of control charts under a Binomial distribution (PEA_0 and $ARL_0 = 200$).

δ	Charts	EWMA			EEWMA			NEEWMA		
	α_0	5	10	50	5	10	50	5	10	50
No change	0	199.87	199.95	199.82	199.88	199.85	199.15	199.99	199.93	199.95
Small	1	130.68	148.59	176.76	127.67	146.63	175.86	128.11	147.20	177.40
	2	92.58	114.43	157.20	89.67	112.59	156.51	90.76	113.79	158.74
	3	69.46	90.75	140.52	67.27	89.63	140.30	68.84	91.36	143.16
	4	54.40	73.76	126.22	52.93	73.46	126.61	54.79	75.56	130.01
	5	44.04	61.21	113.90	43.16	61.64	114.95	45.17	63.99	118.83
Medium	10	20.83	30.00	72.44	21.25	32.41	76.55	23.32	35.01	81.84
	15	12.98	18.43	50.09	13.60	21.27	56.05	15.46	23.60	61.76
	20	9.25	12.84	36.91	9.83	15.62	43.77	11.48	17.65	49.45
	25	7.13	9.65	28.57	7.61	12.26	35.73	9.08	14.03	41.20
Large	50	3.22	3.93	12.18	3.35	5.66	18.35	4.14	6.74	22.46
	100	1.40	1.51	5.12	1.41	2.44	9.13	1.62	3.17	11.70

TABLE A-43 Effect of the parameters in process on the ARL of control charts under a Negative Binomial distribution (PEA_0 and $ARL_0 = 200$).

δ	Charts	EWMA			EEWMA			NEEWMA		
	β_0	5	10	50	5	10	50	5	10	50
No change	0	199.96	199.92	199.83	199.97	199.98	199.94	199.98	199.98	199.96
Small	1	29.51	44.28	90.48	44.25	61.87	109.56	54.23	74.15	123.89
	2	13.98	22.31	54.47	26.29	39.21	79.06	34.10	50.00	95.54
	3	9.09	14.98	38.95	19.07	29.66	64.23	25.35	38.88	80.46
	4	6.69	11.34	30.73	15.02	24.13	55.26	20.28	32.16	70.76
	5	5.26	9.13	25.69	12.40	20.44	49.10	16.93	27.55	63.79
Medium	10	2.40	4.56	15.01	6.56	11.72	33.40	9.23	16.28	44.95
	15	1.43	2.95	10.94	4.38	8.21	26.11	6.28	11.58	35.69
	20	0.99	2.13	8.66	3.23	6.28	21.64	4.71	8.96	29.88
	25	0.77	1.64	7.18	2.52	5.06	18.56	3.73	7.28	25.81
Large	50	0.00	0.69	3.84	1.01	2.38	10.94	1.80	3.67	15.54
	100	0.00	0.00	1.97	0.01	1.00	5.97	0.84	1.82	8.67

Appendix B

Appendix B
Table of Results with $ARL_0 = 500$

TABLE B-1 ARL of the MEWMA control chart using JDA compared to MC under a standard normal process, with $ARL_0 = 500$ and $k = 0.1$

δ	$\lambda_1 = 0.1$			$\lambda_1 = 0.7$		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	5.9925 (2.23)	5.9586±1.6265 (0.13)	0.0339	1.408 (2.16)	1.4228±1.1791 (0.03)	0.0148
-2.00	11.1413 (2.12)	11.1895±2.451 (0.19)	0.0482	5.074 (2.2)	5.108±2.2109 (0.08)	0.0340
-1.00	37.2467 (2.17)	37.2817±5.367 (0.57)	0.0350	38.4247 (2.13)	38.9092±6.199 (0.59)	0.4845
-0.90	45.2553 (2.14)	45.773±6.0149 (0.68)	0.5177	50.082 (2.17)	50.2211±7.0412 (0.75)	0.1391
-0.70	71.5918 (2.16)	71.7901±7.7956 (1.08)	0.1983	88.3043 (2.07)	88.1676±9.3319 (1.33)	0.1367
-0.50	126.2654 (2.12)	126.6588±10.7346 (1.88)	0.3934	162.3353 (2.15)	162.2103±12.7163 (2.44)	0.1250
-0.30	246.6884 (2.16)	246.1849±15.5306 (3.66)	0.5035	298.1717 (2.08)	298.4886±17.3561 (4.45)	0.3169
-0.10	450.0011 (2.18)	450.714±20.8162 (6.54)	0.7129	466.8476 (2.16)	466.3744±21.4576 (6.65)	0.4732
-0.07	474.2444 (2.19)	474.5699±21.7002 (7)	0.3255	483.3024 (2.18)	483.0498±22.0882 (7.07)	0.2526
-0.05	486.5453 (2.11)	486.1291±21.9108 (7.08)	0.4162	491.4106 (2.22)	491.8297±22.3819 (7.32)	0.4191
-0.03	495.093 (2.19)	495.9456±21.884 (7.17)	0.8526	496.9532 (2.16)	496.6745±22.3 (7.15)	0.2787
-0.01	499.4754 (2.17)	499.1392±22.116 (7.21)	0.3362	499.7668 (2.17)	499.4221±22.6191 (7.34)	0.3447
0.00	500.0276 (2.12)	500.3895±22.4659 (7.31)	0.3619	500.1204 (2.16)	500.8024±22.0533 (7.22)	0.6820
0.01	499.473 (2.11)	499.0997±21.7214 (7.02)	0.3733	499.7666 (2.18)	499.1954±22.2971 (7.24)	0.5712
0.03	495.0859 (2.15)	495.7677±22.1983 (7.28)	0.6818	496.9526 (2.18)	496.3089±22.3753 (7.25)	0.6437
0.05	486.5337 (2.23)	486.2978±21.7961 (7.11)	0.2359	491.4096 (2.16)	491.6794±22.1258 (7.18)	0.2698
0.07	474.2285 (2.11)	474.9658±21.6096 (6.91)	0.7373	483.301 (2.16)	483.3713±22.0005 (7.08)	0.0703
0.10	449.9794 (2.15)	449.4062±20.851 (6.55)	0.5732	466.8457 (2.15)	466.9478±21.6458 (6.84)	0.1021
0.30	246.6511 (2.11)	246.1091±15.4106 (3.61)	0.5420	298.1677 (2.1)	298.4301±17.6359 (4.47)	0.2624
0.50	126.2312 (2.14)	126.1011±10.7021 (1.86)	0.1301	162.3312 (2.22)	162.9856±12.7593 (2.42)	0.6544
0.70	71.5628 (2.13)	71.5788±7.9529 (1.08)	0.0160	88.3006 (2.12)	88.211±9.3739 (1.32)	0.0896
0.90	45.2306 (2.13)	45.4218±6.0601 (0.72)	0.1912	50.0786 (2.12)	50.9508±7.034 (0.75)	0.8722
1.00	37.2238 (2.14)	37.3814±5.4752 (0.58)	0.1576	38.4215 (2.14)	38.6009±6.2049 (0.6)	0.1794
2.00	11.1284 (2.21)	11.1255±2.4526 (0.21)	0.0029	5.072 (2.2)	5.0747±2.1761 (0.09)	0.0027
3.00	5.9836 (2.09)	5.9693±1.6164 (0.13)	0.0143	1.4066 (2.14)	1.4064±1.1663 (0.03)	0.0002

TABLE B-2 ARL of the MEWMA control chart using JDA compared to MC under a standard normal process, with $ARL_0 = 500$ and $k = 0.7$

δ	$\lambda_1 = 0.1$			$\lambda_1 = 0.7$		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	85.93 (2.22)	85.1767±8.6633 (1.28)	0.7533	11.2082 (2.11)	11.1635±3.2646 (0.2)	0.0447
-2.00	163.1428 (2.1)	163.4631±12.494 (2.39)	0.3203	35.5569 (2.09)	35.236±5.8805 (0.55)	0.3209
-1.00	333.8336 (2.09)	333.9786±17.953 (4.85)	0.1450	154.8856 (2.09)	154.2058±12.3352 (2.3)	0.6798
-0.90	356.6231 (2.07)	356.5611±18.4575 (5.11)	0.0620	181.7641 (2.09)	181.2122±13.5541 (2.71)	0.5519
-0.70	402.5749 (2.19)	402.4485±20.0671 (5.89)	0.1264	249.4888 (2.14)	249.9776±15.5862 (3.58)	0.4888
-0.50	445.2442 (2.16)	445.5251±21.0672 (6.5)	0.2809	335.2923 (2.13)	335.9146±18.1728 (4.89)	0.6223
-0.30	478.8689 (2.16)	478.0169±21.9904 (6.93)	0.8520	426.7031 (2.2)	426.4546±20.7115 (6.3)	0.2485
-0.10	497.5825 (2.14)	497.6245±22.2394 (7.2)	0.0420	490.7876 (2.08)	490.3187±21.9644 (7.1)	0.4689
-0.07	498.823 (2.12)	498.4713±22.2264 (7.25)	0.3517	495.4573 (2.12)	495.2471±21.9187 (7.22)	0.2102
-0.05	499.4087 (2.16)	499.2058±22.1222 (7.14)	0.2029	497.6826 (2.22)	497.3127±22.4946 (7.41)	0.3699
-0.03	499.7998 (2.17)	499.1482±22.3755 (7.4)	0.6516	499.1761 (2.2)	499.1832±22.2935 (7.35)	0.0071
-0.01	499.9953 (2.22)	499.6352±22.011 (7.27)	0.3601	499.9258 (2.13)	499.0044±22.394 (7.42)	0.9214
0.00	500.0195 (2.25)	500.8829±22.2911 (7.29)	0.8634	500.0196 (2.2)	500.4851±22.0424 (7.25)	0.4655
0.01	499.9948 (2.19)	499.0091±22.3608 (7.28)	0.9857	499.9257 (2.22)	499.9742±22.1466 (7.31)	0.0485
0.03	499.7983 (2.17)	499.2356±22.1618 (7.19)	0.5627	499.1758 (2.19)	499.6844±22.5243 (7.36)	0.5086
0.05	499.4062 (2.22)	499.6569±22.0861 (7.17)	0.2507	497.6821 (2.16)	497.3553±22.0752 (7.28)	0.3268
0.07	498.8195 (2.19)	498.4206±21.8666 (7.09)	0.3989	495.4566 (2.14)	495.5968±22.2742 (7.35)	0.1402
0.10	497.5775 (2.19)	497.3382±22.1382 (7.2)	0.2393	490.7866 (2.15)	490.094±22.1436 (7.14)	0.6926
0.30	478.8544 (2.2)	478.9848±21.6109 (6.95)	0.1304	426.7004 (2.16)	426.4413±20.5677 (6.25)	0.2591
0.50	445.2216 (2.11)	445.619±20.9573 (6.36)	0.3974	335.2886 (2.2)	335.9174±18.3253 (4.99)	0.6288
0.70	402.5462 (2.08)	402.6044±19.7329 (5.78)	0.0582	249.4847 (2.06)	249.2212±15.7624 (3.7)	0.2635
0.90	356.5901 (2.13)	356.2284±18.6432 (5.19)	0.3617	181.76 (2.14)	181.2763±13.4381 (2.72)	0.4837
1.00	333.7991 (2.2)	333.7594±18.0687 (4.91)	0.0397	154.8815 (2.14)	154.9801±12.5309 (2.36)	0.0986
2.00	163.1067 (2.2)	163.679±12.475 (2.48)	0.5723	35.5538 (2.12)	35.5869±5.9345 (0.55)	0.0331
3.00	85.8993 (2.21)	85.8565±8.8827 (1.36)	0.0428	11.2058 (2.16)	11.2333±3.307 (0.2)	0.0275

TABLE B-3 ARL of the MEWMA control chart using JDA compared to MC under a standard normal process, with $ARL_0 = 500$ and $k = 1.0$

δ	$\lambda_1 = 0.1$			$\lambda_1 = 0.7$		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	147.5258 (2.09)	147.0979±11.658 (2.14)	0.4279	22.0863 (2.19)	22.1245±4.6605 (0.36)	0.0382
-2.00	245.9721 (2.19)	245.4245±15.3074 (3.62)	0.5476	62.8937 (2.15)	62.7523±7.9361 (0.97)	0.1414
-1.00	399.1529 (2.1)	399.5662±19.8447 (5.89)	0.4133	215.378 (2.08)	215.9102±14.5962 (3.2)	0.5322
-0.90	415.167 (2.12)	415.4245±20.2279 (6.08)	0.2575	244.0232 (2.08)	244.7547±15.6986 (3.56)	0.7315
-0.70	445.1245 (2.18)	445.9657±20.6753 (6.45)	0.8412	310.0319 (2.16)	310.1047±17.4757 (4.47)	0.0728
-0.50	470.4708 (2.23)	470.784±21.5362 (6.89)	0.3132	383.3605 (2.06)	383.2158±19.496 (5.45)	0.1447
-0.30	488.9707 (2.22)	488.8516±21.9554 (7.16)	0.1191	451.4679 (2.08)	451.7056±21.1056 (6.31)	0.2377
-0.10	498.7555 (2.19)	498.9194±22.0797 (7.34)	0.1639	494.175 (2.09)	494.5671±22.3606 (7.19)	0.3921
-0.07	499.3918 (2.22)	499.1123±22.2457 (7.33)	0.2795	497.1434 (2.18)	497.2341±22.3651 (7.19)	0.0907
-0.05	499.6917 (2.2)	499.5357±22.3252 (7.33)	0.1560	498.5513 (2.11)	498.7142±22.096 (7.14)	0.1629
-0.03	499.8917 (2.22)	499.2618±22.214 (7.29)	0.6299	499.494 (2.12)	499.8178±22.161 (7.21)	0.3238
-0.01	499.9916 (2.22)	499.1322±22.1963 (7.23)	0.8594	499.9664 (2.11)	499.2768±22.5728 (7.3)	0.6896
0.00	500.0039 (2.21)	500.9005±22.1534 (7.18)	0.8966	500.0255 (2.14)	500.7737±22.4452 (7.22)	0.7482
0.01	499.9912 (2.19)	499.6158±22.1108 (7.3)	0.3754	499.9664 (2.12)	499.2715±22.295 (6.96)	0.6949
0.03	499.8906 (2.2)	499.9717±22.2068 (7.34)	0.0811	499.4937 (2.18)	499.7774±22.3515 (7.27)	0.2837
0.05	499.6899 (2.13)	499.9657±22.2083 (7.2)	0.2758	498.5509 (2.13)	498.2994±22.5169 (7.22)	0.2515
0.07	499.3893 (2.11)	499.7805±22.2005 (7.23)	0.3912	497.1428 (2.21)	497.8077±22.3045 (7.17)	0.6649
0.10	498.7519 (2.2)	498.4686±22.4314 (7.27)	0.2833	494.1742 (2.14)	494.4629±22.122 (7.15)	0.2887
0.30	488.9602 (2.17)	488.252±21.9757 (7.03)	0.7082	451.4656 (2.17)	451.0051±21.2555 (6.53)	0.4605
0.50	470.4538 (2.15)	470.8073±21.5516 (6.76)	0.3535	383.3572 (2.16)	383.3491±19.5506 (5.61)	0.0081
0.70	445.102 (2.14)	445.0038±20.8371 (6.48)	0.0982	310.028 (2.09)	310.2971±17.7721 (4.6)	0.2691
0.90	415.1399 (2.19)	415.6886±20.0168 (5.91)	0.5487	244.0191 (2.16)	244.7095±15.5792 (3.58)	0.6904
1.00	399.1238 (2.22)	399.5513±19.869 (5.89)	0.4275	215.3738 (2.14)	215.7622±14.6673 (3.2)	0.3884
2.00	245.9348 (2.18)	245.9432±15.492 (3.63)	0.0084	62.8901 (2.11)	62.1529±7.992 (0.97)	0.7372
3.00	147.4903 (2.19)	147.0078±11.854 (2.2)	0.4825	22.0835 (2.08)	22.023±4.6642 (0.36)	0.0605

TABLE B-4 ARL of the MEWMA control chart using PEA_1 compared to MC under a standard normal process, with $ARL_0 = 500$ and $k = 0.1$

δ	$\lambda_1 = 0.1$			$\lambda_1 = 0.7$		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	5.0909 (6.58)	5.0614±1.4672 (0.62)	0.0295	1.3723 (6.58)	1.3657±1.1608 (0.23)	0.0066
-2.00	9.3423 (6.55)	9.3176±2.1618 (1.08)	0.0247	4.9528 (6.61)	4.9079±2.1809 (0.61)	0.0449
-1.00	30.3389 (6.55)	30.4423±4.7499 (3.14)	0.1034	37.6545 (6.61)	37.7217±6.1279 (3.88)	0.0672
-0.90	36.8213 (6.59)	36.8864±5.3788 (3.84)	0.0651	49.1329 (6.58)	49.7523±6.998 (5)	0.6194
-0.70	58.517 (6.52)	58.8399±7.0316 (5.83)	0.3229	86.8806 (6.54)	86.3917±9.2533 (8.52)	0.4889
-0.50	105.5163 (6.56)	105.4892±9.7324 (10.44)	0.0271	160.3535 (6.55)	160.2471±12.4487 (15.26)	0.1064
-0.30	218.1725 (6.62)	218.451±14.5181 (21.18)	0.2785	296.1253 (6.58)	296.0414±17.3341 (28.86)	0.0839
-0.10	438.9481 (6.56)	438.7127±20.5432 (40.79)	0.2354	466.2495 (6.59)	466.7068±21.5419 (44.27)	0.4573
-0.07	468.1697 (6.6)	468.5086±21.6005 (44.86)	0.3389	482.9331 (6.57)	482.5141±21.8763 (45.42)	0.4190
-0.05	483.2572 (6.61)	483.0103±21.6757 (45.4)	0.2469	491.1598 (6.58)	491.1734±22.0191 (46.21)	0.0136
-0.03	493.847 (6.6)	493.0104±21.8441 (45.97)	0.8366	496.7855 (6.58)	496.8162±22.3127 (47.13)	0.0307
-0.01	499.3103 (6.6)	499.5607±21.9272 (46.85)	0.2504	499.642 (6.61)	499.3515±21.9726 (46.53)	0.2905
0.00	500.0005 (6.58)	500.3797±22.2207 (47)	0.3792	500.0011 (6.54)	500.169±22.2469 (47.32)	0.1679
0.01	499.3077 (6.54)	499.5603±22.0892 (47.08)	0.2526	499.6418 (6.56)	499.3927±22.5046 (47.39)	0.2491
0.03	493.8391 (6.56)	493.3296±22.173 (46.97)	0.5095	496.7849 (6.53)	496.861±22.3599 (47.25)	0.0761
0.05	483.2443 (6.6)	483.0717±21.9776 (45.61)	0.1726	491.1588 (6.52)	491.6823±22.3553 (47.31)	0.5235
0.07	468.1521 (6.59)	468.4749±21.501 (44.72)	0.3228	482.9317 (6.52)	482.1706±22.1959 (46.53)	0.7611
0.10	438.9244 (6.61)	438.8274±20.8012 (41.31)	0.0970	466.2476 (6.54)	466.6726±21.5688 (44.21)	0.4250
0.30	218.1351 (6.6)	218.147±14.4511 (21.39)	0.0119	296.1214 (6.57)	296.1071±17.2948 (28.41)	0.0143
0.50	105.4837 (6.61)	105.133±9.933 (10.54)	0.3507	160.3494 (6.56)	160.4393±12.5806 (15.72)	0.0899
0.70	58.4899 (6.57)	58.1111±7.1103 (5.98)	0.3788	86.8768 (6.58)	86.2892±9.3429 (8.69)	0.5876
0.90	36.7985 (6.59)	36.3739±5.3254 (3.77)	0.4246	49.1296 (6.53)	49.6153±7.0484 (5.12)	0.4857
1.00	30.3178 (6.56)	30.1849±4.7092 (3.21)	0.1329	37.6513 (6.55)	37.7803±6.1012 (3.95)	0.1290
2.00	9.3306 (6.54)	9.2568±2.1493 (1.08)	0.0738	4.9508 (6.53)	4.9669±2.1753 (0.66)	0.0161
3.00	5.0829 (6.52)	5.0852±1.4505 (0.64)	0.0023	1.3709 (6.56)	1.3773±1.1617 (0.25)	0.0064

TABLE B-5 ARL of the MEWMA control chart using PEA_1 compared to MC under a standard normal process, with $ARL_0 = 500$ and $k = 0.7$

δ	$\lambda_1 = 0.1$			$\lambda_1 = 0.7$		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	51.5638 (6.61)	51.3943±6.5674 (5.22)	0.1695	7.9773 (6.61)	7.1908±2.8011 (0.95)	0.7865
-2.00	105.5163 (6.64)	105.0818±9.915 (10.68)	0.4345	26.3217 (6.64)	26.1299±5.0815 (2.75)	0.1918
-1.00	264.7838 (6.57)	264.4059±15.9453 (25.38)	0.3779	128.3705 (6.59)	128.5011±11.4191 (12.77)	0.1306
-0.90	291.325 (6.56)	291.5049±16.7891 (28.08)	0.1799	153.3466 (6.58)	153.1978±12.444 (15.06)	0.1488
-0.70	349.6939 (6.55)	349.2623±18.5011 (33.75)	0.4316	219.1506 (6.58)	219.3085±14.5923 (20.95)	0.1579
-0.50	410.55 (6.59)	410.5072±20.2146 (39.42)	0.0428	308.4592 (6.58)	308.6102±17.4858 (29.08)	0.1510
-0.30	463.789 (6.58)	463.4172±21.4374 (44.47)	0.3718	411.2989 (6.61)	411.4807±20.5459 (39.72)	0.1818
-0.10	495.7119 (6.55)	495.6689±22.1982 (47.37)	0.0430	488.5083 (6.56)	488.2615±22.0135 (46.28)	0.2468
-0.07	497.8907 (6.52)	497.5122±22.0563 (46.59)	0.3785	494.3091 (6.56)	494.8987±22.365 (47.19)	0.5896
-0.05	498.9222 (6.5)	498.7717±22.0507 (46.84)	0.1505	497.0819 (6.56)	497.2906±22.2555 (47.47)	0.2087
-0.03	499.612 (6.55)	499.1973±22.1828 (47.33)	0.4147	498.946 (6.58)	498.8602±22.28 (47.74)	0.0858
-0.01	499.9573 (6.59)	499.2196±22.2262 (46.89)	0.7377	499.8828 (6.57)	499.3426±22.4977 (47.97)	0.5402
0.00	500.0002 (6.62)	500.3603±22.411 (47.33)	0.3601	500 (6.55)	500.7934±22.5468 (48.5)	0.7934
0.01	499.9566 (6.58)	499.8078±22.0532 (47.49)	0.1488	499.8826 (6.61)	499.2367±22.5044 (47.28)	0.6459
0.03	499.61 (6.59)	499.4351±22.0588 (46.89)	0.1749	498.9457 (6.56)	498.1852±22.3383 (47.14)	0.7605
0.05	498.9189 (6.67)	498.6256±22.2303 (47.08)	0.2933	497.0813 (6.55)	497.769±22.1798 (46.97)	0.6877
0.07	497.886 (6.59)	497.5895±22.0273 (47.52)	0.2965	494.3083 (6.55)	494.9727±22.1131 (46.48)	0.6644
0.10	495.7053 (6.55)	495.4177±22.2344 (47.51)	0.2876	488.5072 (6.61)	488.6099±22.0411 (46.06)	0.1027
0.30	463.7703 (6.56)	463.776±21.4433 (44.1)	0.0057	411.2959 (6.66)	411.1054±20.3453 (38.94)	0.1905
0.50	410.5223 (6.61)	410.3808±20.164 (39.03)	0.1415	308.4554 (6.64)	308.8237±17.5648 (29.81)	0.3683
0.70	349.6604 (6.61)	349.621±18.6265 (33.65)	0.0394	219.1464 (6.55)	219.9471±14.7994 (21.26)	0.8007
0.90	291.2886 (6.58)	291.2892±16.8474 (28.1)	0.0006	153.3425 (6.6)	153.3419±12.3003 (14.9)	0.0006
1.00	264.7467 (6.61)	264.2±15.9626 (25.4)	0.5467	128.3665 (6.54)	128.8724±11.3461 (12.73)	0.5059
2.00	105.4836 (6.58)	105.9853±9.8804 (10.59)	0.5017	26.3188 (6.55)	26.535±4.9951 (2.72)	0.2162
3.00	51.5379 (6.57)	51.5867±6.5603 (5.31)	0.0488	7.9751 (6.6)	7.9897±2.7479 (0.94)	0.0146

TABLE B-6 ARL of the MEWMA control chart using PEA_1 compared to MC under a standard normal process, with $ARL_0 = 500$ and $k = 1.0$

δ	$\lambda_1 = 0.1$			$\lambda_1 = 0.7$		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	91.1427 (6.58)	91.8265±9.0656 (9.08)	0.6838	14.3513 (6.54)	14.2462±3.7382 (1.58)	0.1051
-2.00	171.0447 (6.5)	171.7662±12.7887 (16.42)	0.7215	43.966 (6.58)	43.844±6.5967 (4.45)	0.1220
-1.00	341.4277 (6.56)	341.2084±18.0492 (32.5)	0.2193	175.8246 (6.57)	175.3834±13.2363 (16.89)	0.4412
-0.90	363.5831 (6.58)	363.7424±18.5813 (34.15)	0.1593	203.698 (6.54)	203.6975±14.0178 (19.35)	0.0005
-0.70	407.8534 (6.55)	407.5515±20.0602 (39.25)	0.3019	271.6781 (6.54)	271.8825±16.3512 (25.91)	0.2044
-0.50	448.4885 (6.66)	448.1744±21.1576 (42.76)	0.3141	353.6907 (6.56)	353.8358±18.8913 (34.41)	0.1451
-0.30	480.1957 (6.53)	480.7963±21.7869 (45.43)	0.6006	436.5693 (6.55)	436.9823±20.9349 (41.47)	0.4130
-0.10	497.7239 (6.57)	497.938±22.1673 (47.75)	0.2141	492.1543 (6.6)	492.8162±22.0594 (46.6)	0.6619
-0.07	498.8829 (6.6)	498.0684±22.4415 (47.73)	0.8145	496.1279 (6.58)	496.446±22.5554 (48.04)	0.3181
-0.05	499.4299 (6.58)	499.8549±22.4312 (47.63)	0.4250	498.0178 (6.57)	498.7057±22.5458 (47.79)	0.6879
-0.03	499.7951 (6.57)	499.2498±22.0677 (46.79)	0.5453	499.2849 (6.55)	499.286±22.3584 (46.89)	0.0011
-0.01	499.9777 (6.56)	499.7458±22.2859 (47.37)	0.2319	499.9206 (6.55)	499.9982±22.2027 (47.5)	0.0776
0.00	500.0003 (6.61)	500.1025±22.1565 (47.24)	0.1022	500.0002 (6.59)	500.4723±22.0901 (47.16)	0.4721
0.01	499.9772 (6.62)	499.7788±22.2325 (47.49)	0.1984	499.9206 (6.56)	499.0916±22.2998 (46.63)	0.8290
0.03	499.7937 (6.53)	499.2535±22.1679 (48.15)	0.5402	499.2847 (6.57)	499.5569±22.0806 (46.89)	0.2722
0.05	499.4275 (6.63)	499.7566±21.9631 (46.72)	0.3291	498.0173 (6.5)	498.9522±22.5833 (47.41)	0.9349
0.07	498.8795 (6.58)	498.5264±22.1405 (47.26)	0.3531	496.1272 (6.55)	496.8482±22.2347 (46.73)	0.7210
0.10	497.7191 (6.55)	497.5276±22.1851 (47.25)	0.1915	492.1533 (6.59)	492.3495±22.1204 (46.08)	0.1962
0.30	480.1817 (6.54)	480.5494±21.5507 (45.06)	0.3677	436.5667 (6.58)	436.3829±20.6526 (41.06)	0.1838
0.50	448.4666 (6.68)	448.9632±21.1989 (42.17)	0.4966	353.6871 (6.53)	353.7475±18.7172 (33.6)	0.0604
0.70	407.8253 (6.56)	407.7589±19.7425 (38.02)	0.0664	271.6741 (6.61)	271.4527±16.5112 (26.47)	0.2214
0.90	363.5506 (6.59)	363.1386±18.8127 (34.19)	0.4120	203.6938 (6.6)	203.6564±14.1805 (19.54)	0.0374
1.00	341.3937 (6.56)	341.646±18.3163 (32.44)	0.2523	175.8205 (6.57)	175.3185±13.3154 (17.22)	0.5020
2.00	171.0083 (6.56)	171.7362±12.7734 (16.84)	0.7279	43.9627 (6.57)	43.062±6.6181 (4.54)	0.9007
3.00	91.1114 (6.57)	91.8811±9.1359 (9.22)	0.7697	14.3488 (6.58)	14.9132±3.6819 (1.54)	0.5644

TABLE B-7 ARL of the EEWMA and NEEWMA control chart using PEA_0 compared to MC under a standard normal process, with $ARL_0 = 500$ and $\lambda_1 = 0.1$

δ	*EEWMA (0.01)*			*NEEWMA (0.01,0.0025)*		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	1.9609 (7.31)	1.9602±0.8194 (0.22)	0.0007	1.9853 (7.55)	1.9879±0.8233 (0.27)	0.0026
-2.00	3.4839 (7.4)	3.4923±1.1175 (0.4)	0.0084	3.5161 (7.4)	3.4996±1.1165 (0.35)	0.0165
-1.00	9.4433 (7.46)	9.3773±2.1486 (0.83)	0.066	9.4746 (7.35)	9.331±2.1175 (0.85)	0.1436
-0.90	16.5943 (7.48)	16.404±3.1546 (1.44)	0.1903	16.5756 (7.46)	16.5932±3.1384 (1.48)	0.0176
-0.70	29.5935 (7.31)	29.4226±4.5922 (2.45)	0.1709	29.4208 (7.42)	29.472±4.5885 (2.47)	0.0512
-0.50	73.9634 (7.44)	73.0804±7.9278 (5.86)	0.883	73.1618 (7.47)	73.3452±7.9525 (5.83)	0.1834
-0.30	312.0743 (7.47)	312.3087±17.3534 (20.53)	0.2344	310.0401 (7.51)	310.8619±17.3597 (20.72)	0.8218
-0.10	386.7594 (7.46)	386.2653±19.4639 (25.2)	0.4941	385.2128 (7.57)	385.1546±19.5395 (25.71)	0.0582
-0.07	435.2371 (7.42)	435.8148±21.0182 (28.92)	0.5777	434.2331 (7.58)	434.0165±20.7103 (28.85)	0.2166
-0.05	474.6458 (7.52)	474.7917±21.807 (31.17)	0.1459	474.2146 (7.51)	474.5769±21.5275 (30.83)	0.3623
-0.03	497.0565 (7.36)	497.9333±21.8745 (32.2)	0.8768	497.0038 (7.55)	497.0521±21.8268 (32.22)	0.0483
-0.01	500.0007 (7.44)	500.82±22.0058 (32.27)	0.8193	500.0006 (7.51)	500.2649±22.2143 (32.88)	0.2643
0.00	497.0505 (7.47)	497.9907±22.0338 (32.26)	0.9402	496.9976 (7.47)	496.3085±21.9428 (31.85)	0.6891
0.01	474.6284 (7.39)	474.3254±21.5715 (30.71)	0.303	474.1966 (7.49)	474.896±21.5651 (30.92)	0.6994
0.03	435.2104 (7.54)	435.9051±21.1073 (28.91)	0.6947	434.2055 (7.5)	434.164±20.6373 (28.14)	0.0415
0.05	386.726 (7.58)	386.1748±19.2722 (25.43)	0.5512	385.1784 (7.47)	385.4791±19.4315 (25.42)	0.3007
0.07	312.0352 (7.42)	312.608±17.6003 (21.2)	0.5728	309.9998 (7.46)	309.219±17.4266 (21.19)	0.7808
0.10	73.9315 (7.46)	73.8251±8.0136 (5.68)	0.1064	73.1294 (7.49)	73.9342±7.8742 (5.75)	0.8048
0.30	29.5712 (7.53)	29.5004±4.5704 (2.52)	0.0708	29.3981 (7.43)	29.7026±4.6134 (2.68)	0.3045
0.50	16.5774 (7.55)	16.4211±3.1282 (1.4)	0.1563	16.5584 (7.48)	16.6019±3.1437 (1.52)	0.0435
0.70	9.431 (7.47)	9.3982±2.1499 (0.82)	0.0328	9.4621 (7.28)	9.4496±2.1622 (0.77)	0.0125
0.90	3.4776 (7.56)	3.4664±1.1225 (0.33)	0.0112	3.5097 (7.37)	3.4984±1.1177 (0.39)	0.0113
1.00	1.9565 (7.45)	1.9582±0.8198 (0.25)	0.0017	1.9809 (7.56)	1.9874±0.821 (0.24)	0.0065
2.00	1.9609 (7.31)	1.9602±0.8194 (0.22)	0.0007	1.9853 (7.55)	1.9879±0.8233 (0.27)	0.0026
3.00	3.4839 (7.4)	3.4923±1.1175 (0.4)	0.0084	3.5161 (7.4)	3.4996±1.1165 (0.35)	0.0165

TABLE B-8 ARL of the EEWMA and NEEWMA control chart using PEA_0 compared to MC under a standard normal process, with $ARL_0 = 500$ and $\lambda_1 = 0.1$

δ	*EEWMA (0.025)*			*NEEWMA (0.025,0.00625)*		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	2.1218 (7.39)	2.1243±0.826 (0.27)	0.0025	2.2006 (7.44)	2.1969±0.8282 (0.26)	0.0037
-2.00	3.6988 (7.51)	3.707±1.1266 (0.39)	0.0082	3.8068 (7.43)	3.8147±1.1347 (0.39)	0.0079
-1.00	9.6742 (7.44)	9.7019±2.1073 (0.88)	0.0277	9.8066 (7.32)	9.8129±2.0844 (0.8)	0.0063
-0.90	16.5477 (7.44)	16.4657±3.0352 (1.41)	0.082	16.5823 (7.58)	16.5332±3.0003 (1.39)	0.0491
-0.70	28.6538 (7.39)	28.3098±4.3208 (2.5)	0.344	28.3407 (7.52)	28.0177±4.2957 (2.32)	0.323
-0.50	69.2445 (7.42)	69.4832±7.5209 (5.5)	0.2387	67.3586 (7.6)	67.991±7.4298 (5.5)	0.6324
-0.30	299.1396 (7.53)	299.5087±17.0003 (20.13)	0.3691	293.182 (7.55)	293.8229±16.6337 (19.79)	0.6409
-0.10	376.7519 (7.53)	376.2318±19.3064 (24.82)	0.5201	371.9969 (7.58)	371.4237±19.0691 (24.72)	0.5732
-0.07	428.6681 (7.53)	428.7839±20.5073 (28.22)	0.1158	425.4842 (7.57)	425.1231±20.3753 (27.9)	0.3611
-0.05	471.7994 (7.5)	471.0758±21.2785 (30.81)	0.7236	470.3973 (7.54)	470.9053±21.096 (30.3)	0.508
-0.03	496.7078 (7.48)	496.7425±22.1248 (32.17)	0.0347	496.534 (7.5)	496.7373±21.7079 (32)	0.2033
-0.01	500.0007 (7.52)	500.8199±22.5602 (33.19)	0.8192	500.0003 (7.56)	500.7653±22.4204 (32.56)	0.765
0.00	496.7002 (7.5)	496.4044±22.1975 (32.26)	0.2958	496.5257 (7.64)	496.1887±22.0494 (32.25)	0.337
0.01	471.7779 (7.52)	471.9979±21.6267 (31.55)	0.22	470.3736 (7.46)	470.2488±21.3806 (30.5)	0.1248
0.03	428.6354 (7.46)	428.8723±20.1265 (27.38)	0.2369	425.4483 (7.54)	425.0276±20.3283 (27.96)	0.4207
0.05	376.7115 (7.52)	376.6527±19.0292 (25.06)	0.0588	371.9526 (7.53)	371.1664±19.0449 (24.67)	0.7862
0.07	299.0931 (7.37)	299.3516±16.9806 (20.3)	0.2585	293.1316 (7.61)	293.9061±16.7548 (19.97)	0.7745
0.10	69.2089 (7.4)	69.5744±7.5008 (5.53)	0.3655	67.321 (7.55)	67.4341±7.4327 (5.64)	0.1131
0.30	28.6292 (7.5)	28.514±4.423 (2.38)	0.1152	28.315 (7.5)	28.2379±4.3301 (2.45)	0.0771
0.50	16.5292 (7.44)	16.5144±3.0619 (1.37)	0.0148	16.563 (7.52)	16.5234±3.0021 (1.4)	0.0396
0.70	9.6608 (7.47)	9.672±2.1207 (0.81)	0.0112	9.7927 (7.36)	9.9003±2.1087 (0.84)	0.1076
0.90	3.692 (7.52)	3.6959±1.1273 (0.37)	0.0039	3.7997 (7.38)	3.7829±1.1264 (0.39)	0.0168
1.00	2.1172 (7.38)	2.1102±0.8191 (0.21)	0.007	2.1958 (7.42)	2.1971±0.8256 (0.28)	0.0013
2.00	2.1218 (7.39)	2.1243±0.826 (0.27)	0.0025	2.2006 (7.44)	2.1969±0.8282 (0.26)	0.0037
3.00	3.6988 (7.51)	3.707±1.1266 (0.39)	0.0082	3.8068 (7.43)	3.8147±1.1347 (0.39)	0.0079

TABLE B-9 ARL of the EEWMA and NEEWMA control chart using PEA_0 compared to MC under a standard normal process, with $ARL_0 = 500$ and $\lambda_1 = 0.7$

δ	*EEWMA (0.01)*			*NEEWMA (0.01,0.0025)*		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	0.8652 (7.37)	0.8641±0.9674 (0.14)	0.0011	0.8651 (7.33)	0.8683±0.9598 (0.16)	0.0032
-2.00	3.2178 (7.49)	3.2499±1.7468 (0.32)	0.0321	3.206 (7.45)	3.1944±1.7212 (0.36)	0.0116
-1.00	25.7021 (7.27)	25.9789±5.033 (2.23)	0.2768	25.5496 (7.41)	25.6406±5.0232 (2.09)	0.091
-0.90	63.565 (7.54)	63.8403±8.0571 (5.07)	0.2753	63.2263 (7.66)	63.2946±7.8926 (5.23)	0.0683
-0.70	126.1893 (7.54)	126.9352±11.0903 (9.3)	0.7459	125.6466 (7.6)	125.5934±11.2601 (9.4)	0.0532
-0.50	258.3112 (7.51)	258.9881±16.0141 (17.46)	0.6769	257.6429 (7.58)	257.2002±15.9694 (17.24)	0.4427
-0.30	455.9158 (7.5)	455.8797±21.5475 (30.31)	0.0361	455.7115 (7.56)	455.0568±21.4806 (30.17)	0.6547
-0.10	477.4854 (7.65)	477.4506±21.6614 (30.82)	0.0348	477.3766 (7.57)	477.2609±21.8997 (31.61)	0.1157
-0.07	488.2801 (7.49)	488.6687±22.2571 (32.34)	0.3886	488.2224 (7.54)	488.7166±22.0973 (32.19)	0.4942
-0.05	495.7233 (7.56)	495.0476±22.5477 (32.55)	0.6757	495.7021 (7.52)	495.4705±22.5541 (32.57)	0.2316
-0.03	499.5218 (7.41)	499.1906±22.439 (32.54)	0.3312	499.5196 (7.52)	499.2307±22.6092 (32.59)	0.2889
-0.01	500.0003 (7.44)	500.6648±22.3596 (32.49)	0.6645	500.0005 (7.61)	500.6514±22.2851 (32.97)	0.6509
0.00	499.5216 (7.51)	499.0886±22.34 (32.63)	0.433	499.5194 (7.56)	499.454±22.2953 (32.94)	0.0654
0.01	495.7226 (7.42)	495.3159±22.3088 (32.65)	0.4067	495.7014 (7.49)	495.3435±22.1193 (32.32)	0.3579
0.03	488.2789 (7.47)	488.9612±22.0758 (32)	0.6823	488.2212 (7.63)	488.4261±22.232 (32.14)	0.2049
0.05	477.4838 (7.52)	477.1574±21.8442 (31.22)	0.3264	477.375 (7.59)	477.9655±21.9191 (31.49)	0.5905
0.07	455.9136 (7.47)	455.4589±21.6408 (30.34)	0.4547	455.7092 (7.55)	455.4025±21.2894 (30.08)	0.3067
0.10	258.307 (7.58)	258.2142±16.1268 (17.78)	0.0928	257.6387 (7.54)	257.9948±15.9321 (17.58)	0.3561
0.30	126.1852 (7.47)	126.7522±11.1453 (9.28)	0.567	125.6425 (7.56)	125.5554±11.1543 (9.33)	0.0871
0.50	63.5613 (7.6)	63.1397±7.9187 (5.09)	0.4216	63.2227 (7.56)	63.9068±7.9201 (5.3)	0.6841
0.70	25.6992 (7.36)	25.7906±4.9969 (2.2)	0.0914	25.5466 (7.61)	25.76±5.0153 (2.2)	0.2134
0.90	3.216 (7.44)	3.2305±1.7339 (0.39)	0.0145	3.2042 (7.39)	3.2303±1.7172 (0.4)	0.0261
1.00	0.864 (7.28)	0.8634±0.964 (0.14)	0.0006	0.8638 (7.57)	0.8724±0.9632 (0.16)	0.0086
2.00	0.8652 (7.37)	0.8641±0.9674 (0.14)	0.0011	0.8651 (7.33)	0.8683±0.9598 (0.16)	0.0032
3.00	3.2178 (7.49)	3.2499±1.7468 (0.32)	0.0321	3.206 (7.45)	3.1944±1.7212 (0.36)	0.0116

TABLE B-10 ARL of the EEWMA and NEEWMA control chart using PEA_0 compared to MC under a standard normal process, with $ARL_0 = 500$ and $\lambda_1 = 0.7$

δ	*EEWMA (0.01)*			*NEEWMA (0.01,0.00625)*		
	MCA	***MC***	***%diff***	***MCA***	***MC***	***%diff***
-3.00	0.8648 (7.48)	0.8753±0.9608 (0.14)	0.0105	0.8649 (7.53)	0.8701±0.9513 (0.14)	0.0052
-2.00	3.1488 (7.36)	3.1231±1.6941 (0.31)	0.0257	3.1214 (7.52)	3.1157±1.6805 (0.35)	0.0057
-1.00	24.8011 (7.39)	24.9578±4.9366 (2.05)	0.1567	24.4355 (7.46)	24.0322±4.8524 (1.94)	0.4033
-0.90	61.5533 (7.58)	61.2678±7.8228 (4.95)	0.2855	60.7293 (7.52)	60.9923±7.8247 (5.17)	0.263
-0.70	122.9485 (7.45)	122.4069±10.9898 (8.94)	0.5416	121.6091 (7.58)	121.4161±11.0862 (9.26)	0.193
-0.50	254.2893 (7.45)	254.4226±15.8719 (17.35)	0.1333	252.6051 (7.67)	252.4591±15.8966 (17.43)	0.146
-0.30	454.673 (7.5)	454.3834±21.0068 (29.26)	0.2896	454.1435 (7.67)	454.7377±21.3702 (30.01)	0.5942
-0.10	476.8225 (7.56)	476.3505±21.9742 (31.03)	0.472	476.5394 (7.51)	476.0965±22.0619 (31.88)	0.4429
-0.07	487.9278 (7.63)	487.2678±21.9391 (31.48)	0.66	487.7771 (7.62)	487.4309±22.049 (32.17)	0.3462
-0.05	495.593 (7.45)	495.2612±22.2521 (32.3)	0.3318	495.5371 (7.57)	495.4111±22.142 (32.51)	0.126
-0.03	499.5074 (7.47)	499.97±22.4227 (32.44)	0.4626	499.5009 (7.7)	499.7599±22.3659 (32.54)	0.259
-0.01	500.0005 (7.45)	500.7098±22.3003 (32.47)	0.7093	500.0003 (7.65)	500.6895±22.4385 (33.16)	0.6892
0.00	499.5071 (7.3)	499.5897±22.2964 (32.55)	0.0826	499.5007 (7.62)	499.0866±22.4977 (33.19)	0.4141
0.01	495.5923 (7.55)	495.3818±22.2897 (31.96)	0.2105	495.5363 (7.66)	495.4301±21.8812 (32.12)	0.1062
0.03	487.9265 (7.5)	487.0535±21.8736 (31.08)	0.873	487.7758 (7.56)	487.0854±22.0006 (32.21)	0.6904
0.05	476.8208 (7.41)	476.1251±22.193 (31.8)	0.6957	476.5377 (7.58)	476.1661±21.9915 (31.76)	0.3716
0.07	454.6707 (7.47)	454.7292±21.3395 (29.86)	0.0585	454.1411 (7.59)	454.0673±20.917 (29.14)	0.0738
0.10	254.2849 (7.45)	254.1226±16.1412 (17.5)	0.1623	252.6006 (7.58)	252.1262±15.8323 (17.5)	0.4744
0.30	122.9442 (7.44)	122.5227±11.0379 (9.2)	0.4215	121.6048 (7.6)	121.3262±10.9379 (9.2)	0.2786
0.50	61.5496 (7.46)	61.8404±7.8007 (5.07)	0.2908	60.7255 (7.59)	60.9395±7.6634 (5.16)	0.214
0.70	24.798 (7.5)	24.4175±4.895 (2.05)	0.3805	24.4324 (7.64)	24.5959±4.9087 (2.02)	0.1635
0.90	3.147 (7.54)	3.1112±1.6906 (0.36)	0.0358	3.1196 (7.53)	3.1053±1.6871 (0.34)	0.0143
1.00	0.8635 (7.36)	0.8681±0.9523 (0.17)	0.0046	0.8636 (7.42)	0.8547±0.9563 (0.18)	0.0089
2.00	0.8648 (7.48)	0.8753±0.9608 (0.14)	0.0105	0.8649 (7.53)	0.8701±0.9513 (0.14)	0.0052
3.00	3.1488 (7.36)	3.1231±1.6941 (0.31)	0.0257	3.1214 (7.52)	3.1157±1.6805 (0.35)	0.0057

TABLE B-11 ARL of the EEWMA and NEEWMA control chart using PEA_0 compared to MC under a Normal distribution, with $ARL_0 = 500$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MCA	MCA	MCA	MCA	MCA
-1.00	ARL	9.3423	9.3162	9.4433	9.5044	9.4746	9.5520
	SDRL	4.7576	4.6984	4.6324	4.6859	4.6024	4.6701
	CPU Time	3.33	8.12	2.97	8.35	2.95	8.99
-0.70	ARL	16.7061	16.6686	16.5943	16.5799	16.5756	16.5517
	SDRL	10.4316	10.2669	9.9788	9.0241	9.8686	9.7369
	CPU Time	3.05	13.69	2.76	14.27	3.23	15.59
-0.50	ARL	30.3389	30.4631	29.5935	29.5797	29.4208	29.3726
	SDRL	22.5328	22.5126	21.2804	21.1472	20.9717	20.6831
	CPU Time	3.01	23.62	2.97	22.73	3.37	24.51
-0.10	ARL	319.7803	319.1421	312.0743	312.4764	310.0399	310.7805
	SDRL	311.2618	311.5013	302.6271	302.4229	300.3327	300.8835
	CPU Time	3.26	223.44	3.22	214.46	3.44	222.65
-0.07	ARL	392.5308	392.8208	386.7594	386.3050	385.2127	385.7156
	SDRL	384.5144	384.6113	377.8680	377.4919	376.0758	376.9615
	CPU Time	3.36	267.30	3.19	267.39	3.47	277.82
-0.05	ARL	438.9487	438.8370	435.2371	435.5298	434.2329	434.4627
	SDRL	431.2671	431.3344	426.7263	426.8209	425.4895	425.6625
	CPU Time	3.31	296.84	3.22	302.00	3.48	310.07
-0.01	ARL	497.2490	497.3540	497.0565	497.8291	497.0036	497.0817
	SDRL	490.0028	490.3076	489.0503	489.3619	488.7846	488.3217
	CPU Time	3.33	345.16	3.22	332.08	3.42	355.04
0	ARL	500.0012	500.9558	500.0007	500.5819	500.0004	500.5863
	SDRL	492.7780	492.5504	492.0213	492.7135	491.8094	491.6310
	CPU Time	3.27	346.68	3.21	345.28	3.42	357.25
0.01	ARL	497.2437	497.4209	497.0505	497.9511	496.9974	496.7793
	SDRL	490.0027	490.8620	489.0502	489.7930	488.7846	488.3712
	CPU Time	3.32	337.25	3.30	333.17	3.48	357.37
0.05	ARL	438.9250	438.6752	435.2104	435.8198	434.2054	434.5927
	SDRL	431.2668	431.5387	426.7259	426.2261	425.4891	425.4016
	CPU Time	3.29	303.70	3.33	302.36	3.45	311.59
0.07	ARL	392.5010	392.1263	386.7260	386.1939	385.1782	385.8014
	SDRL	384.5139	384.3328	377.8675	377.7275	376.0753	376.4287
	CPU Time	3.35	269.37	3.25	270.47	3.48	276.32
0.10	ARL	319.7450	319.7212	312.0352	312.5034	309.9997	309.3437
	SDRL	311.2612	311.5882	302.6265	302.0459	300.3320	300.5279
	CPU Time	3.36	224.43	3.26	220.47	3.37	226.03
0.50	ARL	30.3178	30.9399	29.5712	29.2493	29.3981	29.9022
	SDRL	22.5316	22.0583	21.2791	21.0947	20.9703	20.3409
	CPU Time	3.01	25.11	3.30	22.98	3.39	24.26
0.70	ARL	16.6901	16.7862	16.5774	16.3205	16.5584	16.4110
	SDRL	10.4304	10.4043	9.9775	9.7700	9.8673	9.8149
	CPU Time	3.24	14.34	3.05	13.21	3.04	14.56
1.00	ARL	9.3306	9.3690	9.4310	9.4310	9.4621	9.4884
	SDRL	4.7566	4.7571	4.6313	4.6779	4.6013	4.6179
	CPU Time	3.18	8.44	3.11	8.56	3.18	9.24

TABLE B-12 ARL of the EEWMA and NEEWMA control chart using PEA_0 compared to MC under a Logistic distribution, with $ARL_0 = 500$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MCA	MCA	MCA	MCA	MCA
-1.00	ARL	27.5141	27.3268	26.5890	26.7376	26.3793	26.9695
	SDRL	19.7218	19.7040	18.3607	18.6165	18.0348	18.6119
	CPU Time	2.75	21.89	2.69	20.88	2.81	21.23
-0.70	ARL	54.2200	54.9082	51.3985	51.1390	50.7293	50.2931
	SDRL	45.1912	45.8087	41.6995	41.3172	40.8507	40.6181
	CPU Time	2.90	40.81	2.93	39.14	3.22	40.98
-0.50	ARL	100.1461	100.2179	94.3052	94.1030	92.8803	92.7495
	SDRL	90.6989	90.7331	83.9896	83.7871	82.3286	82.6155
	CPU Time	2.78	73.08	2.90	70.78	3.21	70.39
-0.10	ARL	437.5481	437.6961	432.5636	432.2815	431.2194	431.7261
	SDRL	430.6162	430.2377	424.7691	424.5186	423.1828	423.9819
	CPU Time	3.17	301.23	3.19	304.08	3.28	306.00
-0.07	ARL	467.4268	467.8842	464.6282	464.5351	463.8677	463.8155
	SDRL	460.7806	460.8572	457.1627	457.4200	456.1723	456.2308
	CPU Time	3.17	317.64	3.16	320.65	3.37	327.16
-0.05	ARL	482.8631	482.4465	481.3364	481.2806	480.9199	480.0032
	SDRL	476.3661	476.2457	474.0444	474.6041	473.4050	473.6472
	CPU Time	3.22	326.82	3.31	333.68	3.22	340.36
-0.01	ARL	499.2942	499.9226	499.2289	499.8727	499.2111	499.3426
	SDRL	492.9590	492.2452	492.1264	492.5757	491.8937	491.4470
	CPU Time	3.14	340.95	3.22	341.23	3.26	353.21
0	ARL	500.0008	500.2376	500.0008	500.1540	500.0009	500.6552
	SDRL	493.6736	493.1414	492.9077	492.1238	492.6933	492.4533
	CPU Time	3.18	348.33	3.19	343.77	3.33	354.74
0.01	ARL	499.2916	499.7495	499.2259	499.1627	499.2080	499.6767
	SDRL	492.9590	492.6594	492.1264	492.0997	491.8937	491.4741
	CPU Time	3.16	341.69	3.15	344.57	3.24	356.89
0.05	ARL	482.8506	482.6097	481.3220	481.7088	480.9050	480.3600
	SDRL	476.3660	476.4452	474.0441	474.3576	473.4047	473.2157
	CPU Time	3.23	328.94	3.21	332.44	3.33	343.23
0.07	ARL	467.4098	467.8532	464.6087	464.1548	463.8475	463.9305
	SDRL	460.7804	460.9841	457.1624	457.9246	456.1720	456.6498
	CPU Time	3.18	322.31	3.19	317.92	3.27	326.47
0.10	ARL	437.5252	437.9145	432.5375	432.6253	431.1925	431.1709
	SDRL	430.6158	430.5042	424.7686	424.9521	423.1823	423.0145
	CPU Time	3.16	303.83	3.16	300.36	3.22	301.86
0.50	ARL	100.1148	100.1539	94.2716	94.9578	92.8462	92.8008
	SDRL	90.6980	90.3952	83.9885	83.7407	82.3275	82.0116
	CPU Time	3.17	74.70	3.21	72.07	3.34	71.76
0.70	ARL	54.1942	54.9517	51.3710	51.2871	50.7014	50.6186
	SDRL	45.1902	45.0139	41.6984	41.3850	40.8495	40.1278
	CPU Time	3.15	41.93	3.14	39.62	2.99	39.67
1.00	ARL	27.4941	27.6937	26.5680	26.5817	26.3580	26.5064
	SDRL	19.7207	19.8274	18.3594	18.5700	18.0335	18.7378
	CPU Time	3.20	22.62	3.03	21.47	3.25	21.80

TABLE B-13 ARL of the EEWMA and NEEWMA control chart using PEA_0 compared to MC under a Laplace distribution, with $ARL_0 = 500$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MCA	MCA	MCA	MCA	MCA
-1.00	ARL	19.1809	19.1553	18.5304	18.4197	18.3886	18.3881
	SDRL	12.0546	12.0843	11.1306	11.0148	10.9164	10.8545
	CPU Time	2.67	17.14	2.74	16.49	3.16	17.61
-0.70	ARL	38.3666	38.4349	36.0658	36.8190	35.5359	35.4273
	SDRL	29.6071	29.2091	26.7862	26.6683	26.1204	26.9362
	CPU Time	3.11	33.76	2.84	32.07	3.31	32.50
-0.50	ARL	74.1588	74.4382	68.7193	68.8652	67.4253	67.8262
	SDRL	64.6795	64.8479	58.4484	58.9360	56.9436	56.7567
	CPU Time	3.10	62.78	2.92	58.55	3.00	58.53
-0.10	ARL	419.5462	419.8393	412.0245	412.1219	409.9893	409.8988
	SDRL	413.1798	413.4661	404.7567	404.9291	402.4671	402.9456
	CPU Time	3.13	325.78	3.22	314.65	3.37	326.73
-0.07	ARL	457.2954	457.0337	452.9031	452.5379	451.7006	451.3183
	SDRL	451.3702	451.3722	446.1458	446.8667	444.7083	444.6508
	CPU Time	3.18	352.34	3.14	347.71	3.32	359.70
-0.05	ARL	477.3262	477.0052	474.8803	474.7146	474.2063	474.0350
	SDRL	471.6369	471.8767	468.4000	468.6450	467.5029	467.9725
	CPU Time	3.16	368.45	3.11	361.03	3.25	379.70
-0.01	ARL	499.0565	499.4358	498.9496	498.4047	498.9196	498.1902
	SDRL	493.6265	493.3221	492.7772	492.6938	492.5381	492.5435
	CPU Time	3.13	381.15	3.16	385.18	3.33	399.60
0	ARL	500.0008	500.4193	500.0010	500.1759	500.0006	500.7611
	SDRL	494.5833	494.2197	493.8434	493.8912	493.6347	493.2754
	CPU Time	3.12	383.11	3.18	389.11	3.39	397.82
0.01	ARL	499.0536	499.8600	498.9463	498.5571	498.9161	498.5663
	SDRL	493.6264	493.0121	492.7771	492.3769	492.5381	492.7417
	CPU Time	3.24	381.54	3.17	388.08	3.31	390.83
0.05	ARL	477.3126	477.6176	474.8644	474.0067	474.1898	474.0207
	SDRL	471.6367	471.6784	468.3997	468.1899	467.5026	467.0502
	CPU Time	3.18	376.17	3.19	368.15	3.27	375.26
0.07	ARL	457.2770	457.4854	452.8818	452.9263	451.6784	451.9481
	SDRL	451.3700	451.3185	446.1454	446.9800	444.7079	444.0156
	CPU Time	3.17	350.59	3.13	345.86	3.35	361.72
0.10	ARL	419.5218	419.5272	411.9965	411.5742	409.9604	409.3570
	SDRL	413.1794	413.6392	404.7562	404.2413	402.4666	402.4160
	CPU Time	3.24	321.25	3.09	318.75	3.32	327.81
0.50	ARL	74.1310	74.0407	68.6897	68.0958	67.3953	67.2470
	SDRL	64.6787	64.9519	58.4473	58.2938	56.9426	56.6989
	CPU Time	3.14	64.53	3.13	58.79	3.31	59.46
0.70	ARL	38.3444	38.8058	36.0424	36.9883	35.5121	35.7881
	SDRL	29.6062	29.3516	26.7851	26.0681	26.1191	26.5079
	CPU Time	3.25	33.72	3.14	31.39	3.31	32.25
1.00	ARL	19.1641	19.1256	18.5128	18.3614	18.3708	18.4854
	SDRL	12.0536	12.0904	11.1293	11.8446	10.9151	10.9224
	CPU Time	3.09	17.56	3.08	16.53	3.03	17.67

TABLE B-14 Effect of control charts parameters on ARL under a Normal distribution (PEA_0 and $ARL_0 = 500$).

δ	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
Large	-3.00	1.87	1.16	0.87	1.96	2.12	2.50	1.99	2.20	2.79
	-2.00	3.37	2.55	3.27	3.48	3.70	4.23	3.52	3.81	4.65
	-1.00	9.34	10.97	26.32	9.44	9.67	10.40	9.47	9.81	11.05
Medium	-0.90	11.03	14.01	34.98	11.10	11.29	11.98	11.12	11.41	12.64
	-0.70	16.71	25.32	64.93	16.59	16.55	16.96	16.58	16.58	17.58
	-0.50	30.34	54.50	128.37	29.59	28.65	27.80	29.42	28.34	27.94
	-0.30	77.21	143.94	260.98	73.96	69.24	62.17	73.16	67.36	59.39
	-0.10	319.78	402.20	456.72	312.07	299.14	272.85	310.04	293.18	256.85
Small	-0.09	343.63	417.95	464.42	336.38	324.09	298.56	334.46	318.37	282.58
	-0.07	392.53	447.19	477.92	386.76	376.75	354.99	385.21	372.00	340.58
	-0.05	438.95	471.70	488.51	435.24	428.67	413.73	434.23	425.48	403.29
	-0.03	476.23	489.46	495.81	474.65	471.80	465.09	474.21	470.40	460.17
	-0.01	497.25	498.81	499.53	497.06	496.71	495.87	497.00	496.53	495.24
No change	0.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00
Small	0.01	497.24	498.81	499.53	497.05	496.70	495.86	497.00	496.53	495.22
	0.03	476.21	489.46	495.81	474.63	471.78	465.05	474.20	470.37	460.13
	0.05	438.93	471.69	488.51	435.21	428.64	413.68	434.21	425.45	403.22
	0.07	392.50	447.18	477.91	386.73	376.71	354.93	385.18	371.95	340.50
	0.09	343.59	417.94	464.42	336.34	324.05	298.49	334.42	318.32	282.50
Medium	0.10	319.75	402.19	456.72	312.04	299.09	272.79	310.00	293.13	256.77
	0.30	77.18	143.93	260.97	73.93	69.21	62.13	73.13	67.32	59.33
	0.50	30.32	54.49	128.37	29.57	28.63	27.77	29.40	28.32	27.90
	0.70	16.69	25.31	64.93	16.58	16.53	16.94	16.56	16.56	17.55
	0.90	11.02	14.00	34.98	11.08	11.27	11.96	11.11	11.39	12.62
Large	1.00	9.33	10.97	26.32	9.43	9.66	10.38	9.46	9.79	11.03
	2.00	3.36	2.54	3.26	3.48	3.69	4.22	3.51	3.80	4.64
	3.00	1.87	1.16	0.86	1.96	2.12	2.49	1.98	2.20	2.79

TABLE B-15 Effect of control charts parameters on ARL under a Logistic distribution (PEA_0 and $ARL_0 = 500$).

δ	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
Large	-3.00	4.56	4.57	17.25	4.67	4.88	5.44	4.70	4.99	5.91
	-2.00	8.36	12.21	61.95	8.43	8.61	9.23	8.45	8.72	9.81
	-1.00	27.51	70.83	231.26	26.59	25.47	24.50	26.38	25.11	24.63
Medium	-0.90	33.57	88.93	261.16	32.23	30.54	28.77	31.92	29.96	28.60
	-0.70	54.22	143.05	327.50	51.40	47.62	42.80	50.73	46.22	41.36
	-0.50	100.15	232.02	397.18	94.31	86.00	73.85	92.88	82.74	68.99
	-0.30	212.97	360.46	458.35	202.71	186.88	159.67	200.11	180.14	146.13
	-0.10	437.55	480.12	495.06	432.56	423.81	404.34	431.22	419.60	391.15
Small	-0.09	448.25	483.79	496.00	444.00	436.50	419.61	442.86	432.88	407.97
	-0.07	467.43	490.08	497.57	464.63	459.63	448.09	463.87	457.19	439.91
	-0.05	482.86	494.90	498.76	481.34	478.58	472.11	480.92	477.23	467.40
	-0.03	493.70	498.15	499.55	493.13	492.08	489.59	492.97	491.57	487.75
	-0.01	499.29	499.79	499.95	499.23	499.11	498.82	499.21	499.05	498.61
No change	0.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00
Small	0.01	499.29	499.79	499.95	499.23	499.11	498.82	499.21	499.05	498.60
	0.03	493.69	498.15	499.55	493.12	492.07	489.58	492.96	491.55	487.73
	0.05	482.85	494.89	498.76	481.32	478.57	472.08	480.90	477.21	467.36
	0.07	467.41	490.08	497.57	464.61	459.60	448.05	463.85	457.16	439.86
	0.09	448.22	483.78	496.00	443.98	436.47	419.56	442.83	432.85	407.91
Medium	0.10	437.53	480.11	495.06	432.54	423.77	404.29	431.19	419.57	391.08
	0.30	212.93	360.45	458.35	202.67	186.83	159.61	200.07	180.10	146.06
	0.50	100.11	232.01	397.18	94.27	85.96	73.80	92.85	82.70	68.94
	0.70	54.19	143.04	327.50	51.37	47.59	42.76	50.70	46.19	41.32
	0.90	33.55	88.92	261.16	32.21	30.51	28.74	31.90	29.93	28.57
Large	1.00	27.49	70.82	231.26	26.57	25.45	24.47	26.36	25.09	24.60
	2.00	8.35	12.21	61.95	8.42	8.59	9.21	8.44	8.70	9.79
	3.00	4.55	4.57	17.25	4.66	4.87	5.43	4.69	4.98	5.89

TABLE B-16 Effect of control charts parameters on ARL under a Laplace distribution (PEA_0 and $ARL_0 = 500$).

δ	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
	-3.00	3.32	3.24	15.72	3.40	3.57	4.00	3.43	3.65	4.36
Large	-2.00	6.00	9.05	60.15	6.07	6.23	6.74	6.09	6.32	7.22
	-1.00	19.18	61.15	229.74	18.53	17.81	17.40	18.39	17.61	17.71
	-0.90	23.43	78.20	259.76	22.44	21.28	20.33	22.22	20.92	20.47
	-0.70	38.37	130.42	326.41	36.07	33.17	29.95	35.54	32.16	29.29
Medium	-0.50	74.16	218.99	396.48	68.72	61.39	51.80	67.43	58.65	48.57
	-0.30	174.16	351.23	458.05	162.61	145.44	118.42	159.73	138.43	106.52
	-0.10	419.55	478.49	495.03	412.02	398.75	369.52	409.99	392.38	350.39
	-0.09	432.91	482.45	495.97	426.42	414.85	388.88	424.65	409.25	371.46
	-0.07	457.30	489.25	497.55	452.90	444.95	426.42	451.70	441.03	413.40
Small	-0.05	477.33	494.47	498.75	474.88	470.39	459.59	474.21	468.15	451.70
	-0.03	491.61	498.00	499.55	490.68	488.94	484.68	490.42	488.07	481.46
	-0.01	499.06	499.78	499.95	498.95	498.75	498.25	498.92	498.65	497.87
No change	0.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00
	0.01	499.05	499.78	499.95	498.95	498.75	498.25	498.92	498.64	497.86
	0.03	491.60	497.99	499.55	490.67	488.93	484.66	490.41	488.05	481.43
Small	0.05	477.31	494.47	498.75	474.86	470.37	459.56	474.19	468.13	451.65
	0.07	457.28	489.25	497.55	452.88	444.92	426.37	451.68	441.00	413.34
	0.09	432.89	482.45	495.97	426.39	414.82	388.83	424.62	409.22	371.39
	0.10	419.52	478.48	495.03	412.00	398.72	369.47	409.96	392.34	350.32
	0.30	174.12	351.22	458.05	162.57	145.40	118.37	159.69	138.39	106.46
Medium	0.50	74.13	218.98	396.48	68.69	61.35	51.76	67.40	58.62	48.52
	0.70	38.34	130.41	326.41	36.04	33.14	29.92	35.51	32.14	29.25
	0.90	23.41	78.19	259.76	22.42	21.26	20.31	22.20	20.90	20.45
	1.00	19.16	61.14	229.74	18.51	17.79	17.38	18.37	17.59	17.69
Large	2.00	5.99	9.05	60.15	6.06	6.22	6.73	6.08	6.31	7.20
	3.00	3.31	3.23	15.71	3.40	3.56	4.00	3.42	3.65	4.36

TABLE B-17 Effect of the location parameters in process on the ARL of control charts under a Normal distribution (PEA_0 and $ARL_0 = 500$).

δ	Charts	EMWA			EEWMA			NEEWMA		
	μ_0	0	5	10	0	5	10	0	5	10
Large	-3.00	1.87	1.87	1.87	1.96	1.96	1.96	1.99	1.99	1.99
	-2.00	3.37	3.37	3.37	3.48	3.48	3.48	3.52	3.52	3.52
	-1.00	9.34	9.34	9.34	9.44	9.44	9.44	9.47	9.47	9.47
Medium	-0.90	11.03	11.03	11.03	11.10	11.10	11.10	11.12	11.12	11.12
	-0.70	16.71	16.71	16.71	16.59	16.59	16.59	16.58	16.58	16.58
	-0.50	30.34	30.34	30.34	29.59	29.59	29.59	29.42	29.42	29.42
	-0.30	77.21	77.21	77.21	73.96	73.96	73.96	73.16	73.16	73.16
	-0.10	319.78	319.78	319.78	312.07	312.07	312.07	310.04	310.04	310.04
Small	-0.09	343.63	343.63	343.63	336.38	336.38	336.38	334.46	334.46	334.46
	-0.07	392.53	392.53	392.53	386.76	386.76	386.76	385.21	385.21	385.21
	-0.05	438.95	438.95	438.95	435.24	435.24	435.24	434.23	434.23	434.23
	-0.03	476.23	476.23	476.23	474.65	474.65	474.65	474.21	474.21	474.21
	-0.01	497.25	497.25	497.25	497.06	497.06	497.06	497.00	497.00	497.00
No change	0.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00
Small	0.01	497.24	497.24	497.24	497.05	497.05	497.05	497.00	497.00	497.00
	0.03	476.21	476.21	476.21	474.63	474.63	474.63	474.20	474.20	474.20
	0.05	438.93	438.93	438.93	435.21	435.21	435.21	434.21	434.21	434.21
	0.07	392.50	392.50	392.50	386.73	386.73	386.73	385.18	385.18	385.18
	0.09	343.59	343.59	343.59	336.34	336.34	336.34	334.42	334.42	334.42
Medium	0.10	319.75	319.75	319.75	312.04	312.04	312.04	310.00	310.00	310.00
	0.30	77.18	77.18	77.18	73.93	73.93	73.93	73.13	73.13	73.13
	0.50	30.32	30.32	30.32	29.57	29.57	29.57	29.40	29.40	29.40
	0.70	16.69	16.69	16.69	16.58	16.58	16.58	16.56	16.56	16.56
	0.90	11.02	11.02	11.02	11.08	11.08	11.08	11.11	11.11	11.11
Large	1.00	9.33	9.33	9.33	9.43	9.43	9.43	9.46	9.46	9.46
	2.00	3.36	3.36	3.36	3.48	3.48	3.48	3.51	3.51	3.51
	3.00	1.87	1.87	1.87	1.96	1.96	1.96	1.98	1.98	1.98

TABLE B-18 Effect of the location parameters in process on the ARL of control charts under a Logistic distribution (PEA_0 and $ARL_0 = 500$).

δ	Charts	EMWA			EEWMA			NEEWMA		
	μ_0	0	5	10	0	5	10	0	5	10
Large	-3.00	4.56	4.56	4.56	4.67	4.67	4.67	4.70	4.70	4.70
	-2.00	8.36	8.36	8.36	8.43	8.43	8.43	8.45	8.45	8.45
	-1.00	27.51	27.51	27.51	26.59	26.59	26.59	26.38	26.38	26.38
Medium	-0.90	33.57	33.57	33.57	32.23	32.23	32.23	31.92	31.92	31.92
	-0.70	54.22	54.22	54.22	51.40	51.40	51.40	50.73	50.73	50.73
	-0.50	100.15	100.15	100.15	94.31	94.31	94.31	92.88	92.88	92.88
	-0.30	212.97	212.97	212.97	202.71	202.71	202.71	200.11	200.11	200.11
	-0.10	437.55	437.55	437.55	432.56	432.56	432.56	431.22	431.22	431.22
Small	-0.09	448.25	448.25	448.25	444.00	444.00	444.00	442.86	442.86	442.86
	-0.07	467.43	467.43	467.43	464.63	464.63	464.63	463.87	463.87	463.87
	-0.05	482.86	482.86	482.86	481.34	481.34	481.34	480.92	480.92	480.92
	-0.03	493.70	493.70	493.70	493.13	493.13	493.13	492.97	492.97	492.97
	-0.01	499.29	499.29	499.29	499.23	499.23	499.23	499.21	499.21	499.21
No change	0.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00
Small	0.01	499.29	499.29	499.29	499.23	499.23	499.23	499.21	499.21	499.21
	0.03	493.69	493.69	493.69	493.12	493.12	493.12	492.96	492.96	492.96
	0.05	482.85	482.85	482.85	481.32	481.32	481.32	480.90	480.90	480.90
	0.07	467.41	467.41	467.41	464.61	464.61	464.61	463.85	463.85	463.85
	0.09	448.22	448.22	448.22	443.98	443.98	443.98	442.83	442.83	442.83
Medium	0.10	437.53	437.53	437.53	432.54	432.54	432.54	431.19	431.19	431.19
	0.30	212.93	212.93	212.93	202.67	202.67	202.67	200.07	200.07	200.07
	0.50	100.11	100.11	100.11	94.27	94.27	94.27	92.85	92.85	92.85
	0.70	54.19	54.19	54.19	51.37	51.37	51.37	50.70	50.70	50.70
	0.90	33.55	33.55	33.55	32.21	32.21	32.21	31.90	31.90	31.90
Large	1.00	27.49	27.49	27.49	26.57	26.57	26.57	26.36	26.36	26.36
	2.00	8.35	8.35	8.35	8.42	8.42	8.42	8.44	8.44	8.44
	3.00	4.55	4.55	4.55	4.66	4.66	4.66	4.69	4.69	4.69

TABLE B-19 Effect of the location parameters in process on the ARL of control charts under a Laplace distribution (PEA_0 and $ARL_0 = 500$).

δ	Charts	EMWA			EEWMA			NEEWMA		
	μ_0	0	5	10	0	5	10	0	5	10
Large	-3.00	3.32	3.32	3.32	3.40	3.40	3.40	3.43	3.43	3.43
	-2.00	6.00	6.00	6.00	6.07	6.07	6.07	6.09	6.09	6.09
	-1.00	19.18	19.18	19.18	18.53	18.53	18.53	18.39	18.39	18.39
Medium	-0.90	23.43	23.43	23.43	22.44	22.44	22.44	22.22	22.22	22.22
	-0.70	38.37	38.37	38.37	36.07	36.07	36.07	35.54	35.54	35.54
	-0.50	74.16	74.16	74.16	68.72	68.72	68.72	67.43	67.43	67.43
	-0.30	174.16	174.16	174.16	162.61	162.61	162.61	159.73	159.73	159.73
	-0.10	419.55	419.55	419.55	412.02	412.02	412.02	409.99	409.99	409.99
Small	-0.09	432.91	432.91	432.91	426.42	426.42	426.42	424.65	424.65	424.65
	-0.07	457.30	457.30	457.30	452.90	452.90	452.90	451.70	451.70	451.70
	-0.05	477.33	477.33	477.33	474.88	474.88	474.88	474.21	474.21	474.21
	-0.03	491.61	491.61	491.61	490.68	490.68	490.68	490.42	490.42	490.42
	-0.01	499.06	499.06	499.06	498.95	498.95	498.95	498.92	498.92	498.92
No change	0.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00
Small	0.01	499.05	499.05	499.05	498.95	498.95	498.95	498.92	498.92	498.92
	0.03	491.60	491.60	491.60	490.67	490.67	490.67	490.41	490.41	490.41
	0.05	477.31	477.31	477.31	474.86	474.86	474.86	474.19	474.19	474.19
	0.07	457.28	457.28	457.28	452.88	452.88	452.88	451.68	451.68	451.68
	0.09	432.89	432.89	432.89	426.39	426.39	426.39	424.62	424.62	424.62
Medium	0.10	419.52	419.52	419.52	412.00	412.00	412.00	409.96	409.96	409.96
	0.30	174.12	174.12	174.12	162.57	162.57	162.57	159.69	159.69	159.69
	0.50	74.13	74.13	74.13	68.69	68.69	68.69	67.40	67.40	67.40
	0.70	38.34	38.34	38.34	36.04	36.04	36.04	35.51	35.51	35.51
	0.90	23.41	23.41	23.41	22.42	22.42	22.42	22.20	22.20	22.20
Large	1.00	19.16	19.16	19.16	18.51	18.51	18.51	18.37	18.37	18.37
	2.00	5.99	5.99	5.99	6.06	6.06	6.06	6.08	6.08	6.08
	3.00	3.31	3.31	3.31	3.40	3.40	3.40	3.42	3.42	3.42

TABLE B-20 Effect of the scale parameters in process on the ARL of control charts under a Normal distribution (PEA_0 and $ARL_0 = 500$).

δ	Charts	EMWA			EEWMA			NEEWMA		
	β_0	1	5	10	1	5	10	1	5	10
Large	-3.00	1.87	21.84	77.21	1.96	21.52	73.96	1.99	21.45	73.16
	-2.00	3.37	45.78	149.48	3.48	44.20	143.21	3.52	43.82	141.62
	-1.00	9.34	149.48	319.78	9.44	143.21	312.07	9.47	141.62	310.04
Medium	-0.90	11.03	173.40	343.63	11.10	166.44	336.38	11.12	164.66	334.46
	-0.70	16.71	235.76	392.53	16.59	227.74	386.76	16.58	225.66	385.21
	-0.50	30.34	319.78	438.95	29.59	312.07	435.24	29.42	310.04	434.23
	-0.30	77.21	416.43	476.23	73.96	411.64	474.65	73.16	410.35	474.21
	-0.10	319.78	489.16	497.25	312.07	488.42	497.06	310.04	488.21	497.00
Small	-0.09	343.63	491.19	497.77	336.38	490.58	497.61	334.46	490.41	497.57
	-0.07	392.53	494.63	498.65	386.76	494.26	498.55	385.21	494.16	498.53
	-0.05	438.95	497.25	499.31	435.24	497.06	499.26	434.23	497.00	499.25
	-0.03	476.23	499.01	499.75	474.65	498.94	499.73	474.21	498.92	499.73
	-0.01	497.25	499.89	499.97	497.06	499.88	499.97	497.00	499.88	499.97
No change	0.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00
Small	0.01	497.24	499.89	499.97	497.05	499.88	499.97	497.00	499.88	499.97
	0.03	476.21	499.00	499.75	474.63	498.93	499.73	474.20	498.91	499.73
	0.05	438.93	497.24	499.31	435.21	497.05	499.26	434.21	497.00	499.25
	0.07	392.50	494.63	498.65	386.73	494.25	498.55	385.18	494.15	498.52
	0.09	343.59	491.18	497.77	336.34	490.57	497.61	334.42	490.40	497.56
Medium	0.10	319.75	489.15	497.24	312.04	488.40	497.05	310.00	488.20	497.00
	0.30	77.18	416.40	476.21	73.93	411.61	474.63	73.13	410.32	474.20
	0.50	30.32	319.75	438.93	29.57	312.04	435.21	29.40	310.00	434.21
	0.70	16.69	235.73	392.50	16.58	227.70	386.73	16.56	225.62	385.18
	0.90	11.02	173.36	343.59	11.08	166.40	336.34	11.11	164.62	334.42
Large	1.00	9.33	149.44	319.75	9.43	143.17	312.04	9.46	141.58	310.00
	2.00	3.36	45.75	149.44	3.48	44.17	143.17	3.51	43.79	141.58
	3.00	1.87	21.83	77.18	1.96	21.50	73.93	1.98	21.43	73.13

TABLE B-21 Effect of the scale parameters in process on the ARL of control charts under a Logistic distribution (PEA_0 and ARL_0 = 500).

δ	Charts	EMWA			EEWMA			NEEWMA		
	β_0	1	5	10	1	5	10	1	5	10
Large	-3.00	4.56	72.34	212.97	4.67	68.26	202.71	4.70	67.28	200.11
	-2.00	8.36	143.88	316.01	8.43	135.79	305.78	8.45	133.78	303.11
	-1.00	27.51	316.01	437.55	26.59	305.78	432.56	26.38	303.11	431.22
Medium	-0.90	33.57	340.28	448.25	32.23	330.64	444.00	31.92	328.11	442.86
	-0.70	54.22	390.14	467.43	51.40	382.42	464.63	50.73	380.37	463.87
	-0.50	100.15	437.55	482.86	94.31	432.56	481.34	92.88	431.22	480.92
	-0.30	212.97	475.67	493.70	202.71	473.54	493.13	200.11	472.96	492.97
	-0.10	437.55	497.18	499.29	432.56	496.92	499.23	431.22	496.85	499.21
Small	-0.09	448.25	497.72	499.43	444.00	497.51	499.38	442.86	497.45	499.36
	-0.07	467.43	498.62	499.65	464.63	498.49	499.62	463.87	498.45	499.61
	-0.05	482.86	499.29	499.82	481.34	499.23	499.81	480.92	499.21	499.80
	-0.03	493.70	499.75	499.94	493.13	499.72	499.93	492.97	499.72	499.93
	-0.01	499.29	499.97	499.99	499.23	499.97	499.99	499.21	499.97	499.99
No change	0.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00
Small	0.01	499.29	499.97	499.99	499.23	499.97	499.99	499.21	499.97	499.99
	0.03	493.69	499.74	499.94	493.12	499.72	499.93	492.96	499.71	499.93
	0.05	482.85	499.29	499.82	481.32	499.23	499.81	480.90	499.21	499.80
	0.07	467.41	498.61	499.65	464.61	498.49	499.62	463.85	498.45	499.61
	0.09	448.22	497.71	499.43	443.98	497.50	499.37	442.83	497.44	499.36
Medium	0.10	437.53	497.18	499.29	432.54	496.92	499.23	431.19	496.85	499.21
	0.30	212.93	475.66	493.69	202.67	473.52	493.12	200.07	472.94	492.96
	0.50	100.11	437.53	482.85	94.27	432.54	481.32	92.85	431.19	480.90
	0.70	54.19	390.11	467.41	51.37	382.39	464.61	50.70	380.33	463.85
	0.90	33.55	340.25	448.22	32.21	330.61	443.98	31.90	328.07	442.83
Large	1.00	27.49	315.97	437.53	26.57	305.74	432.54	26.36	303.07	431.19
	2.00	8.35	143.85	315.97	8.42	135.75	305.74	8.44	133.74	303.07
	3.00	4.55	72.31	212.93	4.66	68.23	202.67	4.69	67.25	200.07

TABLE B-22 Effect of the scale parameters in process on the ARL of control charts under a Laplace distribution (PEA_0 and $ARL_0 = 500$).

δ	Charts	EMWA			EEWMA			NEEWMA		
	β_0	1	5	10	1	5	10	1	5	10
Large	-3.00	3.32	52.09	174.16	3.40	48.55	162.61	3.43	47.72	159.73
	-2.00	6.00	111.05	278.83	6.07	102.85	265.63	6.09	100.86	262.22
	-1.00	19.18	278.83	419.55	18.53	265.63	412.02	18.39	262.22	409.99
Medium	-0.90	23.43	305.38	432.91	22.44	292.55	426.42	22.22	289.20	424.65
	-0.70	38.37	362.29	457.30	36.07	351.34	452.90	35.54	348.43	451.70
	-0.50	74.16	419.55	477.33	68.72	412.02	474.88	67.43	409.99	474.21
	-0.30	174.16	467.95	491.61	162.61	464.56	490.68	159.73	463.64	490.42
	-0.10	419.55	496.24	499.06	412.02	495.82	498.95	409.99	495.70	498.92
Small	-0.09	432.91	496.95	499.24	426.42	496.61	499.15	424.65	496.51	499.12
	-0.07	457.30	498.15	499.54	452.90	497.94	499.49	451.70	497.88	499.47
	-0.05	477.33	499.06	499.76	474.88	498.95	499.74	474.21	498.92	499.73
	-0.03	491.61	499.66	499.92	490.68	499.62	499.91	490.42	499.61	499.90
	-0.01	499.06	499.96	499.99	498.95	499.96	499.99	498.92	499.96	499.99
No change	0.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00
Small	0.01	499.05	499.96	499.99	498.95	499.96	499.99	498.92	499.96	499.99
	0.03	491.60	499.66	499.92	490.67	499.62	499.91	490.41	499.61	499.90
	0.05	477.31	499.05	499.76	474.86	498.95	499.74	474.19	498.92	499.73
	0.07	457.28	498.15	499.54	452.88	497.94	499.48	451.68	497.88	499.47
	0.09	432.89	496.95	499.23	426.39	496.60	499.15	424.62	496.51	499.12
Medium	0.10	419.52	496.23	499.05	412.00	495.81	498.95	409.96	495.69	498.92
	0.30	174.12	467.93	491.60	162.57	464.55	490.67	159.69	463.62	490.41
	0.50	74.13	419.52	477.31	68.69	412.00	474.86	67.40	409.96	474.19
	0.70	38.34	362.27	457.28	36.04	351.31	452.88	35.51	348.40	451.68
	0.90	23.41	305.35	432.89	22.42	292.51	426.39	22.20	289.16	424.62
Large	1.00	19.16	278.80	419.52	18.51	265.60	412.00	18.37	262.18	409.96
	2.00	5.99	111.02	278.80	6.06	102.81	265.60	6.08	100.82	262.18
	3.00	3.31	52.07	174.12	3.40	48.53	162.57	3.42	47.70	159.69

TABLE B-23 ARL of the EWMA EEWMA and NEEWMA control chart using PEA_0 compared to MC under a Gamma distribution, with $ARL_0 = 500$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MC	MCA	MC	MCA	MC
0.00	ARL	500.0000	500.8135	500.0000	500.1324	500.0000	500.2497
	SDRL	497.5131	497.3915	489.3186	489.6580	487.3781	487.6473
	CPU Time	3.39	35.43	3.02	36.15	3.11	35.70
0.01	ARL	447.9877	447.2730	447.3656	447.2578	447.1906	447.5474
	SDRL	445.2612	445.0151	436.6703	436.0722	434.5989	434.6023
	CPU Time	3.33	31.33	3.30	31.91	3.35	31.98
0.03	ARL	363.4401	363.3132	362.2429	362.0037	361.9050	361.4270
	SDRL	360.2970	360.3167	351.5539	351.8017	349.4035	349.9494
	CPU Time	3.37	26.56	3.31	26.41	3.38	26.37
0.05	ARL	298.7782	298.9147	297.5392	297.0189	297.1871	297.7103
	SDRL	295.2929	295.2864	286.8965	286.0770	284.8108	284.2810
	CPU Time	3.41	21.70	3.36	21.54	3.27	21.76
0.07	ARL	248.6649	248.0838	247.6523	247.7876	247.3607	247.5868
	SDRL	244.9015	244.6947	237.0899	237.0048	235.1391	235.3641
	CPU Time	3.39	18.14	3.30	18.44	3.30	18.11
0.09	ARL	209.3399	209.2949	208.6731	208.0465	208.4752	208.2542
	SDRL	205.3539	205.5067	198.2198	198.5437	196.4328	196.9853
	CPU Time	3.39	15.57	3.28	15.23	3.36	15.65
0.10	ARL	192.8517	192.2136	192.3751	192.9283	192.2289	192.1411
	SDRL	188.7728	188.6417	181.9856	181.0869	180.2839	180.3303
	CPU Time	3.36	14.54	3.30	14.25	3.29	14.37
0.30	ARL	57.4216	57.8399	59.3001	59.2056	59.7978	59.1873
	SDRL	52.9922	52.5641	50.7871	50.7212	50.2332	50.3531
	CPU Time	3.41	4.58	3.34	4.61	3.33	4.72
0.50	ARL	28.2350	28.9793	30.3586	30.3280	30.9295	30.1176
	SDRL	24.4683	24.6835	23.7084	23.0011	23.5173	23.9340
	CPU Time	3.36	2.29	3.31	2.47	3.20	2.47
0.70	ARL	17.7156	17.8037	19.6492	19.6682	20.1738	20.3446
	SDRL	14.6505	14.8116	14.4063	14.3242	14.3446	14.4890
	CPU Time	3.36	1.55	3.34	1.64	3.22	1.69
0.90	ARL	12.6640	12.7793	14.3660	14.4023	14.8313	14.0007
	SDRL	10.1716	10.1054	10.1456	10.1431	10.1388	10.2665
	CPU Time	3.08	1.08	3.03	1.27	3.21	1.27
1.00	ARL	11.0452	11.1876	12.6413	12.6500	13.0790	13.2256
	SDRL	8.7900	8.9056	8.8247	8.7421	8.8334	8.9788
	CPU Time	3.07	0.97	3.33	1.09	3.23	1.13
2.00	ARL	4.7527	4.7872	5.6965	5.7069	5.9603	5.9209
	SDRL	3.8044	3.8924	3.9864	3.9635	4.0332	4.0232
	CPU Time	3.07	0.48	3.28	0.58	3.30	0.54
3.00	ARL	3.0083	3.0286	3.6691	3.6269	3.8555	3.8653
	SDRL	2.5614	2.6074	2.7389	2.7464	2.7851	2.7848
	CPU Time	3.38	0.32	2.86	0.36	3.00	0.39

TABLE B-24 ARL of the EWMA EEWMA and NEEWMA control chart using PEA_0 compared to MC under a Weibull distribution, with $ARL_0 = 500$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MC	MCA	MC	MCA	MC
0.00	ARL	500.0000	500.9867	500.0000	500.4646	500.0000	500.8757
	SDRL	497.5131	497.5462	489.3186	489.0411	487.3781	487.1745
	CPU Time	3.01	34.97	2.94	35.33	3.00	34.69
0.01	ARL	447.9877	447.2161	447.3656	447.9991	447.1906	447.1023
	SDRL	445.2612	445.5778	436.6703	436.6612	434.5989	434.7860
	CPU Time	3.24	31.09	3.20	31.64	3.23	31.56
0.03	ARL	363.4401	363.9357	362.2429	362.8112	361.9050	361.8997
	SDRL	360.2970	360.4784	351.5539	351.6632	349.4035	349.8427
	CPU Time	3.28	26.21	3.22	25.96	3.27	26.06
0.05	ARL	298.7782	298.0270	297.5392	297.4534	297.1871	297.7921
	SDRL	295.2929	295.8793	286.8965	286.1589	284.8108	284.8348
	CPU Time	3.25	21.14	3.23	21.14	3.21	21.65
0.07	ARL	248.6649	248.9852	247.6523	247.5654	247.3607	247.0726
	SDRL	244.9015	244.1016	237.0899	237.6393	235.1391	235.8823
	CPU Time	3.18	17.58	3.19	17.75	3.28	17.66
0.09	ARL	209.3399	209.1654	208.6731	208.0374	208.4752	208.5859
	SDRL	205.3539	205.6471	198.2198	198.4283	196.4328	196.2576
	CPU Time	3.20	15.35	3.30	15.28	3.22	15.03
0.10	ARL	192.8517	192.8323	192.3751	192.3928	192.2289	192.1236
	SDRL	188.7728	188.6621	181.9856	181.5395	180.2839	180.6499
	CPU Time	3.22	13.90	3.18	14.22	3.20	13.94
0.30	ARL	57.4216	57.7944	59.3001	59.2483	59.7978	59.1095
	SDRL	52.9922	52.8031	50.7871	50.5336	50.2332	50.1473
	CPU Time	3.30	4.41	3.24	4.50	3.34	4.72
0.50	ARL	28.2350	28.1385	30.3586	30.2363	30.9295	30.7341
	SDRL	24.4683	24.2468	23.7084	23.3692	23.5173	23.3836
	CPU Time	3.25	2.23	3.12	2.41	3.17	2.47
0.70	ARL	17.7156	17.6176	19.6492	19.8128	20.1738	20.1742
	SDRL	14.6505	14.5771	14.4063	14.5867	14.3446	14.2383
	CPU Time	3.22	1.45	3.15	1.61	3.21	1.75
0.90	ARL	12.6640	12.6599	14.3660	14.2602	14.8313	14.9509
	SDRL	10.1716	10.1802	10.1456	10.0621	10.1388	10.1768
	CPU Time	2.77	1.06	3.13	1.19	3.21	1.22
1.00	ARL	11.0452	11.0643	12.6413	12.5850	13.0790	13.0770
	SDRL	8.7900	8.9085	8.8247	8.8682	8.8334	8.8509
	CPU Time	2.75	0.92	3.14	1.12	2.96	1.11
2.00	ARL	4.7527	4.7591	5.6965	5.7297	5.9603	5.9804
	SDRL	3.8044	3.8558	3.9864	3.0100	4.0332	4.0625
	CPU Time	2.75	0.44	3.14	0.55	3.20	0.56
3.00	ARL	3.0083	3.9931	3.6691	3.6944	3.8555	3.8663
	SDRL	2.5614	2.5517	2.7389	2.7290	2.7851	2.8121
	CPU Time	2.80	0.33	2.91	0.37	3.01	0.38

TABLE B-25 ARL of the EWMA EEWMA and NEEWMA control chart using PEA_0 compared to MC under a Gumbel distribution, with $ARL_0 = 500$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MC	MCA	MC	MCA	MC
0.00	ARL	500.0000	500.1989	500.0000	500.3977	500.0000	500.7524
	SDRL	495.7885	495.3330	491.9558	491.0129	490.7233	490.6135
	CPU Time	3.09	38.58	3.25	39.33	3.40	38.98
0.01	ARL	460.2335	460.4157	459.7943	459.9790	459.8156	459.6781
	SDRL	456.0073	456.2873	451.7853	451.5323	450.5958	450.6236
	CPU Time	3.42	36.16	3.46	36.73	3.37	35.91
0.03	ARL	392.7189	392.2683	391.8738	391.5845	391.9584	391.3872
	SDRL	388.4732	388.1215	383.9481	383.4173	382.8640	382.1871
	CPU Time	3.42	30.23	3.38	30.76	3.44	31.03
0.05	ARL	338.1211	338.2577	337.2622	337.1652	337.4229	337.3833
	SDRL	333.8686	333.6936	329.4350	329.5319	328.4671	328.3880
	CPU Time	3.44	26.56	3.45	26.50	3.47	26.72
0.07	ARL	293.5537	293.0475	292.8904	292.6377	293.1291	293.0117
	SDRL	289.3063	289.5784	285.1744	285.7799	284.3229	284.9329
	CPU Time	3.35	22.92	3.44	23.17	3.48	23.13
0.09	ARL	256.8510	256.2881	256.4846	256.8696	256.7974	256.7316
	SDRL	252.6195	252.2016	248.8901	248.9422	248.1495	248.9297
	CPU Time	3.31	20.48	3.49	20.75	3.45	20.48
0.10	ARL	240.9196	240.3633	240.7186	240.2411	241.0660	241.5869
	SDRL	236.6997	236.8637	233.1883	233.1677	232.5000	232.1104
	CPU Time	3.44	19.48	3.36	19.14	3.33	19.52
0.30	ARL	90.1013	90.7059	92.2083	92.0629	92.8936	92.2977
	SDRL	86.4314	86.0075	86.1428	86.8222	86.0755	86.0354
	CPU Time	3.42	7.45	3.48	7.71	3.44	7.64
0.50	ARL	48.3776	48.9515	50.9081	50.6970	51.6070	51.5599
	SDRL	45.4244	45.8286	46.1893	46.2668	46.3204	46.7516
	CPU Time	3.46	4.19	3.43	4.38	3.46	4.52
0.70	ARL	31.4108	31.0605	33.8483	33.0514	34.4980	34.8767
	SDRL	29.0790	29.4944	30.1759	30.4215	30.3841	30.7312
	CPU Time	3.43	2.71	3.44	3.00	3.45	3.10
0.90	ARL	22.7390	22.4857	24.9832	24.8564	25.5787	25.2646
	SDRL	20.8978	20.5344	22.0956	22.0275	22.3396	22.8397
	CPU Time	3.23	2.05	3.43	2.32	3.47	2.32
1.00	ARL	19.8780	19.6851	22.0225	22.8088	22.5929	22.2873
	SDRL	18.2389	18.2735	19.4504	19.0972	19.7048	19.3263
	CPU Time	3.39	1.81	3.26	2.08	3.44	2.08
2.00	ARL	8.3558	8.5428	9.7631	9.6829	10.1588	10.2034
	SDRL	7.8245	7.0135	8.8512	8.6784	9.1144	9.1335
	CPU Time	3.23	0.86	3.30	0.98	3.37	1.07
3.00	ARL	5.0551	5.9869	6.0609	6.0441	6.3567	6.4812
	SDRL	4.9352	4.8992	5.7345	5.7524	5.9587	5.0104
	CPU Time	3.35	0.54	3.34	0.63	3.09	0.64

TABLE B-26 ARL of the EWMA EEWMA and NEEWMA control chart using PEA_0 compared to MC under a TP Reyleigh distribution, with $ARL_0 = 500$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MC	MCA	MC	MCA	MC
0.00	ARL	500.0000	500.8730	500.0000	500.9579	500.0000	500.3074
	SDRL	493.6891	493.8031	481.6057	481.7371	478.9455	478.6057
	CPU Time	2.81	37.72	2.83	37.38	2.91	37.39
0.01	ARL	427.5819	427.7871	401.0119	401.7359	401.0018	401.8363
	SDRL	421.3718	421.2733	382.8991	382.7818	380.3188	380.1094
	CPU Time	3.31	31.92	3.32	30.38	3.40	29.91
0.03	ARL	304.2570	304.1339	269.0087	269.1014	269.2515	269.2665
	SDRL	297.9594	297.6572	251.5297	251.8307	249.3716	249.3622
	CPU Time	3.37	23.35	3.36	20.47	3.42	20.59
0.05	ARL	216.6510	216.0956	189.8050	189.6012	190.3595	190.2743
	SDRL	210.1749	210.6216	173.0283	173.6385	171.3379	171.6872
	CPU Time	3.40	16.92	3.42	14.61	3.41	15.02
0.07	ARL	157.6946	157.3541	139.9631	139.4690	140.7703	140.7515
	SDRL	151.1021	151.7762	123.9289	123.9330	122.6342	122.9879
	CPU Time	3.39	12.49	3.30	11.17	3.32	11.25
0.09	ARL	118.1109	118.0887	107.2221	107.9268	108.2073	108.2054
	SDRL	111.4971	111.4727	91.9462	91.5234	90.9603	90.3969
	CPU Time	3.42	9.55	3.39	8.67	3.30	8.78
0.10	ARL	103.3429	103.7119	95.0343	95.1975	96.0838	96.9751
	SDRL	96.7517	96.6631	80.1373	80.0074	79.2768	79.8411
	CPU Time	2.95	8.27	3.36	7.58	3.32	7.79
0.30	ARL	20.3413	20.3359	23.4112	23.4916	24.4986	24.6673
	SDRL	15.7557	15.5998	14.5001	14.4416	14.4525	14.5389
	CPU Time	3.23	1.84	3.45	2.08	3.36	2.24
0.50	ARL	9.7260	9.7301	12.5111	12.4724	13.3478	13.2834
	SDRL	6.7361	6.7463	6.6590	6.5318	6.7014	6.5870
	CPU Time	3.00	0.94	3.30	1.21	3.18	1.28
0.70	ARL	6.1748	6.1390	8.4461	8.5226	9.1135	9.1824
	SDRL	4.1071	4.0022	4.2487	4.2531	4.3073	4.3378
	CPU Time	3.02	0.65	3.40	0.87	3.24	0.91
0.90	ARL	4.4482	4.4304	6.3346	6.3975	6.8874	6.8886
	SDRL	2.9525	2.9999	3.1466	3.2157	3.2068	3.2318
	CPU Time	3.05	0.45	2.92	0.60	3.31	0.70
1.00	ARL	3.8831	3.8332	5.6185	5.6594	6.1273	6.1070
	SDRL	2.5970	2.5430	2.7982	2.8031	2.8577	2.9075
	CPU Time	2.93	0.42	2.90	0.61	3.45	0.58
2.00	ARL	1.5741	1.5613	2.5299	2.5483	2.8124	2.8092
	SDRL	1.2825	1.2668	1.4477	1.4526	1.4927	1.5101
	CPU Time	2.90	0.22	3.17	0.33	2.91	0.34
3.00	ARL	0.8918	0.8937	1.5408	1.5562	1.7379	1.7373
	SDRL	0.9234	0.9280	1.0723	1.0765	1.1062	1.1060
	CPU Time	2.89	0.17	2.92	0.22	2.94	0.28

TABLE B-27 Effect of control charts parameters on ARL under a Gamma distribution (PEA_0 and $ARL_0 = 500$).

	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
δ	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
No change	0.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00
Small	0.01	447.99	460.95	468.16	447.37	446.23	444.33	447.19	445.71	444.18
	0.03	363.44	393.94	412.04	362.24	360.07	356.99	361.91	359.13	357.79
	0.05	298.78	339.05	364.43	297.54	295.31	292.95	297.19	294.40	295.38
	0.07	248.66	293.74	323.83	247.65	245.85	245.00	247.36	245.18	249.16
	0.09	209.34	256.08	289.03	208.67	207.50	208.40	208.48	207.17	214.14
Medium	0.10	192.85	239.63	273.48	192.38	191.55	193.33	192.23	191.39	199.76
	0.30	57.42	81.94	108.52	59.30	62.66	73.01	59.80	64.46	84.24
	0.50	28.24	39.41	55.29	30.36	34.17	44.78	30.93	36.15	55.18
	0.70	17.72	23.30	33.07	19.65	23.16	32.75	20.17	24.97	41.90
	0.90	12.66	15.68	22.04	14.37	17.49	26.03	14.83	19.11	34.11
Large	1.00	11.05	13.31	18.54	12.64	15.59	23.65	13.08	17.12	31.28
	2.00	4.75	4.86	6.08	5.70	7.50	12.58	5.96	8.45	17.43
	3.00	3.01	2.88	3.33	3.67	4.95	8.64	3.86	5.64	12.20

TABLE B-28 Effect of control charts parameters on ARL under a Weibull distribution (PEA_0 and $ARL_0 = 500$).

	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
δ	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
No change	0.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00
Small	0.01	447.99	460.95	468.16	447.37	446.23	444.33	447.19	445.71	444.18
	0.03	363.44	393.94	412.04	362.24	360.07	356.99	361.91	359.13	357.79
	0.05	298.78	339.05	364.43	297.54	295.31	292.95	297.19	294.40	295.38
	0.07	248.66	293.74	323.83	247.65	245.85	245.00	247.36	245.18	249.16
	0.09	209.34	256.08	289.03	208.67	207.50	208.40	208.48	207.17	214.14
Medium	0.10	192.85	239.63	273.48	192.38	191.55	193.33	192.23	191.39	199.76
	0.30	57.42	81.94	108.52	59.30	62.66	73.01	59.80	64.46	84.24
	0.50	28.24	39.41	55.29	30.36	34.17	44.78	30.93	36.15	55.18
	0.70	17.72	23.30	33.07	19.65	23.16	32.75	20.17	24.97	41.90
	0.90	12.66	15.68	22.04	14.37	17.49	26.03	14.83	19.11	34.11
Large	1.00	11.05	13.31	18.54	12.64	15.59	23.65	13.08	17.12	31.28
	2.00	4.75	4.86	6.08	5.70	7.50	12.58	5.96	8.45	17.43
	3.00	3.01	2.88	3.33	3.67	4.95	8.64	3.86	5.64	12.20

TABLE B-29 Effect of control charts parameters on ARL under a Gumbel distribution (PEA_0 and $ARL_0 = 500$).

	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
δ	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
No change	0.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00
Small	0.01	460.23	464.63	468.93	459.79	459.89	460.61	459.82	459.97	461.77
	0.03	392.72	403.27	414.03	391.87	392.32	394.59	391.96	392.62	397.82
	0.05	338.12	352.25	367.33	337.26	338.16	341.96	337.42	338.71	346.95
	0.07	293.55	309.55	327.39	292.89	294.26	299.47	293.13	295.06	305.91
	0.09	256.85	273.58	293.06	256.48	258.31	264.77	256.80	259.35	272.39
Medium	0.10	240.92	257.72	277.70	240.72	242.76	249.78	241.07	243.90	257.90
	0.30	90.10	98.13	113.05	92.21	96.35	107.95	92.89	98.47	119.36
	0.50	48.38	50.63	58.86	50.91	55.12	66.31	51.61	57.21	76.92
	0.70	31.41	31.27	35.89	33.85	37.73	47.77	34.50	39.63	57.20
	0.90	22.74	21.60	24.35	24.98	28.52	37.46	25.58	30.22	45.83
Large	1.00	19.88	18.48	20.66	22.02	25.41	33.86	22.59	27.01	41.77
	2.00	8.36	6.71	6.89	9.76	12.17	17.64	10.16	13.26	22.69
	3.00	5.06	3.75	3.52	6.06	7.95	12.16	6.36	8.82	15.88

TABLE B-30 Effect of control charts parameters on ARL under a TP Reyleigh distribution (PEA_0 and $ARL_0 = 500$).

	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
δ	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
No change	0.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00
Small	0.01	427.58	421.20	435.66	401.01	400.66	400.63	401.00	400.45	403.03
	0.03	304.26	304.81	335.01	269.01	270.00	273.88	269.25	270.40	281.45
	0.05	216.65	226.08	261.75	189.80	192.69	201.02	190.36	193.98	212.45
	0.07	157.69	171.56	207.55	139.96	144.44	156.21	140.77	146.47	170.06
	0.09	118.11	132.94	166.83	107.22	112.85	126.95	108.21	115.41	142.18
Medium	0.10	103.34	117.88	150.30	95.03	101.09	116.01	96.08	103.84	131.67
	0.30	20.34	22.30	31.05	23.41	30.08	45.72	24.50	33.19	59.89
	0.50	9.73	9.24	11.84	12.51	17.79	30.52	13.35	20.31	42.02
	0.70	6.17	5.31	6.19	8.45	12.73	23.38	9.11	14.83	33.06
	0.90	4.45	3.58	3.85	6.33	9.93	19.08	6.89	11.72	27.46
Large	1.00	3.88	3.04	3.17	5.62	8.95	17.50	6.13	10.61	25.35
	2.00	1.57	1.06	0.93	2.53	4.43	9.62	2.81	5.42	14.51
	3.00	0.89	0.56	0.46	1.54	2.88	6.61	1.74	3.58	10.17

TABLE B-31 Effect of the shape parameters in process on the ARL of control charts under a Gamma distribution (PEA_0 and $ARL_0 = 500$).

δ	Chart	EWMA			EEWMA			NEEWMA		
	α_0	1	5	10	1	5	10	1	5	10
No change	0.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00
Small	0.01	447.99	415.30	402.63	447.37	396.73	363.87	447.19	396.57	363.78
	0.03	363.44	282.60	242.99	362.24	260.90	209.03	361.91	260.92	209.43
	0.05	298.78	195.49	149.36	297.54	180.94	132.23	297.19	181.30	133.13
	0.07	248.66	139.64	97.41	247.65	131.52	90.69	247.36	132.19	91.91
	0.09	209.34	103.26	67.52	208.67	99.60	66.43	208.48	100.48	67.83
Medium	0.10	192.85	89.90	57.38	192.38	87.86	58.03	192.23	88.82	59.47
	0.30	57.42	17.19	10.19	59.30	21.07	14.63	59.80	22.16	15.80
	0.50	28.24	8.20	4.96	30.36	11.28	8.19	30.93	12.11	9.05
	0.70	17.72	5.19	3.14	19.65	7.62	5.63	20.17	8.28	6.30
	0.90	12.66	3.72	2.23	14.37	5.71	4.24	14.83	6.26	4.80
Large	1.00	11.05	3.23	1.92	12.64	5.06	3.76	13.08	5.57	4.27
	2.00	4.75	1.26	0.64	5.70	2.25	1.64	5.96	2.53	1.92
	3.00	3.01	0.67	0.25	3.67	1.34	0.94	3.86	1.54	1.14

TABLE B-32 Effect of the shape parameters in process on the ARL of control charts under a Weibull distribution (PEA_0 and $ARL_0 = 500$).

δ	Chart	EWMA			EEWMA			NEEWMA		
	α_0	1	5	10	1	5	10	1	5	10
No change	0.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00
Small	0.01	447.99	705.48	914.39	447.37	716.61	945.48	447.19	719.70	954.70
	0.03	363.44	>705.48	>914.39	362.24	>716.61	>945.48	361.91	>719.70	>954.70
	0.05	298.78	>705.48	>914.39	297.54	>716.61	>945.48	297.19	>719.70	>954.70
	0.07	248.66	>705.48	>914.39	247.65	>716.61	>945.48	247.36	>719.70	>954.70
	0.09	209.34	>705.48	>914.39	208.67	>716.61	>945.48	208.48	>719.70	>954.70
Medium	0.10	192.85	>705.48	>914.39	192.38	>716.61	>945.48	192.23	>719.70	>954.70
	0.30	57.42	>705.48	>914.39	59.30	>716.61	>945.48	59.80	>719.70	>954.70
	0.50	28.24	>705.48	>914.39	30.36	>716.61	>945.48	30.93	>719.70	>954.70
	0.70	17.72	>705.48	>914.39	19.65	>716.61	>945.48	20.17	>719.70	>954.70
	0.90	12.66	>705.48	>914.39	14.37	>716.61	>945.48	14.83	>719.70	>954.70
Large	1.00	11.05	>705.48	>914.39	12.64	>716.61	>945.48	13.08	>719.70	>954.70
	2.00	4.75	>705.48	>914.39	5.70	>716.61	>945.48	5.96	>719.70	>954.70
	3.00	3.01	>705.48	>914.39	3.67	>716.61	>945.48	3.86	>719.70	>954.70

TABLE B-33 Effect of the location parameters in process on the ARL of control charts under a Gumbel distribution (PEA_0 and $ARL_0 = 500$).

δ	Chart	EWMA			EEWMA			NEEWMA		
	θ_0	1	5	10	1	5	10	1	5	10
No change	0.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00
Small	0.01	460.23	460.23	460.23	459.79	461.28	461.91	459.82	461.49	462.15
	0.03	392.72	392.72	392.72	391.87	395.77	397.42	391.96	396.33	398.07
	0.05	338.12	338.12	338.12	337.26	342.96	345.40	337.42	343.83	346.40
	0.07	293.55	293.55	293.55	292.89	299.94	302.99	293.13	301.07	304.29
	0.09	256.85	256.85	256.85	256.48	264.56	268.08	256.80	265.90	269.63
Medium	0.10	240.92	240.92	240.92	240.72	249.21	252.93	241.07	250.64	254.58
	0.30	90.10	90.10	90.10	92.21	102.93	108.06	92.89	105.14	110.63
	0.50	48.38	48.38	48.38	50.91	60.91	66.03	51.61	63.13	68.67
	0.70	31.41	31.41	31.41	33.85	42.92	47.82	34.50	45.02	50.39
	0.90	22.74	22.74	22.74	24.98	33.25	37.90	25.58	35.21	40.35
Large	1.00	19.88	19.88	19.88	22.02	29.94	34.46	22.59	31.83	36.86
	2.00	8.36	8.36	8.36	9.76	15.49	19.02	10.16	16.88	20.93
	3.00	5.06	5.06	5.06	6.06	10.75	13.62	6.36	11.84	15.21

TABLE B-34 Effect of the scale parameters in process on the ARL of control charts under a TP Reyleigh distribution (PEA_0 and $ARL_0 = 500$).

δ	Chart	EWMA			EEWMA			NEEWMA		
	θ_0	1	5	10	1	5	10	1	5	10
No change	0.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00
Small	0.01	427.58	427.58	427.58	401.01	404.50	405.95	401.00	404.76	406.28
	0.03	304.26	304.26	304.26	269.01	276.68	279.92	269.25	277.54	280.94
	0.05	216.65	216.65	216.65	189.80	199.56	203.76	190.36	200.93	205.33
	0.07	157.69	157.69	157.69	139.96	150.74	155.45	140.77	152.48	157.43
	0.09	118.11	118.11	118.11	107.22	118.45	123.44	108.21	120.45	125.70
Medium	0.10	103.34	103.34	103.34	95.03	106.36	111.44	96.08	108.45	113.80
	0.30	20.34	20.34	20.34	23.41	32.97	37.93	24.50	35.28	40.60
	0.50	9.73	9.73	9.73	12.51	20.27	24.73	13.35	22.29	27.16
	0.70	6.17	6.17	6.17	8.45	14.96	18.99	9.11	16.74	21.21
	0.90	4.45	4.45	4.45	6.33	11.96	15.63	6.89	13.54	17.67
Large	1.00	3.88	3.88	3.88	5.62	10.88	14.40	6.13	12.39	16.36
	2.00	1.57	1.57	1.57	2.53	5.74	8.21	2.81	6.74	9.64
	3.00	0.89	0.89	0.89	1.54	3.86	5.76	1.74	4.61	6.89

TABLE B-35 ARL of the EWMA EEWMA and NEEWMA control chart using PEA_0 compared to MC under a Poisson distribution, with $ARL_0 = 500$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MC	MCA	MC	MCA	MC
0	ARL	498.9152	498.4023	499.9046	499.1877	499.9518	499.2316
	SDRL	476.2835	476.8986	472.7128	472.0288	471.4777	471.8271
	CPU Time	2.90	33.72	2.82	34.40	2.87	33.86
1	ARL	37.5698	37.4158	39.0593	39.5007	39.4916	39.6787
	SDRL	23.0317	23.9697	22.0576	22.1614	21.8078	21.9999
	CPU Time	3.46	2.96	3.41	3.01	3.41	2.97
2	ARL	15.7961	15.7491	17.3093	17.4005	17.7343	17.7886
	SDRL	6.5209	6.4711	6.4413	6.4692	6.4227	6.4233
	CPU Time	3.33	1.30	3.24	1.48	3.36	1.42
3	ARL	9.9387	9.9621	11.1720	11.1669	11.5181	11.5206
	SDRL	3.3774	3.4300	3.4128	3.3883	3.4224	3.4311
	CPU Time	3.41	0.81	3.21	0.92	3.06	0.95
4	ARL	7.2192	7.2445	8.2411	8.2735	8.5289	8.5437
	SDRL	2.2244	2.2243	2.2792	2.2845	2.2930	2.2809
	CPU Time	3.05	0.65	3.45	0.67	3.33	0.77
5	ARL	5.6365	5.6013	6.5052	6.5007	6.7514	6.7437
	SDRL	1.6493	1.6382	1.7053	1.6957	1.7190	1.7205
	CPU Time	3.39	0.55	2.83	0.58	3.19	0.64
10	ARL	2.5476	2.5502	3.0393	3.0287	3.1821	3.1889
	SDRL	0.7363	0.7280	0.7762	0.7715	0.7859	0.7774
	CPU Time	3.41	0.28	2.83	0.31	3.17	0.33
15	ARL	1.5178	1.5162	1.8855	1.8882	1.9828	1.9774
	SDRL	0.5431	0.5418	0.5382	0.5431	0.5348	0.5319
	CPU Time	3.28	0.19	2.92	0.22	3.20	0.22
20	ARL	1.0375	1.0374	1.2484	1.2488	1.3226	1.3233
	SDRL	0.3224	0.3290	0.4392	0.4399	0.4742	0.4731
	CPU Time	3.29	0.16	2.92	0.17	3.30	0.17
25	ARL	0.8003	0.7976	0.9914	0.9912	1.0157	1.0156
	SDRL	0.4072	0.4107	0.2365	0.2322	0.2442	0.2416
	CPU Time	3.31	0.17	2.95	0.14	3.29	0.16
50	ARL	0.0016	0.0023	0.0213	0.0202	0.0300	0.0280
	SDRL	0.0401	0.0479	0.1442	0.1407	0.1707	0.1650
	CPU Time	3.64	0.11	3.02	0.12	3.34	0.08
100	ARL	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	SDRL	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	CPU Time	3.76	0.08	3.35	0.08	3.49	0.07

TABLE B-36 ARL of the EWMA EEWMA and NEEWMA control chart using PEA_0 compared to MC under a Binomial distribution, with $ARL_0 = 500$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MC	MCA	MC	MCA	MC
0	ARL	497.8036	497.6132	499.7323	499.9205	499.4231	499.6256
	SDRL	512.4834	512.8275	515.2806	515.3160	514.6637	514.4220
	CPU Time	3.14	37.38	2.86	34.37	3.17	34.42
1	ARL	296.0185	296.9323	293.7741	293.1848	292.7211	292.9443
	SDRL	306.7864	306.8879	304.8724	304.6806	303.4826	303.9285
	CPU Time	3.59	21.56	3.49	21.05	3.43	20.45
2	ARL	193.8335	193.2771	190.9796	190.2556	190.0007	190.0358
	SDRL	201.9236	201.7185	199.1050	199.1085	197.7930	197.7756
	CPU Time	3.52	14.38	3.42	14.06	3.30	13.65
3	ARL	136.1325	136.9760	133.5421	133.2487	132.7675	132.1805
	SDRL	142.3166	142.6566	139.5989	139.9811	138.5087	138.4800
	CPU Time	3.53	10.44	3.42	9.72	3.41	9.47
4	ARL	100.8039	100.6602	98.6424	98.0959	98.0563	98.0103
	SDRL	105.5926	105.1880	103.2166	103.4337	102.3375	102.1807
	CPU Time	3.58	7.69	3.36	7.23	3.34	7.30
5	ARL	77.7777	77.1494	76.0207	76.8561	75.5830	75.7162
	SDRL	81.5226	81.7447	79.5078	79.1361	78.8010	78.7808
	CPU Time	3.48	5.89	3.38	5.83	3.37	5.60
10	ARL	31.2739	31.3198	30.6428	30.3413	30.5462	30.1987
	SDRL	32.4525	32.2266	31.5782	31.6924	31.3112	31.0524
	CPU Time	3.55	2.43	3.39	2.33	3.43	2.44
15	ARL	17.7715	17.5538	17.5068	17.6297	17.4967	17.5730
	SDRL	18.1185	18.0669	17.6865	17.1386	17.5681	17.9501
	CPU Time	3.47	1.39	2.90	1.41	3.31	1.50
20	ARL	11.9364	11.8854	11.8107	11.0730	11.8249	11.0523
	SDRL	11.9736	11.8216	11.7367	11.0744	11.6824	11.9722
	CPU Time	3.22	1.03	2.94	1.03	3.35	0.98
25	ARL	8.8123	8.7540	8.7473	8.7698	8.7672	8.8048
	SDRL	8.7288	8.5768	8.5896	8.5023	8.5675	8.5693
	CPU Time	3.54	0.70	2.92	0.75	2.88	0.75
50	ARL	3.5393	3.5566	3.5349	3.5424	3.5431	3.5097
	SDRL	3.4257	3.4240	3.4099	3.4370	3.4212	3.4113
	CPU Time	3.06	0.36	2.95	0.35	2.90	0.35
100	ARL	1.4302	1.4326	1.4302	1.4417	1.4306	1.4279
	SDRL	1.4603	1.4709	1.4601	1.4753	1.4617	1.4277
	CPU Time	3.59	0.25	3.01	0.19	3.01	0.19

TABLE B-37 ARL of the EWMA EEWMA and NEEWMA control chart using PEA_0 compared to MC under a Negative Binomial distribution, with $ARL_0 = 500$.

δ	Measure	EWMA (0.1)		EEWMA (0.1, 0.01)		NEEWMA (0.1, 0.01, 0.0025)	
		MCA	MC	MCA	MC	MCA	MC
0	ARL	499.8378	499.0496	499.8469	499.6421	499.8194	499.3836
	SDRL	478.4612	478.4403	473.9240	473.2264	472.6209	472.7124
	CPU Time	3.11	36.03	3.30	35.75	3.22	35.88
1	ARL	41.5487	41.9296	42.5366	42.3882	42.8584	42.7202
	SDRL	26.8092	26.9673	25.2839	25.7876	24.9091	24.7546
	CPU Time	3.64	3.39	3.61	3.45	3.51	3.47
2	ARL	16.7778	16.7973	18.1943	18.2121	18.6001	18.5384
	SDRL	7.1947	7.1849	7.0123	7.0241	6.9715	6.9373
	CPU Time	3.46	1.50	3.64	1.52	3.66	1.55
3	ARL	10.3748	10.3837	11.5760	11.5858	11.9164	11.9309
	SDRL	3.5916	3.5685	3.5975	3.6143	3.6018	3.6109
	CPU Time	3.25	0.94	3.39	1.01	3.35	1.08
4	ARL	7.4729	7.4577	8.4800	8.4875	8.7649	8.7803
	SDRL	2.3191	2.3018	2.3624	2.3629	2.3752	2.3685
	CPU Time	3.52	0.75	3.69	0.76	3.39	0.77
5	ARL	5.8064	5.8071	6.6672	6.6503	6.9107	6.9118
	SDRL	1.6999	1.6942	1.7502	1.7589	1.7643	1.7672
	CPU Time	3.54	0.54	3.68	0.62	3.33	0.62
10	ARL	2.6014	2.6105	3.0940	3.0769	3.2338	3.2374
	SDRL	0.7449	0.7521	0.7822	0.7854	0.7919	0.7950
	CPU Time	3.62	0.30	3.72	0.33	3.25	0.31
15	ARL	1.5566	1.5471	1.9139	1.9087	2.0099	2.0146
	SDRL	0.5419	0.5465	0.5344	0.5357	0.5338	0.5386
	CPU Time	3.34	0.20	3.70	0.24	3.22	0.24
20	ARL	1.0486	1.0504	1.2716	1.2794	1.3535	1.3444
	SDRL	0.3233	0.3245	0.4536	0.4578	0.4846	0.4817
	CPU Time	3.33	0.16	3.34	0.17	3.15	0.21
25	ARL	0.8386	0.8369	0.9900	0.9887	1.0200	1.0215
	SDRL	0.3741	0.3754	0.2386	0.2328	0.2428	0.2422
	CPU Time	3.28	0.17	3.19	0.17	3.17	0.16
50	ARL	0.0014	0.0013	0.0203	0.0193	0.0364	0.0372
	SDRL	0.0374	0.0360	0.1412	0.1376	0.1874	0.1893
	CPU Time	3.58	0.08	3.11	0.08	3.04	0.10
100	ARL	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	SDRL	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	CPU Time	3.19	0.09	2.98	0.08	3.04	0.08

TABLE B-38 Effect of control charts parameters on ARL under a Poisson distribution (PEA_0 and $ARL_0 = 500$).

δ	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
No change	0	498.92	499.94	499.77	499.90	499.35	499.69	499.95	499.90	499.75
Small	1	37.57	47.00	84.29	39.06	42.26	53.00	39.49	44.15	64.25
	2	15.80	13.17	23.35	17.31	20.28	29.04	17.73	21.90	37.60
	3	9.94	6.33	9.10	11.17	13.58	20.55	11.52	14.87	27.33
	4	7.22	3.90	4.45	8.24	10.23	16.01	8.53	11.30	21.62
	5	5.64	2.71	2.51	6.51	8.20	13.13	6.75	9.11	17.92
Medium	10	2.55	0.87	0.32	3.04	4.01	6.85	3.18	4.53	9.64
	15	1.52	0.30	0.04	1.89	2.55	4.55	1.98	2.92	6.52
	20	1.04	0.06	0.00	1.25	1.83	3.35	1.32	2.09	4.88
	25	0.80	0.01	0.00	0.99	1.30	2.62	1.02	1.59	3.87
Large	50	0.00	0.00	0.00	0.02	0.32	1.02	0.03	0.64	1.87
	100	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.90

TABLE B-39 Effect of control charts parameters on ARL under a Binomial distribution (PEA_0 and $ARL_0 = 500$).

δ	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
No change	0	497.80	496.19	313.34	499.73	499.72	499.99	499.42	500.00	499.90
Small	1	296.02	320.42	219.24	293.77	289.19	279.74	292.72	286.64	275.21
	2	193.83	223.08	162.78	190.98	186.37	177.68	190.00	183.77	174.31
	3	136.13	163.98	126.16	133.54	129.79	123.34	132.77	127.65	121.51
	4	100.80	125.59	101.00	98.64	95.76	91.33	98.06	94.10	90.65
	5	77.78	99.32	82.94	76.02	73.86	70.97	75.58	72.58	71.06
Medium	10	31.27	41.85	39.56	30.64	30.21	30.56	30.55	29.85	31.85
	15	17.77	23.61	23.82	17.51	17.53	18.54	17.50	17.41	19.83
	20	11.94	15.51	16.24	11.81	11.96	13.09	11.82	11.92	14.23
	25	8.81	11.18	11.95	8.75	8.93	10.03	8.77	8.91	11.03
Large	50	3.54	4.09	4.48	3.53	3.63	4.37	3.54	3.63	5.09
	100	1.43	1.51	1.62	1.43	1.44	1.71	1.43	1.44	2.35

TABLE B-40 Effect of control charts parameters on ARL under a Negative Binomial distribution (PEA_0 and $ARL_0 = 500$).

δ	λ_1	0.100	0.300	0.700	0.100	0.100	0.100	0.100	0.100	0.100
	λ_2	-	-	-	0.010	0.025	0.050	0.010	0.025	0.050
	λ_3	-	-	-	-	-	-	0.0025	0.00625	0.0125
No change	0	499.84	499.81	499.87	499.85	499.93	499.94	499.82	499.88	499.95
Small	1	41.55	63.66	126.37	42.54	45.07	54.79	42.86	46.67	65.60
	2	16.78	17.48	41.08	18.19	21.04	29.60	18.60	22.60	38.08
	3	10.37	7.87	16.53	11.58	13.94	20.85	11.92	15.21	27.60
	4	7.47	4.63	7.91	8.48	10.45	16.20	8.76	11.52	21.80
	5	5.81	3.13	4.32	6.67	8.36	13.27	6.91	9.26	18.06
Medium	10	2.60	0.93	0.53	3.09	4.07	6.91	3.23	4.59	9.70
	15	1.56	0.33	0.07	1.91	2.59	4.59	2.01	2.96	6.56
	20	1.05	0.06	0.00	1.27	1.86	3.38	1.35	2.11	4.91
	25	0.84	0.01	0.00	0.99	1.33	2.65	1.02	1.63	3.89
Large	50	0.00	0.00	0.00	0.02	0.35	1.02	0.04	0.67	1.88
	100	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.91

TABLE B-41 Effect of the parameters in process on the ARL of control charts under a Poisson distribution (PEA_0 and $ARL_0 = 500$).

δ	Charts	EWMA			EEWMA			NEEWMA		
	θ_0	5	10	50	5	10	50	5	10	50
No change	0	498.92	499.35	499.35	499.69	497.35	496.31	499.75	498.73	499.94
Small	1	37.57	62.11	161.42	53.00	78.57	170.42	64.25	92.39	186.09
	2	15.80	26.37	78.15	29.04	44.30	100.75	37.60	56.27	119.53
	3	9.94	16.76	49.40	20.55	32.33	75.20	27.33	42.36	93.52
	4	7.22	12.38	36.42	16.01	25.87	62.07	21.62	34.49	79.31
	5	5.64	9.84	29.27	13.13	21.70	53.88	17.92	29.27	70.01
Medium	10	2.55	4.81	16.02	6.85	12.21	35.14	9.64	16.97	47.40
	15	1.52	3.10	11.47	4.55	8.50	27.11	6.52	11.99	37.15
	20	1.04	2.22	9.01	3.35	6.48	22.32	4.88	9.24	30.89
	25	0.80	1.72	7.44	2.62	5.20	19.06	3.87	7.49	26.56
Large	50	0.00	0.76	3.93	1.02	2.47	11.14	1.87	3.78	15.84
	100	0.00	0.00	1.99	0.02	1.00	6.02	0.90	1.88	8.80

TABLE B-42 Effect of the parameters in process on the ARL of control charts under a Binomial distribution (PEA_0 and $ARL_0 = 500$).

δ	Charts	EWMA			EEWMA			NEEWMA		
	α_0	5	10	50	5	10	50	5	10	50
No change	0	497.80	499.72	498.62	499.99	499.67	498.96	499.90	499.98	499.81
Small	1	296.02	347.63	428.34	279.74	332.68	419.91	275.21	328.93	418.62
	2	193.83	252.49	370.30	177.68	235.26	357.11	174.31	231.81	355.10
	3	136.13	190.09	322.06	123.34	174.69	306.71	121.51	172.50	304.78
	4	100.80	147.51	281.73	91.33	134.96	265.88	90.65	134.02	264.46
	5	77.78	117.45	247.81	70.97	107.71	232.48	71.06	107.79	231.78
Medium	10	31.27	49.54	140.59	30.56	48.20	132.75	31.85	50.42	135.61
	15	17.77	27.86	88.58	18.54	29.16	87.33	19.83	31.59	92.10
	20	11.94	18.36	60.59	13.09	20.46	63.27	14.23	22.72	68.78
	25	8.81	13.33	44.19	10.03	15.60	48.99	11.03	17.64	54.65
Large	50	3.54	5.12	16.04	4.37	6.89	22.32	5.09	8.15	26.85
	100	1.43	1.98	6.16	1.71	3.07	10.46	2.35	3.69	13.28

TABLE B-43 Effect of the parameters in process on the ARL of control charts under a Negative Binomial distribution (PEA_0 and $ARL_0 = 500$).

δ	Charts	EWMA			EEWMA			NEEWMA		
	β_0	5	10	50	5	10	50	5	10	50
No change	0	499.84	499.70	499.29	499.94	499.91	499.79	499.95	499.92	499.83
Small	1	41.55	67.16	166.63	54.79	80.94	173.70	65.60	94.10	188.13
	2	16.78	27.79	80.94	29.60	45.07	102.27	38.08	56.89	120.58
	3	10.37	17.38	50.87	20.85	32.74	76.03	27.60	42.72	94.15
	4	7.47	12.73	37.27	16.20	26.14	62.61	21.80	34.74	79.75
	5	5.81	10.07	29.82	13.27	21.90	54.27	18.06	29.47	70.35
Medium	10	2.60	4.88	16.17	6.91	12.29	35.31	9.70	17.06	47.57
	15	1.56	3.14	11.55	4.59	8.55	27.22	6.56	12.05	37.27
	20	1.05	2.25	9.06	3.38	6.52	22.41	4.91	9.29	30.99
	25	0.84	1.74	7.47	2.65	5.23	19.14	3.89	7.53	26.65
Large	50	0.00	0.78	3.94	1.02	2.50	11.18	1.88	3.80	15.90
	100	0.00	0.00	1.99	0.02	1.00	6.03	0.91	1.89	8.83

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