



STATISTICAL INFERENCES FOR PARAMETERS OF ZERO-INFLATED
SKEWED DISTRIBUTIONS

MR. WASURAT KHUMPASEE

A DISSERTATION SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS

FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN
APPLIED STATISTICS

DEPARTMENT OF APPLIED STATISTICS
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KING MONGKUT'S UNIVERSITY OF TECHNOLOGY NORTH BANGKOK
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ABSTRACT

The Delta-Inverse Gaussian (delta-IG) distribution is a statistical distribution that incorporates a delta parameter to improve the inverse Gaussian distribution. This delta parameter allows modifications to the distribution's skewness or kurtosis, making it easier to handle data with different shapes or variability patterns in circumstances with an excess of zeros and substantially skewed data. In this thesis, confidence intervals will be used to estimate delta-IG distributions for single- and two-parameter simulations. The generalized confidence interval, adjusted generalized confidence interval, bootstrap percentile confidence interval, fiducial confidence interval, fiducial highest posterior density confidence interval, Bayesian credible interval, Bayesian highest posterior density credible interval, method of variance estimates recovery, and normal approximation will be used as estimation methods. The purpose of this thesis is to calculate the confidence intervals for the function (the single, difference, and ratio) of the variance of the delta-IG distributions. Next, create the confidence intervals for the function of coefficient of variation (CV) for the delta-IG distributions, as well as the confidence intervals of common CV for the delta-IG distributions and the simultaneous confidence intervals for difference of CV for the delta-IG distributions. The optimal confidence interval has a coverage probability better than 0.95 and the smallest Monte Carlo simulation average width. To verify the recommended methods, all methods were tested on traffic fatality data.

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บทคัดย่อ

การแจกแจงเดลตาอินเวอร์สเกาส์เซียน เป็นการแจกแจงทางสถิติที่มีการเพิ่มพารามิเตอร์เดลตาเข้าไปเพื่อปรับปรุงการแจกแจงอินเวอร์สเกาส์เซียน พารามิเตอร์เดลตานี้ช่วยให้สามารถปรับลักษณะความเบ้หรือความโด่งของการแจกแจงได้ ทำให้เหมาะสมต่อการวิเคราะห์ข้อมูลที่มีรูปแบบหลากหลายหรือมีความแปรปรวนแตกต่างกัน โดยเฉพาะในกรณีที่ข้อมูลมีศูนย์จำนวนมากและมีความเบ้อย่างมาก วิทยานิพนธ์ฉบับนี้ศึกษาการประมาณช่วงความเชื่อมั่นสำหรับการแจกแจงเดลตาอินเวอร์สเกาส์เซียน ภายใต้การจำลองแบบหนึ่งพารามิเตอร์และสองพารามิเตอร์ วัตถุประสงค์ของวิทยานิพนธ์นี้คือการสร้างช่วงความเชื่อมั่นสำหรับฟังก์ชันของความแปรปรวน ได้แก่ ค่าเดียว ผลต่าง และอัตราส่วน สำหรับการแจกแจงเดลตาอินเวอร์สเกาส์เซียน รวมถึงการสร้างช่วงความเชื่อมั่นสำหรับฟังก์ชันของสัมประสิทธิ์การแปรผันของการแจกแจงเดลตาอินเวอร์สเกาส์เซียน ตลอดจนช่วงความเชื่อมั่นของสัมประสิทธิ์การแปรผันร่วม และช่วงความเชื่อมั่นพร้อมกันสำหรับผลต่างของสัมประสิทธิ์การแปรผันของการแจกแจงเดลตาอินเวอร์สเกาส์เซียน ช่วงความเชื่อมั่นที่ประสิทธิภาพมากที่สุดถูกเลือกโดยพิจารณาจากความน่าจะเป็นคุ่มรวม ซึ่งต้องมากกว่าหรือใกล้เคียงระดับความเชื่อมั่นที่ 0.95 และมีความกว้างเฉลี่ยที่สั้นที่สุดโดยใช้การจำลองแบบมอนติคาร์โล และเพื่อยืนยันประสิทธิภาพของวิธีการที่แนะนำ ทุกวิธีการสร้างช่วงความเชื่อมั่นได้ถูกนำไปประยุกต์ใช้กับข้อมูลจำนวนผู้เสียชีวิตจากอุบัติเหตุทางถนน

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Wasurat Khumpasee

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LIST OF ABBREVIATIONS AND SYMBOLS

Abbreviations

CI	Confidence Interval
SCI	Simultaneous confidence interval
CV	Coefficient of variation
IG	Inverse Gaussian
Delta-IG	Delta-Inverse Gaussian
CP	Coverage Probability
AW	Average Width
GCI	Generalized confidence interval
AGCI	Adjusted generalized confidence interval
PBPCI	Parametric bootstrap percentile confidence interval
FCI	Fiducial confidence interval
HPD	Highest posterior density
F-HPDCI	Fiducial-highest posterior density confidence interval
BCI	Bayesian credible interval
B-HPDCI	Bayesian-highest posterior density confidence interval
MOVER	Method of variance estimates recovery
NA	Normal approximation
VST	Variance-stabilizing transformation
PDF	Probability density function
CDF	Cumulative distribution function
MLE	Maximum likelihood estimator
AIC	Akaike information criterion
BIC	Bayesian information criterion
GPQ	Generalized pivotal quantity

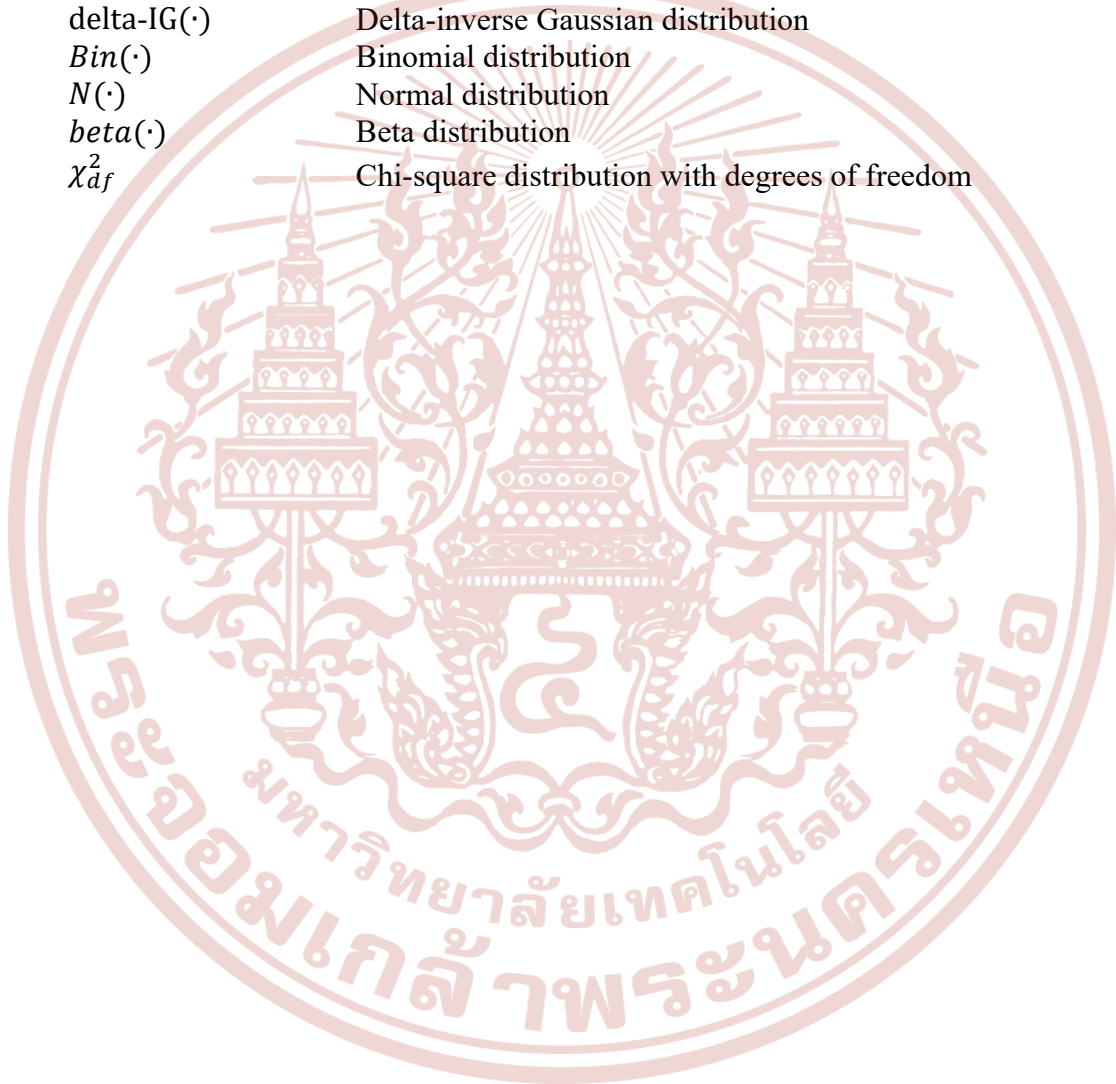
Symbols

μ	Mean parameter
λ	Shape parameter
δ	The proportion of zero values
$n_{(1)}$	The number of positive values
$n_{(0)}$	The number of zero values
n	Total number of observations
θ	The coefficient of variation of delta-inverse Gaussian distribution
θ_{diff}	The difference between the coefficients of variation of delta-inverse Gaussian distributions
θ_{ratio}	The ratio between the coefficients of variation of delta-inverse Gaussian distributions
θ_{common}	The common coefficient of variation of delta-inverse Gaussian distributions
η	The variance of delta-inverse Gaussian distribution

LIST OF ABBREVIATIONS AND SYMBOLS (CONTINUED)

Symbols

η_{diff}	The difference between the variances of delta-inverse Gaussian distributions
η_{ratio}	The ratio between the variances of delta-inverse Gaussian distributions
$IG(\cdot)$	Inverse Gaussian distribution
$\text{delta-IG}(\cdot)$	Delta-inverse Gaussian distribution
$Bin(\cdot)$	Binomial distribution
$N(\cdot)$	Normal distribution
$\text{beta}(\cdot)$	Beta distribution
χ^2_{df}	Chi-square distribution with degrees of freedom





CHAPTER 1

AN OVERVIEW

1.1 Introduction

Parameter estimation is the process of determining unknown constants that characterize a population by using information obtained from a random sample. It can generally be conducted in two forms: point estimation and confidence interval (CI) estimation. Point estimation provides a single numerical value as an estimate of a population parameter; however, this estimate is subject to sampling variability and does not convey the associated uncertainty. In contrast, confidence intervals offer a range of plausible values for the parameter, incorporating sampling error and providing a specified level of confidence that the true parameter lies within this range. For this reason, confidence interval estimation is often preferred in statistical inference. Population parameters, such as the mean, variance, and coefficient of variation, describe fundamental characteristics of a distribution. Since it is typically impractical to observe every unit in a large population, parameter estimation based on random samples plays a crucial role in various fields, including medicine, economics, engineering, and environmental sciences.

The Inverse Gaussian distribution is a continuous probability distribution for positive-valued data and exhibits several properties analogous to those of the Gaussian distribution. Despite its name, the inverse Gaussian distribution is not simply the mathematical inverse of the Gaussian distribution. Rather, the term “inverse” arises from a fundamental stochastic relationship: while the Gaussian distribution describes the position of a Brownian motion at a fixed time, the inverse Gaussian distribution characterizes the distribution of the first passage time, that is, the time required for a Brownian motion with positive drift to reach a predetermined level. This relationship is conceptually analogous to that between the binomial distribution, which models the number of successes in a fixed number of trials, and the negative binomial distribution, which models the number of trials required to achieve a fixed number of successes.

An additional theoretical connection between the Gaussian and inverse Gaussian distributions lies in their cumulant generating functions, whose graphs are reflections of each other across the line $y = -x$. A similar duality is also observed between the binomial and negative binomial distributions (after appropriate normalization), further highlighting the structural symmetry underlying these distributional pairs.

Historically, the inverse Gaussian distribution has a rich development across multiple scientific domains. It was first derived by Louis Bachelier (1900) in the context of modeling the time at which a stock price first reaches a specified level. Later, it was independently studied by Erwin Schrödinger (1915) and Marian Smoluchowski (1915) as the first passage time of Brownian motion. In biological modeling, the distribution appeared as the Hadwiger function, named after Hugo Hadwiger (1940). Subsequently, Abraham Wald (1944) re-derived the distribution in the context of sequential analysis. The term “inverse Gaussian distribution” was later introduced by Maurice Tweedie (1945), who further developed its statistical properties in subsequent works. A

comprehensive review of the distribution was later provided by Folks and Chhikara (1978).

In this study, we consider an extension of the inverse Gaussian distribution, namely the Delta Inverse Gaussian (delta-IG) distribution, which accommodates datasets containing an excess number of zero observations alongside positive continuous values. Following the framework of Aitchison (1955), the delta-IG distribution combines a point mass at zero with the inverse Gaussian distribution, thereby enhancing its flexibility for modeling real-world data exhibiting both structural zeros and right-skewed positive measurements. This extension broadens the applicability of the delta-IG model to a wider range of practical situations. This thesis is organized into four main parts. A brief outline of the contents is provided as follows.

In Chapter 2, methods for constructing confidence intervals for the single coefficient of variation (CV), difference of coefficients of variation, and ratio of coefficients of variation of the delta-IG are presented. The methods for constructing CI for single CV include the generalized confidence interval (GCI), adjusted generalized confidence interval (AGCI), parametric bootstrap percentile confidence interval (PBPCI), fiducial confidence interval (FCI), fiducial highest posterior density confidence interval (F-HPDCI), Bayesian credible interval (BCI), and Bayesian highest posterior density credible interval (B-HPDCI). The methods for constructing CI for difference and ratio of CVs were the AGCI, FCI, BCI, method of variance estimates recovery (MOVER) and normal approximation (NA).

In Chapter 3, confidence intervals for the single variance, standard deviation, difference of variances, and ratio of variances of the delta-IG are presented. The methods for constructing CI for single variance and standard deviation include the AGCI, PBPCI, FCI, and BCI. The AGCI, PBPCI, and FCI will be utilized with estimation methods for proportions which are variance stabilizing transformation (VST), Wilson's score, and Hannig approaches. The methods for constructing CI for difference and ratio of variances were the AGCI, FCI, BCI, MOVER, and NA.

In Chapter 4, confidence intervals are constructed for the common coefficient of variation of the multiple delta-IG distributions. For the common coefficient of variation for the delta-IG distributions, the GCI, AGCI, PBPCI, BCI and B-HPDCI methods are proposed.

In Chapter 5, methods for estimating simultaneous confidence intervals (SCIs) for all pairwise differences between the coefficients of variation of multiple delta-IG distributions are presented. This study uses the GCI, AGCI, FCI, MOVER, and NA methods. The methods employed to calculate the confidence intervals are described in detail in each chapter of the thesis. We executed simulation research employing Monte Carlo simulations in R statistical software to examine the performance of different methods. This study evaluates the coverage probabilities (CPs) and average widths (AWs) of the proposed confidence intervals. The efficacy of the methods was evaluated by applying them to traffic fatality data from many regions of Thailand. Conclusions are provided at the conclusion of each chapter.

1.2 Definition

The definitions provided below are utilized consistently throughout this thesis.

1.2.1 Confidence interval (CI)

Let $X = (X_1, X_2, \dots, X_n)$ be a random sample size n with the probability density function $f_X(x; \varphi)$ where φ is real-valued parameter and let Φ be the parameter space of φ . Let $x = (x_1, x_2, \dots, x_n)$ be an observed value of X , then $L(x) \leq \varphi \leq U(x)$. If there exist the inference $L(x)$ and $U(x)$ then $P[L(x) \leq \varphi \leq U(x)] = 1 - \alpha$, where $0 < 1 - \alpha < 1$. Therefore, the interval estimator $[L(x), U(x)]$ is a $(1 - \alpha)\%$ confidence interval for φ (Casella and Berger, 2002).

1.2.2 Simultaneous confidence interval (SCI)

Let $X_{ij} = (X_{i1}, X_{i2}, \dots, X_{in_i})$ be a random size n_i from the i -th population, and let φ_i represent the parameter of interest for population i . To compare multiple populations, the simultaneous confidence intervals are constructed to evaluate all pairwise differences $\varphi_{il} = \varphi_i - \varphi_l; i \neq l$ where $i, l = 1, 2, \dots, k$.

1.2.3 Coverage probability (CP)

Let $[L(X), U(X)]$ be an interval estimator for the parameter φ . The probability that the random interval $[L(X), U(X)]$ encompasses the true parameter φ is known as the C P of $[L(X), U(X)]$, denoted by $P(\varphi \in [L(X), U(X)] | \varphi)$ (Casella and Berger, 2002). Then, the estimate of the CP can be calculated as follows:

$$CP = \frac{\#[L(X) \leq \varphi \leq U(X)]}{M}$$

where $\#[L(X) \leq \varphi \leq U(X)]$ is the number of simulation runs in which all φ values fall within the interval, and M is the total number of simulation runs.

1.2.4 Average width (AW)

Let $[L(X), U(X)]$ be an interval of a parameter φ ; $L(X)$ and $U(X)$ are the lower and upper limits, respectively. The interval width is the difference between $L(X)$ and $U(X)$, then

$$w(X) = U(X) - L(X).$$

The expected value of the width, denoted as $E[w(X)] = E[U(X) - L(X)]$, represents the average width of the confidence interval. The estimated average width becomes:

$$AW = \sum_{r=1}^M [U_r(X) - L_r(X)] / M$$

where $L_r(X)$ and $U_r(X)$ stands for the lower and upper limits of the confidence interval with r -th simulation run; $r = 1, 2, \dots, M$, respectively, and M is the number of simulation runs.

1.3 Review of the distributions

This thesis examines the following distributions.

1.3.1 Inverse Gaussian (IG) distribution

Suppose that the random variable $X = (X_1, X_2, \dots, X_n)$ follows a IG distribution with mean parameter (μ) and shape parameter (λ). The probability density function (PDF) of the IG distribution, denoted as $X \sim IG(\mu, \lambda)$ is

$$f(x; \mu, \lambda) = \sqrt{\frac{\lambda}{2\pi x^3}} \exp \left[\frac{-\lambda(x-\mu)^2}{2\mu^2 x} \right], x > 0, \mu > 0, \lambda > 0 \quad (1-1)$$

The cumulative distribution function (CDF) of X is given by

$$F(x) = \Phi \left(\sqrt{\frac{\lambda}{x}} \left(\frac{x}{\mu} - 1 \right) \right) + \exp \left(\frac{2\lambda}{\mu} \right) \Phi \left(-\sqrt{\frac{\lambda}{x}} \left(\frac{x}{\mu} + 1 \right) \right), x > 0, \mu > 0, \lambda > 0, \quad (1-2)$$

where $\Phi(\cdot)$ is the CDF of a standard normal distribution.

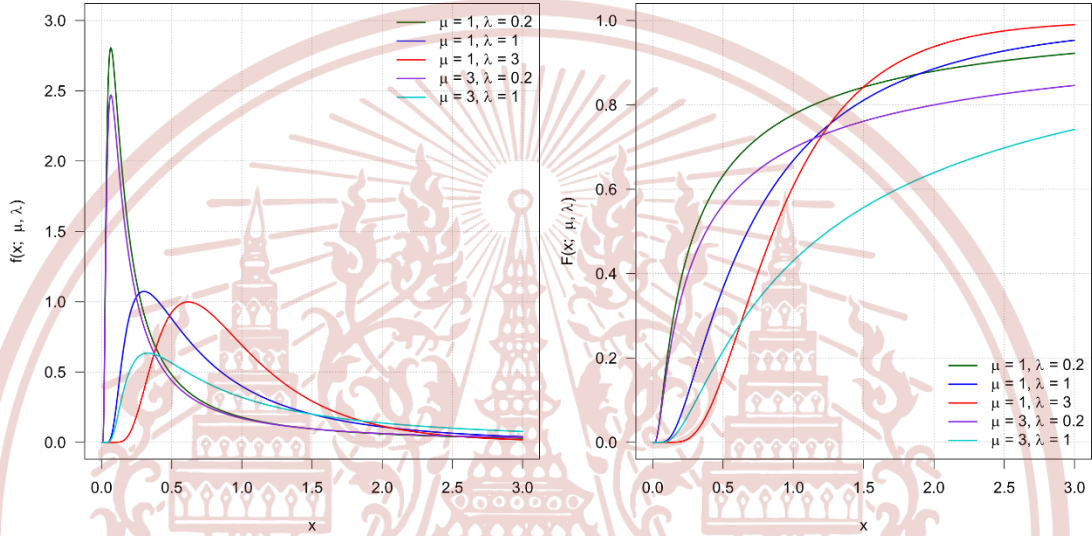


FIGURE 1-1 (Left) PDF and (Right) CDF of the inverse Gaussian distribution

The population mean and variance of the IG distribution are described by

$$E(X) = \mu, \quad (1-3)$$

and

$$\text{Var}(X) = \frac{\mu^3}{\lambda}. \quad (1-4)$$

1.3.2 Delta Inverse Gaussian (delta-IG) distribution

Assume that the random variable $X = (X_1, X_2, \dots, X_n)$ follows a delta-IG distribution with mean parameter (μ), shape parameter (λ), and the probability of zero values (δ). The PDF of the delta-IG distribution, denoted as $X \sim \text{delta-IG}(\mu, \lambda, \delta)$ is

$$g(x; \mu, \lambda, \delta) = \delta I_{\{x=0\}} + (1 - \delta) I_{\{x>0\}} \sqrt{\frac{\lambda}{2\pi x^3}} \exp \left[\frac{-\lambda(x-\mu)^2}{2\mu^2 x} \right], \quad (1-5)$$

where $I(\cdot)$ is an indicator function, $\delta = P(X = 0)$, and $\mu, \lambda > 0$.

The CDF of X is given by

$$G(x; \mu, \lambda, \delta) = \begin{cases} \delta & ; x = 0 \\ \delta + (1 - \delta)F(x; \mu, \lambda) & ; x > 0, \end{cases} \quad (1-6)$$

where $F(x; \mu, \lambda)$ is the CDF of the IG distribution, $\delta = P(X = 0)$ follow the binomial distribution and $n_{(0)} \sim \text{Bin}(n, \delta)$. The maximum likelihood estimator of δ is $\hat{\delta} = n_{(0)}/n$, where $n_{(0)}$ and $n_{(1)}$ are the number of zero observed values and the number of positive observed values, respectively, and $n = n_{(0)} + n_{(1)}$.

The MLEs of the parameters are $\hat{\delta} = \frac{n_{(0)}}{n}$, $\hat{\mu} = \bar{X} = \frac{1}{n_{(1)}} \sum_{j=1}^{n_{(1)}} X_j$, and $n_{(1)} \hat{\lambda}^{-1} = V = \sum_{j=1}^{n_{(1)}} (X_j^{-1} - \bar{X}^{-1})$ (Folks and Chhikara (1978)).

The population mean and variance of the delta-IG distribution are given by

$$E(X) = (1 - \delta)\mu, \quad (1-7)$$

and

$$Var(X) = \mu^2(1 - \delta) \left(\frac{\mu}{\lambda} + \delta \right). \quad (1-8)$$

1.3.3 The parameters of interest

In this thesis, the focus is on the parameter coefficient of variation (CV) and variance of the delta-IG distribution.

The population CV of the delta-IG distribution is

$$\theta = \sqrt{(\mu + \lambda\delta)/(1 - \delta)\lambda}.$$

The population variance of the delta-IG distribution is

$$\eta = \mu^2(1 - \delta) \left(\frac{\mu}{\lambda} + \delta \right).$$

These are extended to functions of the single parameter, difference, ratio, common parameter, and simultaneous estimation of the parameter, as described in Table 1-1.

TABLE 1-1 The parameters relate to this thesis.

Parameters	Transformed parameters	Notations
Coefficient of Variation	The single CV	θ
	The difference between the CVs	$\theta_{diff} = \theta_1 - \theta_2$
	The ratio between the CVs	$\theta_{ratio} = \theta_1/\theta_2$
	The common CV	$\theta_{common} = \frac{\sum_{i=1}^k [\hat{\theta}_i / \hat{V}\hat{ar}(\hat{\theta}_i)]}{\sum_{i=1}^k [1/\hat{V}\hat{ar}(\hat{\theta}_i)]},$ $i, l = 1, 2, \dots, k.$
Variance	All pairwise differences between CVs	$\theta_{il} = \theta_i - \theta_l;$ $i, l = 1, 2, \dots, k$ and $i \neq l.$
	The single variance	η
	The difference between the variances	$\eta_{diff} = \eta_1 - \eta_2$
	The ratio between the variances	$\eta_{ratio} = \eta_1/\eta_2$

1.4 Data processing and management

The CPs and AWs of the confidence intervals are evaluated by a Monte Carlo simulation conducted using the R statistical program (R Core Team, 2022). Comprehensive information on each calculation is available in Appendix A.

1.5 List of publications and presentations

1.5.1 Publications

1.5.1.1. Khumpasee, W., Niwitpong, S.-A., & Niwitpong, S. (2024). “Confidence Intervals for the Coefficient of Variation in Delta Inverse Gaussian Distributions.” *Symmetry*. Vol.16 No.11: 1488.

1.5.1.2 Khumpasee, W., Niwitpong, S.-A., & Niwitpong, S. (2025). “Confidence Intervals for the Variance and Standard Deviation of Delta-Inverse Gaussian Distributions with Application to Traffic Mortality Count.” *Symmetry*. Vol.17 No.3: 387.

1.5.1.3 Khumpasee, W., Niwitpong, S.-A., & Niwitpong, S. (2026). “Confidence Intervals for the Difference and Ratio of Two Coefficients of Variation of Delta-Inverse Gaussian Distributions.” *Computational Journal of Mathematical and Statistical Sciences*. Vol.5 No.1: 331-353.

1.5.1.4 Khumpasee, W., Niwitpong, S.-A., & Niwitpong, S. (2026). “Confidence Intervals for the Difference and Ratio of Two Variances of Delta-Inverse Gaussian Distributions.” *Mathematics*. Vol.14 No.3: 536.

1.5.1.5 Khumpasee, W., Niwitpong, S.-A., & Niwitpong, S. (2026). “Simultaneous Confidence Intervals for All Pairwise Differences of Coefficients of Variation of Delta-Inverse Gaussian Distributions.” *Symmetry*. Vol.18 No.4: 604.

1.5.1.6 Khumpasee, W., Niwitpong, S.-A., & Niwitpong, S. (2026). “Confidence Intervals for the Common Coefficients of Variation of Delta-Inverse Gaussian Distributions.” *Symmetry*. In progress.

1.5.2 Presentations

1.5.2.1 Khumpasee, W., Niwitpong, S.-A. and Niwitpong, S. “Confidence Intervals for the Coefficient of Variation in Delta Inverse Gaussian Distributions with Application to Rainfall Dispersion Data.” In *Proceedings of the 5th International Conference on Informatics, Agriculture, Management, Business administration, Engineering, Sciences and Technology: IAMBEST 2024*. Krabi, Thailand : Deevana Plaza Krabi Aonang, May 29 - 31, 2024 : 88-94.

1.5.2.2 Niwitpong, S., Khumpasee, W., & Niwitpong, S. (2025). “Confidence Intervals for the Variance of Delta-Inverse Gaussian Distribution with Application to Traffic Mortality Count.” *Journal of Physics: Conference Series*. Vol.3042 No.1: 012009.

For more details about these papers and proceedings are given in Appendix B.

CHAPTER 2

CONFIDENCE INTERVALS FOR A SINGLE COEFFICIENT OF VARIATION, THE DIFFERENCE BETWEEN COEFFICIENTS OF VARIATION, AND THE RATIO BETWEEN COEFFICIENTS OF VARIATION OF THE DELTA-INVERSE GAUSSIAN DISTRIBUTIONS

This chapter presents methods for constructing confidence intervals for the single coefficient of variation, the difference between coefficients of variation, and the ratio between coefficients of variation (CVs) of the delta Inverse Gaussian (delta-IG) distribution. The methods for constructing CI for single CV include the generalized confidence interval (GCI), adjusted generalized confidence interval (AGCI), parametric bootstrap percentile confidence interval (PBPCI), fiducial confidence interval (FCI), fiducial highest posterior density confidence interval (F-HPDCI), Bayesian credible interval (BCI), and Bayesian highest posterior density credible interval (B-HPDCI). The methods for constructing CI for difference and ratio of CVs were the AGCI, FCI, BCI, method of variance estimates recovery (MOVER) and normal approximation (NA). A simulation study is conducted to assess the coverage probabilities (CPs) and average widths (AWs) of these intervals. The proposed methods for estimating confidence intervals for the single coefficient of variation, the difference and ratio between coefficients of variation are demonstrated using traffic fatality data from the central region of Thailand.

2.1 Introduction

Observations that are non-negative with an excess of zeros are frequently encountered in diverse fields such as economics, biomedical sciences, life sciences, transportation safety, and meteorology. These data often exhibit asymmetry with a right-skewed distribution and heteroscedasticity, where many observations cluster at zero while positive values follow a skewed pattern (Liu et al., 2019). For instance, in traffic fatality counts, values may be zero on days with no accidents, while non-zero values represent actual recorded fatalities or injuries.

Several distributions have been proposed for modeling such zero-inflated data. Among them, the Inverse Gaussian (IG) distribution has proven particularly effective. Heller et al. (2006) found that the IG distribution outperformed the Gamma distribution for zero-inflated insurance data. Ma and Yan (2014) demonstrated its suitability for modeling traffic accident frequencies, and Punzo (2019) applied it to insurance and economic data. Folks and Chhikara (1978) further noted that the IG distribution offers computational advantages and is simpler than the lognormal distribution. Chatzidiamantis et al. (2011) highlighted its usefulness in wind speed modeling, while Bardsley (1980) noted its reduced complexity compared to the three-parameter Weibull distribution.

The zero-inflated extension of the IG distribution, known as the delta-IG distribution, incorporates an additional parameter $\delta \in [0, 1]$ representing the probability mass at zero, while preserving the IG's ability to model heavy right tails. This makes it suitable for modeling zero-inflated, right-skewed outcomes such as accident fatality counts, insurance claims, rainfall amounts, and COVID-19 deaths (Chhikara & Folks, 1989). Research on confidence intervals (CIs) for parameters of the delta-IG distribution has grown progressively. Jana and Gautam (2024) examined CIs for means and their differences and ratios within this distribution.

A scale-free statistic widely used for comparing variability among groups is the coefficient of variation (CV), defined as the ratio of the standard deviation to the mean. The CV is particularly valuable as it allows comparison of variability across datasets with different units of measurement. Comparative studies often extend beyond a single CV to examine the difference or ratio of two CVs, as these measures directly evaluate relative variability between populations as a distinction crucial in quality control, biostatistics, and risk assessment. Constructing adequate CIs for these functionals under delta-IG sampling is complex due to skewness, heteroscedasticity, and the point mass at zero. Prior research has addressed CIs for the CV, as well as differences and ratios of CVs, under various distributions. Chankham et al. (2019) proposed CIs for the CV of the IG distribution using the GCI approach. Buntao and Niwitpong (2013) studied CIs for the difference of CVs in lognormal and delta-lognormal distributions, finding the Generalized Variable Approach most suitable. La-ongkaew et al. (2021) examined CIs for the difference between two CVs of Weibull distributions using GCI, Bayesian, and MOVER methods. Yosboonruang et al. (2022) utilized fiducial GCI and Bayesian methods for the ratio of CVs of a lognormal distribution with excess zeros. Chankham et al. (2022) developed FCI, F-HPDCI, and MOVER methods for the ratio of CVs of the IG distribution. Janthasuwan et al. (2025) constructed CIs for the ratio of CVs of delta Birnbaum-Saunders distributions using MOVER, bootstrap CI, and GCI methods. Notable contributions on inferential methods for the CV also include Banik and Kibria (2011), Banik et al. (2012), Albatineh et al. (2014), and Akyüz et al. (2024).

Despite this progress, no study has simultaneously addressed CIs for the single CV, the difference, and the ratio of CVs within the delta-IG framework. This study therefore formulates and evaluates a comprehensive set of methods for these three estimation problems. For the single CV, the methods considered are the Generalized Confidence Interval (GCI), Adjusted Generalized Confidence Interval (AGCI), Parametric Bootstrap Percentile Confidence Interval (PBPCI), Fiducial Confidence Interval (FCI), Fiducial-Highest Posterior Density Confidence Interval (F-HPDCI), Bayesian Credible Interval (BCI), and Bayesian-Highest Posterior Density Credible Interval (B-HPDCI). For the difference and ratio of two CVs, the methods considered are the AGCI, FCI, BCI, Method of Variance Estimates Recovery (MOVER), and Normal Approximation (NA). Performance of all methods is evaluated through Monte Carlo simulations focusing on coverage probability (CP) and average width (AW), with practical demonstration using traffic fatality data from the central region of Thailand.

2.2 The confidence interval for the coefficient of variation of delta-IG distribution

Let $X = (X_1, X_2, \dots, X_n)$ be a non-negative random sample drawn from a delta-IG distribution with mean parameter (μ), shape parameter (λ), and the probability of zero values (δ) denoted as $X \sim \text{delta-IG}(\mu, \lambda, \delta)$ is

$$f(x; \mu, \lambda, \delta) = \delta I_{\{x=0\}} + (1 - \delta) I_{\{x>0\}} \sqrt{\frac{\lambda}{2\pi x^3}} \exp \left[\frac{-\lambda(x-\mu)^2}{2\mu^2 x} \right], \quad (2-1)$$

where $I(\cdot)$ is an indicator function, $\delta = P(X = 0)$, and $\mu, \lambda > 0$. The random sample consists of n observations, which can be partitioned into zero observations $n_{(0)}$ and nonzero observations $n_{(1)}$, such that $n = n_{(0)} + n_{(1)}$. The number of zeros follows a binomial distribution, $n_{(0)} \sim \text{Bin}(n, \delta)$.

The maximum likelihood estimators (MLEs) of the parameters are $\hat{\delta} = \frac{n_{(0)}}{n}$, $\hat{\mu} = \bar{X} = \frac{1}{n_{(1)}} \sum_{j=1}^{n_{(1)}} X_j$, and $n_{(1)} \hat{\lambda}^{-1} = V = \sum_{j=1}^{n_{(1)}} (X_j^{-1} - \bar{X}^{-1})$. In this section, the interest parameter is the CV of X which can be calculated as

$$\theta = \sqrt{(\mu + \lambda\delta)/(1 - \delta)\lambda}. \quad (2-2)$$

The following section provides the methods for constructing confidence intervals for θ .

2.2.1 Generalized confidence intervals (GCI)

Weerahandi (1993) proposed the GCI method that offers a flexible approach for interval estimation, particularly in settings where conventional methods are unreliable or difficult to implement. Classical confidence intervals often rely on restrictive assumptions, such as normality or large sample sizes, whereas the GCI framework relaxes these conditions and remains effective in more complex scenarios.

The GCI is constructed based on a generalized pivotal quantity (GPQ), a function whose distribution is free from unknown parameters defined as follows

DEFINITION 2-1 Let $X = (X_1, X_2, \dots, X_n)$ be a random variable with the PDF $f_X(x; \tau, \nu)$, where τ is the parameter of interest and ν is a nuisance parameter. Let $x = (x_1, x_2, \dots, x_n)$ denote the observed value of X . Then, $R = R(X; x, \tau, \nu)$ is considered a GPQ for τ if it meets the following two conditions:

1. The distribution of $R = R(X; x, \tau, \nu)$ does not depend on any unknown parameters.
2. The observed pivotal, defined as $r = r(x; x, \tau, \nu)$, does not depend on nuisance parameter ν .

Then, if $R(X; x, \tau, \nu)$ satisfies the conditions (1) and (2), the quantiles of R form a $(1 - \alpha)$ confidence interval for τ . Now, let $R(\alpha)$ be the α^{th} quantile of $R(X; x, \tau, \nu)$. Subsequently, the $100(1 - \alpha)\%$ two-sided GCIs for parameter τ is $(R_{\alpha/2}, R_{1-\alpha/2})$.

From the study of Ye, Ma, and Wang (2010), consider k independent inverse Gaussian (IG) populations that share a common mean μ , while allowing their scale parameters λ_i , $i = 1, \dots, k$, to differ. Let $X_{i1}, X_{i2}, \dots, X_{in_i}$ denote a random sample drawn from the i -th population, which follows an IG distribution with parameters (μ, λ_i) .

For each population, the maximum likelihood estimators (MLEs) of μ and λ_i are given by $\hat{\mu} = \bar{X}_i$ and $\hat{\lambda}_i^{-1} = \frac{1}{n_i} \sum_{j=1}^{n_i} (X_{ij}^{-1} - \bar{X}_i^{-1})$, respectively. Defining $V_i = \hat{\lambda}_i^{-1}$ for notational convenient, it follows that $\bar{X}_i$ and V_i are mutually independent random variables, with the distributions

$$\bar{X}_i \sim IG(\mu, n_i \lambda_i) \text{ and } n_i \lambda_i V_i \sim \chi_{n_i-1}^2, i = 1, 2, \dots, k.$$

where $\chi_{n_i-1}^2$ denotes the χ^2 distribution with $n_i - 1$ degrees of freedom. These statistics $(\bar{X}_i, V_i)$ jointly form a complete sufficient set for inference on μ and λ_i .

Let $\bar{x}_i$ and v_i denote as the observed values of $\bar{X}_i$ and V_i , respectively. The GPQ for λ_i based on the i -th sample is given by

$$R_{\lambda_i} = \frac{n_i \lambda_i V_i}{n_i v_i} \sim \frac{\chi_{n_i-1}^2}{n_i v_i}, i = 1, 2, \dots, k \quad (2-3)$$

For a given v_i , let $R_{\lambda_i}(\gamma)$ denote the 100γ -th percentile of R_{λ_i} . Then, $(R_{\lambda_i}(\alpha/2), R_{\lambda_i}(1 - \alpha/2))$ is an exact $100(1 - \alpha)\%$ confidence interval for λ_i and it coincides with the usual confidence interval based on the χ^2 distribution. Thus, the GPQ for μ_i based on the i -th sample is described as

$$R_{\mu_i} = \frac{\bar{x}_i}{1 + \frac{\sqrt{n_i \lambda_i (\bar{x}_i - \mu_i)}}{\mu_i \sqrt{\bar{x}_i}} \sqrt{\frac{\bar{x}_i}{n_i R_{\lambda_i}}}} \sim \frac{\bar{x}_i}{1 + Z_i \sqrt{\frac{\bar{x}_i}{n_i R_{\lambda_i}}}}, \quad (2-4)$$

where $\sim$ means approximately distributed and $Z_i \sim N(0, 1)$. From the last expression in (2-4), the distribution of R_{μ_i} is free of any unknown parameters. Therefore, R_{μ_i} satisfies conditions (1) and (2) in Definition 2-1.

Thus, the GPQ for the parameters λ and μ for the delta-IG distribution can be expressed as

$$R_{\lambda} = \frac{n_{(1)} \lambda V}{n_{(1)} v} \sim \frac{\chi_{n_{(1)}-1}^2}{n_{(1)} v} \quad (2-5)$$

and

$$R_{\mu} = \frac{\bar{X}}{1 + \frac{\sqrt{n_{(1)} \lambda (\bar{X} - \mu)}}{\mu \sqrt{\bar{X}}} \sqrt{\frac{\bar{X}}{n_{(1)} R_{\lambda}}}} \sim \frac{\bar{X}}{1 + Z \sqrt{\frac{\bar{X}}{n_{(1)} R_{\lambda}}}}. \quad (2-6)$$

The parameter δ is estimated using a variance-stabilizing transformation (VST). The concept of VST, originally introduced by Dasgupta (2008), was later extended by Wu and Hsieh (2014), who applied the arcsine square-root transformation to stabilize the variance of binomial data. The GPQ for δ is defined as

$$R_{\delta} = \sin^2 \left[\arcsin \sqrt{\hat{\delta}} - \frac{T}{2\sqrt{n}} \right], \quad (2-7)$$

where $T = 2\sqrt{n}(\arcsin \sqrt{\hat{\delta}} - \arcsin \sqrt{\delta}) \sim N(0, 1)$, $n \rightarrow \infty$.

Therefore, from the equations (2-5), (2-6), and (2-7), the GPQ of the single CV (θ) can be expressed as

$$R_{\theta} = \sqrt{(R_{\mu} + R_{\lambda} R_{\delta}) / (1 - R_{\delta}) R_{\lambda}}. \quad (2-8)$$

Consequently, the $100(1 - \alpha)\%$ two-sided confidence interval for θ is determined from the empirical percentiles of the GPQs from the GCI method is

$$CI_{\theta}^{GCI} = [L_{\theta}^{GCI}, U_{\theta}^{GCI}] = [R_{\theta}(\alpha/2), R_{\theta}(1 - \alpha/2)], \quad (2-9)$$

where $R_\theta(\alpha/2)$ and $R_\theta(1 - \alpha/2)$ are the $100\frac{\alpha}{2}$ -th and $100(1-\frac{\alpha}{2})$ -th percentiles of the distribution of R_θ , respectively.

2.2.2 Adjusted generalized confidence interval (AGCI)

Building on the GCI framework for inverse Gaussian distributions, Krishnamoorthy and Tian (2008) later introduced an alternative generalized pivotal quantity (GPQ) for the parameter of interest. The adjusted generalized confidence interval (AGCI) method employed in this study adopts this modified GPQ for μ , replacing in the Equation (2-4) of the conventional form used in the standard GCI approach. The GPQ for μ , as proposed by Krishnamoorthy and Tian, is defined as

$$R_{\tilde{\mu}} = \frac{\bar{x}}{\max\left\{0, 1 + t_{n_{(1)}-1} \sqrt{\frac{\bar{x}v}{n_{(1)}-1}}\right\}}, \quad (2-10)$$

where $t_{n_{(1)}-1}$ is Student's t distribution with $n_{(1)} - 1$ degrees of freedom. The estimation of δ continues to rely on the variance-stabilizing transformation (VST).

Accordingly, the GPQs for the parameters are R_λ , $R_{\tilde{\mu}}$, and R_δ , given in equations (2-5), (2-10), and (2-7), respectively. Using these, the GPQ for the single coefficient of variation (θ) is constructed as

$$R_{\tilde{\theta}} = \sqrt{(R_{\tilde{\mu}} + R_\lambda R_\delta) / (1 - R_\delta) R_\lambda}, \quad (2-11)$$

Consequently, the $100(1 - \alpha)\%$ two-sided confidence interval for θ is determined from the empirical percentiles of the GPQs from the AGCI method is

$$CI_\theta^{AGCI} = [L_\theta^{AGCI}, U_\theta^{AGCI}] = [R_{\tilde{\theta}}(\alpha/2), R_{\tilde{\theta}}(1 - \alpha/2)], \quad (2-12)$$

where $R_{\tilde{\theta}}(\alpha/2)$ and $R_{\tilde{\theta}}(1 - \alpha/2)$ are the $100\frac{\alpha}{2}$ -th and $100(1-\frac{\alpha}{2})$ -th percentiles of the distribution of $R_{\tilde{\theta}}$, respectively.

2.2.3 Parametric Bootstrap Percentile Confidence Interval (PBPCI)

The parametric bootstrap percentile confidence interval (PBPCI), introduced by Efron and Tibshirani (1986), is a simulation-based method for constructing confidence intervals for a parameter of interest. Unlike traditional approaches that rely on asymptotic theory or strict distributional assumptions, this method assumes that the data follow a specified parametric model, such as the normal or Weibull distribution, and estimates the parameters of the model from the observed sample.

The core idea is to generate a large number of bootstrap samples from the fitted parametric distribution rather than resampling directly from the data. For each bootstrap sample, the statistic of interest is computed, producing an empirical distribution. The confidence interval is then obtained from the $\alpha/2$ and $1 - \alpha/2$ percentiles of this distribution. For example, a 95% confidence interval corresponds to the 2.5th and 97.5th percentiles.

From $x = (x_1, x_2, \dots, x_n)$ be a non-negative random sample of size n drawn from a delta-IG(μ, λ, δ) distribution. The MLE of μ and λ denoted as $\hat{\mu}$ and $\hat{\lambda}$, respectively. Let $x^* = (x_1^*, x_2^*, \dots, x_b^*)$ denote the bootstrap samples of size b . Define $b_{(0)}^*$ and $b_{(1)}^*$ as the numbers of zero and nonzero observations from bootstrap samples, respectively, such that $b = b_{(0)}^* + b_{(1)}^*$. Subsequently, $\hat{\mu}^*$ and $\hat{\lambda}^*$ denote the bootstrap estimates of $\hat{\mu}$

and $\hat{\lambda}$, obtained using the Broyden–Fletcher–Goldfarb–Shanno (BFGS) quasi-Newton optimization algorithm applied to the bootstrap sample $x^* = (x_1^*, x_2^*, \dots, x_b^*)$. Let $B(\hat{\mu}, \mu)$ and $B(\hat{\lambda}, \lambda)$ represent the biases of the estimators $\hat{\mu}$ and $\hat{\lambda}$, defined as

$$B(\hat{\mu}, \mu) = E(\hat{\mu}) - \mu \text{ and } B(\hat{\lambda}, \lambda) = E(\hat{\lambda}) - \lambda. \quad (2-13)$$

The corresponding bias estimators are given by

$$B(\hat{\mu}, \mu) = \bar{\mu}^* - \hat{\mu} \text{ and } B(\hat{\lambda}, \lambda) = \bar{\lambda}^* - \hat{\lambda}, \quad (2-14)$$

where $\bar{\mu}^* = \frac{1}{b} \sum_{j=1}^b \hat{\mu}_j^*$, $\bar{\lambda}^* = \frac{1}{b} \sum_{j=1}^b \hat{\lambda}_j^*$, $j = 1, 2, \dots, b$.

The bias-corrected bootstrap estimators for μ and λ are then given by

$$R_{\mu}^{PBPCI} = \hat{\mu}^* - 2\hat{B}(\hat{\mu}, \mu) \text{ and } R_{\lambda}^{PBPCI} = \hat{\lambda}^* - 2\hat{B}(\hat{\lambda}, \lambda) \quad (2-15)$$

The estimator of the single CV (θ) for the delta-IG distribution is given by

$$\hat{\theta}^* = \sqrt{(R_{\mu}^{PBPCI} + R_{\lambda}^{PBPCI} R_{\delta}) / (1 - R_{\delta}) R_{\lambda}^{PBPCI}}, \quad (2-16)$$

Finally, the $100(1-\alpha)\%$ two-sided CI of θ for the delta-IG distribution based on the PBPCI method is given by

$$CI_{\theta}^{PBPCI} = [\hat{\theta}^*(\alpha/2), \hat{\theta}^*(1 - \alpha/2)], \quad (2-17)$$

where $\hat{\theta}^*(\alpha/2)$ and $\hat{\theta}^*(1 - \alpha/2)$ are the $100\frac{\alpha}{2}$ -th and $100(1 - \frac{\alpha}{2})$ -th percentiles of the distribution of $\hat{\theta}^*$.

2.2.4 Fiducial confidence interval (FCI)

In the context of parameter estimation, the fiducial confidence interval (FCI) approach provides a framework for deriving a probability distribution for an unknown parameter directly from the observed data, without relying on prior distributions or repeated sampling arguments. This method was first introduced by Fisher (1930), where a fiducial distribution is constructed to represent the uncertainty of the parameter based solely on the assumed model and the observed sample.

Unlike Bayesian credible intervals, fiducial confidence intervals do not require subjective or objective prior information. However, despite offering an alternative perspective on statistical inference, the fiducial approach has been criticized for its unconventional interpretation of parameter uncertainty.

Let $X = (X_1, X_2, \dots, X_n)$ be a random sample from the delta-IG distribution with parameters μ , λ , and δ . The CV is defined as

$$CV = h(\mu, \lambda, \delta) = \theta = \sqrt{(\mu + \lambda\delta) / (1 - \delta)\lambda}$$

where $h(\cdot)$ is a known function depending on the parameterization of the delta-IG distribution.

To construct the fiducial distribution, the sampling distributions of the MLEs are utilized. Let $\hat{\mu}$ and $\hat{\lambda}$ denote the MLEs of μ and λ , respectively. Based on the properties of the inverse Gaussian distribution, their sampling distributions can be approximated as: $\mu \sim IG(\hat{\mu}, n_{(1)}\hat{\lambda})$ and $\lambda \sim \frac{\hat{\lambda}}{n_{(1)}} \chi_{n_{(1)}-1}^2$.

These distributions are treated as fiducial distributions by replacing the unknown parameters with their corresponding MLEs.

Since the $CV = h(\mu, \lambda, \delta)$ is a function of μ , λ , and δ , the fiducial distribution of the CV can be obtained via simulation. A practical approach is to employ a Monte Carlo or Gibbs sampling scheme (Geman and Geman (1984)):

Step 1: Generate $\mu^{(i)} \sim IG(\hat{\mu}, n_{(1)}\hat{\lambda})$

Step 2: Generate $\lambda^{(i)} \sim (\hat{\lambda}/n_{(1)})\chi_{n_{(1)}-1}^2$

Step 3: Substitute $\mu^{(i)}$, $\lambda^{(i)}$, and δ into $CV^{(i)} = g(\mu^{(i)}, \lambda^{(i)}, \delta)$

Step 4: Repeat for $i = 1, 2, \dots, t$ to obtain the fiducial sample $\{CV^{(i)}\}$

In this study, the fiducial simulation was implemented using a Gibbs sampling scheme with a total of 20,000 iterations. To ensure stability of the simulated fiducial distribution, the first 5,000 iterations were discarded as burn-in. The selection of the burn-in length was determined through visual inspection of trace plots, which confirmed that the sampling chains achieved stable mixing behavior with no systematic trend or drift within the first 5,000 iterations (see Appendix C for details). The remaining 15,000 simulated values of $\{CV^{(i)}\}$ were then used to construct the fiducial confidence interval via the empirical quantiles.

The Gibbs sampling scheme was selected over alternative simulation approaches for the fiducial distribution, such as importance sampling or rejection sampling, for several practical reasons. First, the fiducial distributions of μ and λ derived from the sampling properties of their MLEs are available in closed, standard forms (inverse Gaussian and scaled chi-square, respectively), allowing each parameter to be generated directly without requiring an accept-reject step or numerical optimization. Second, because every generated value is used directly in constructing the fiducial sample $\{CV^{(i)}\}$, this approach makes full and efficient use of every iteration, in contrast to rejection-based methods that discard a portion of generated values. Third, the component-wise generation of $\mu^{(i)}$ and $\lambda^{(i)}$ is well suited to the multi-parameter structure of the delta-IG model, where the fiducial quantity for the CV is constructed as a function of several individually simulated components. Finally, trace plot diagnostics confirmed that the simulated fiducial values achieved stable behavior with no systematic trend within the first 5,000 iterations, indicating rapid and reliable convergence within a computationally feasible number of iterations. These properties made the Gibbs-based simulation scheme a practical and computationally efficient choice for generating the fiducial distribution of the CV in this study.

The $100(1 - \alpha)\%$ FCI for the coefficient of variation (θ) is obtained from the empirical quantiles of the simulated values:

$$CV_L = \text{quantile}_{\alpha/2}(CV^{(i)}), CV_U = \text{quantile}_{1-\alpha/2}(CV^{(i)}). \quad (2-18)$$

Thus, the fiducial confidence interval is given by

$$CI_{\theta}^{FCI} = [L_{FCI}, U_{FCI}] = (CV_L, CV_U). \quad (2-19)$$

2.2.5 Fiducial highest posterior density confidence interval (F-HPDCI)

The highest density interval (HDI), often referred to as the highest posterior density (HPD) interval in Bayesian analysis, is a widely used approach for constructing interval estimates for unknown parameters. In contrast to frequentist confidence intervals, which are based on repeated sampling behavior, HPD intervals are derived

from an underlying distribution of the parameter, such as the posterior distribution in Bayesian inference or the fiducial distribution in fiducial inference, thereby incorporating both prior information and observed data.

An HPD interval is defined as the shortest interval that contains a specified proportion of the distributional mass. It identifies the region where the parameter values have the highest posterior density. This property makes HPD intervals particularly advantageous when the posterior distribution is skewed, multimodal, or deviates from normality, where equal-tailed intervals may not adequately reflect uncertainty. Consequently, HPD intervals often provide a more representative summary of the posterior distribution. In practice, they can be computed in R using the *hdi* function from the *HPDinterval* package.

Accordingly, the $100(1 - \alpha)\%$ confidence interval for θ based on the F-HPDCI method is given by

$$CI_{\theta}^{F-HPD} = [L_{\theta}^{F-HPD}, U_{\theta}^{F-HPD}], \quad (2-20)$$

where L_{θ}^{F-HPD} and U_{θ}^{F-HPD} are obtained from the corresponding distribution of θ using appropriate computational methods such as the *hdi* function in R.

2.2.6 Bayesian credible interval (BCI)

A Bayesian credible interval provides a range of plausible values for an unknown parameter based on Bayesian inference. It is derived from the posterior distribution, which combines prior information with the likelihood obtained from observed data via Bayes' theorem. For the delta-IG distribution, the posterior is obtained by integrating the likelihood function with the chosen prior distributions. The choice of prior is critical, as it can substantially affect the resulting interval, particularly in small samples.

Let $X = (X_1, X_2, \dots, X_n)$ be a random sample from the delta-IG distribution with parameters μ , λ , and δ . The CV is defined as

$$CV = h(\mu, \lambda, \delta) = \theta = \sqrt{(\mu + \lambda\delta)/(1 - \delta)\lambda}$$

where $h(\cdot)$ is a known function depending on the parameterization of the delta-IG distribution.

To construct the posterior distribution, the likelihood function of the delta-IG distribution is combined with the prior distributions of the unknown parameters via Bayes' theorem. The likelihood function is given by

$$L(\theta|X) \propto \delta^{n_{(0)}} (1 - \delta)^{n_{(1)}} \lambda^{n_{(1)}/2} \exp\left(-\frac{\lambda}{2\mu^2} (t_1 - 2n_{(1)}\mu + t_2\mu^2)\right), \quad (2-21)$$

where $t_1 = \sum_{j=1}^{n_{(1)}} X_j$, and $t_2 = \sum_{j=1}^{n_{(1)}} X_j^{-1}$.

In this study, a uniform prior is assigned to μ , a gamma prior to λ , and a beta prior to δ . The posterior density is then proportional to the product of the prior and the likelihood, given by

$$f(\theta|X) \propto f(\theta)L(\theta|X). \quad (2-22)$$

These prior distributions are treated as the basis for posterior inference by combining them with the observed data through the likelihood function. Since $\theta = h(\mu, \lambda, \delta)$ is a function of μ , λ , and δ , the posterior distribution of the CV can be obtained via simulation. Because the resulting posterior distributions are analytically

intractable, a practical approach is to employ MCMC methods, specifically Gibbs sampling:

Step 1: Generate $\mu^{(i)}$ from its full conditional posterior distribution given the current values of $\lambda^{(i-1)}$ and $\delta^{(i-1)}$.

Step 2: Generate $\lambda^{(i)}$ from its full conditional posterior distribution given the current values of $\mu^{(i)}$ and $\delta^{(i-1)}$.

Step 3: Generate $\delta^{(i)}$ from its full conditional posterior distribution given the current values of $\mu^{(i)}$ and $\lambda^{(i)}$.

Step 4: Substitute $\mu^{(i)}$, $\lambda^{(i)}$, and $\delta^{(i)}$ into $\theta^{(i)} = h(\mu^{(i)}, \lambda^{(i)}, \delta^{(i)})$.

Step 5: Repeat for $i = 1, 2, \dots, t$ to obtain the posterior sample $\{\theta^{(i)}\}$.

In this study, the posterior simulation was implemented using a Gibbs sampling scheme via the *R2OpenBUGS* package in the R programming language, with a total of 20,000 iterations. To ensure stability of the simulated posterior distribution, the first 5,000 iterations were discarded as burn-in. The selection of the burn-in length was determined through visual inspection of trace plots, which confirmed that the sampling chains achieved stable mixing behavior with no systematic trend or drift within the first 5,000 iterations. The remaining 15,000 simulated values of $\hat{\theta}^{BS}$ were then used to construct the Bayesian credible interval via the empirical quantiles.

Based on the posterior samples, the estimator of the CV for the delta-IG distribution is defined as

$$\hat{\theta}^{BS} = \sqrt{(\hat{\mu}^{BS} + \hat{\lambda}^{BS} \hat{\delta}^{BS}) / (1 - \hat{\delta}^{BS}) \hat{\lambda}^{BS}}, \quad (2-23)$$

Finally, the $100(1-\alpha)\%$ two-sided CI of θ for the delta-IG distribution based on the BCI method is given by

$$CI_{\theta}^{BCI} = [\hat{\theta}^{BS}(\alpha/2), \hat{\theta}^{BS}(1 - \alpha/2)], \quad (2-24)$$

where $\hat{\theta}^{BS}(\alpha/2)$ and $\hat{\theta}^{BS}(1 - \alpha/2)$ are the $100\frac{\alpha}{2}$ -th and $100(1-\frac{\alpha}{2})$ -th percentiles of the distribution of $\hat{\theta}^{BS}$.

2.2.7 Bayesian highest posterior density credible interval (B-HPDCI)

The HPD method is used to construct credible intervals for an unknown parameter based on its posterior distribution such as equation (2-22). In practice, the highest posterior density confidence interval from Bayesian credible interval can be computed in R using the *hdi* function from the *HPDinterval* package.

Accordingly, the $100(1 - \alpha)\%$ credible interval for θ based on the B-HPDCI method is given by

$$CI_{\theta}^{B-HPD} = [L_{\theta}^{B-HPD}, U_{\theta}^{B-HPD}], \quad (2-25)$$

where L_{θ}^{B-HPD} and U_{θ}^{B-HPD} are obtained using the *hdi* function in R.

2.3 The confidence intervals for the difference and the ratio of the coefficients of variation of delta-IG distributions

This section extends the analysis to confidence intervals for the difference and ratio of two coefficients of variation, developed using the frameworks of AGCI, FCI, BCI, method of variance estimates recovery (MOVER), and normal approximation

(NA). Let $X_{ij} = (X_{i1}, X_{i2}, \dots, X_{in_i})$, for $i = 1, 2, j = 1, 2, \dots, n_i$ be a non-negative random sample from a delta-IG distribution stated as $X_{ij} \sim \text{delta-IG}(\delta_i, \mu_i, \lambda_i)$. The coefficients of variation of delta-IG distributions were

$$CV(X_{ij}) = \theta_i = \sqrt{(\mu_i + \lambda_i \delta_i)/(1 - \delta_i)\lambda_i}, i = 1, 2. \quad (2-26)$$

Therefore, the difference and ratio of the CVs are parameters that we will calculate for its interval, denoted as

$$\theta_{diff} = \theta_1 - \theta_2 = \sqrt{(\mu_1 + \lambda_1 \delta_1)/(1 - \delta_1)\lambda_1} - \sqrt{(\mu_2 + \lambda_2 \delta_2)/(1 - \delta_2)\lambda_2}, \quad (2-27)$$

and

$$\theta_{ratio} = \frac{\theta_1}{\theta_2} = \frac{\sqrt{(\mu_1 + \lambda_1 \delta_1)/(1 - \delta_1)\lambda_1}}{\sqrt{(\mu_2 + \lambda_2 \delta_2)/(1 - \delta_2)\lambda_2}}, \quad (2-28)$$

respectively.

The following section provides the methods for constructing confidence intervals for θ_{diff} and θ_{ratio} .

2.3.1 Adjusted generalized confidence interval (AGCI)

To construct confidence intervals for θ_{diff} and θ_{ratio} using the AGCI method, it is first necessary to define a generalized pivotal quantity (GPQ) for each parameter. The GPQ for the parameters λ_i and μ_i , $i = 1, 2$ for the delta-IG distribution can be expressed as

$$R_{\lambda_i} = \frac{n_{i(1)}\lambda_i V_i}{n_{i(1)}v_i} \sim \frac{\chi_{n_{i(1)}-1}^2}{n_{i(1)}v_i} \quad (2-29)$$

The GPQ for μ , as proposed by Krishnamoorthy and Tian, is defined as

$$R_{\tilde{\mu}_i} = \frac{\bar{X}_i}{\max\left\{0, 1 + t_{n_{i(1)}-1} \sqrt{\frac{\bar{X}_i v_i}{n_{i(1)}-1}}\right\}}, \quad (2-30)$$

where $t_{n_{i(1)}-1}$ is Student's t distribution with $n_{i(1)} - 1$ degrees of freedom. The GPQ for δ_i is defined as

$$R_{\delta_i} = \sin^2 \left[\arcsin \sqrt{\hat{\delta}_i} - \frac{T_i}{2\sqrt{n_i}} \right], \quad (2-31)$$

where $T_i = 2\sqrt{n_i}(\arcsin \sqrt{\hat{\delta}_i} - \arcsin \sqrt{\delta_i}) \sim N(0, 1)$, $n_i \rightarrow \infty$, $i = 1, 2$.

Therefore, from the equations (2-29), (2-30), and (2-31), the GPQ of the difference and the ratio of two CVs (θ_{diff} and θ_{ratio}) can be expressed as

$$R_{\tilde{\theta}_{diff}} = \sqrt{(R_{\tilde{\mu}_1} + R_{\lambda_1} R_{\delta_1})/(1 - R_{\delta_1})R_{\lambda_1}} - \sqrt{(R_{\tilde{\mu}_2} + R_{\lambda_2} R_{\delta_2})/(1 - R_{\delta_2})R_{\lambda_2}}, \quad (2-32)$$

and

$$R_{\tilde{\theta}_{ratio}} = \frac{\sqrt{(R_{\tilde{\mu}_1} + R_{\lambda_1} R_{\delta_1})/(1 - R_{\delta_1})R_{\lambda_1}}}{\sqrt{(R_{\tilde{\mu}_2} + R_{\lambda_2} R_{\delta_2})/(1 - R_{\delta_2})R_{\lambda_2}}}. \quad (2-33)$$

Consequently, the $100(1 - \alpha)\%$ two-sided confidence interval for θ_{diff} and θ_{ratio} is determined from the empirical percentiles of the GPQs from the AGCI method is expressed by

$$CI_{\theta_{diff}}^{AGCI} = [L_{\theta_{diff}}^{AGCI}, U_{\theta_{diff}}^{AGCI}] = [R_{\tilde{\theta}_{diff}}(\alpha/2), R_{\tilde{\theta}_{diff}}(1 - \alpha/2)], \quad (2-34)$$

and

$$CI_{\theta_{ratio}}^{AGCI} = [L_{\theta_{ratio}}^{AGCI}, U_{\theta_{ratio}}^{AGCI}] = [R_{\tilde{\theta}_{ratio}}(\alpha/2), R_{\tilde{\theta}_{ratio}}(1 - \alpha/2)], \quad (2-35)$$

where $R_{\tilde{\theta}_{diff}}(\alpha)$ and $R_{\tilde{\theta}_{ratio}}(\alpha)$ denote the $100(\alpha)^{th}$ percentiles of $R_{\tilde{\theta}_{diff}}$ and $R_{\tilde{\theta}_{ratio}}$, respectively.

2.3.2 Fiducial confidence interval (FCI)

Let $X_i = (X_{i1}, X_{i2}, \dots, X_{in_i}), i = 1, 2$ be a random sample from the delta-IG distribution with parameters μ_i, λ_i , and δ_i . The difference and ratio of two CVs is defined as

$$\theta_{diff} = p(\mu_i, \lambda_i, \delta_i) = \sqrt{(\mu_1 + \lambda_1 \delta_1)/(1 - \delta_1)\lambda_1} - \sqrt{(\mu_2 + \lambda_2 \delta_2)/(1 - \delta_2)\lambda_2}, \quad (2-36)$$

and

$$\theta_{ratio} = q(\mu_i, \lambda_i, \delta_i) = \frac{\sqrt{(\mu_1 + \lambda_1 \delta_1)/(1 - \delta_1)\lambda_1}}{\sqrt{(\mu_2 + \lambda_2 \delta_2)/(1 - \delta_2)\lambda_2}}, \quad (2-37)$$

where $p(\cdot)$ and $q(\cdot)$ are known function depending on the parameterization of the delta-IG distribution.

To construct the fiducial distribution, the sampling distributions of the maximum likelihood estimators (MLEs) are utilized. Let $\hat{\mu}_i$ and $\hat{\lambda}_i$ denote the MLEs of μ_i and λ_i , respectively. Based on the properties of the inverse Gaussian distribution, their sampling distributions can be approximated as: $\mu_i \sim IG(\hat{\mu}_i, n_{i(1)}\hat{\lambda}_i)$ and $\lambda_i \sim \frac{\hat{\lambda}_i}{n_{i(1)}} \chi_{n_{i(1)}-1}^2$.

These distributions are treated as fiducial distributions by replacing the unknown parameters with their corresponding MLEs.

Since the $p(\mu_i, \lambda_i, \delta_i)$ and $q(\mu_i, \lambda_i, \delta_i)$ are function of μ_i, λ_i , and δ_i , the fiducial distribution of the difference and ratio of two CVs can be obtained via simulation. A practical approach is to employ a Monte Carlo or Gibbs sampling scheme:

Step 1: Generate $\mu_i \sim IG(\hat{\mu}_i, n_{i(1)}\hat{\lambda}_i)$

Step 2: Generate $\lambda_i \sim \frac{\hat{\lambda}_i}{n_{i(1)}} \chi_{n_{i(1)}-1}^2$.

Step 3: Substitute $\mu^{(r)}, \lambda^{(r)}$, and δ into $\theta_{diff}^{(r)} = p(\mu_i^{(r)}, \lambda_i^{(r)}, \delta_i)$ and $\theta_{ratio}^{(r)} = q(\mu_i^{(r)}, \lambda_i^{(r)}, \delta_i)$.

Step 4: Repeat for $r = 1, 2, \dots, t$ to obtain the fiducial sample $\{\theta_{diff}^{(r)} \text{ and } \theta_{ratio}^{(r)}\}$.

The $100(1 - \alpha)\%$ FCI for the difference and ratio of two CVs (θ_{diff} and θ_{ratio}) are obtained from the empirical quantiles of the simulated values defined as

$$\theta_{diff_L} = \text{quantile}_{\alpha/2}(\theta_{diff}^{(r)}), \theta_{diff_U} = \text{quantile}_{1-\alpha/2}(\theta_{diff}^{(r)}). \quad (2-38)$$

and

$$\theta_{ratio_L} = \text{quantile}_{\alpha/2}(\theta_{ratio}^{(r)}), \theta_{ratio_U} = \text{quantile}_{1-\alpha/2}(\theta_{ratio}^{(r)}). \quad (2-39)$$

Thus, the fiducial confidence interval of θ_{diff} and θ_{ratio} are given by

$$CI_{\theta_{diff}}^{FCI} = [L_{FCI}, U_{FCI}] = (\theta_{diff_L}, \theta_{diff_U}). \quad (2-40)$$

and

$$CI_{\theta_{ratio}}^{FCI} = [L_{FCI}, U_{FCI}] = (\theta_{ratio_L}, \theta_{ratio_U}). \quad (2-41)$$

2.3.3 Bayesian credible interval (BCI)

Let $X_{ij} = (X_{i1}, X_{i2}, \dots, X_{ini})$, for $i = 1, 2, j = 1, 2, \dots, n_i$ be a non-negative random sample from a delta-IG distribution stated as $X_{ij} \sim \text{delta-IG}(\delta_i, \mu_i, \lambda_i)$. The likelihood function of the delta-IG distribution is given by

$$L(\theta|X_i) \propto \delta_i^{n_{i(0)}} (1 - \delta_i)^{n_{i(1)}} \lambda_i^{n_{i(1)}/2} \exp\left(-\frac{\lambda_i}{2\mu_i^2} (t_1 - 2n_{i(1)}\mu_i + t_2\mu_i^2)\right), \quad (2-42)$$

where $t_1 = \sum_{j=1}^{n_{i(1)}} X_{ij}$, and $t_2 = \sum_{j=1}^{n_{i(1)}} X_{ij}^{-1}$, $i = 1, 2$.

The posterior density is then proportional to the product of the prior and the likelihood, given by

$$f(\theta|X_i) \propto f(\theta)L(\theta|X_i). \quad (2-43)$$

Since the resulting posterior distributions are analytically intractable, Markov Chain Monte Carlo (MCMC) methods, specifically Gibbs sampling, are used to generate the posterior samples. In this study, a uniform prior is assigned to μ , a gamma prior to λ , and a beta prior to δ . The computation is implemented using the *R2OpenBUGS* package in the R programming language.

Based on the posterior samples, the estimator of the difference and ratio of two CVs (θ_{diff} and θ_{ratio}) for the delta-IG distribution are defined as

$$\hat{\theta}_{diff}^{BS} = \sqrt{(\hat{\mu}_1^{BS} + \hat{\lambda}_1^{BS} \hat{\delta}_1^{BS}) / (1 - \hat{\delta}_1^{BS}) \hat{\lambda}_1^{BS}} - \sqrt{(\hat{\mu}_2^{BS} + \hat{\lambda}_2^{BS} \hat{\delta}_2^{BS}) / (1 - \hat{\delta}_2^{BS}) \hat{\lambda}_2^{BS}}, \quad (2-44)$$

and

$$\hat{\theta}_{ratio}^{BS} = \frac{\sqrt{(\hat{\mu}_1^{BS} + \hat{\lambda}_1^{BS} \hat{\delta}_1^{BS}) / (1 - \hat{\delta}_1^{BS}) \hat{\lambda}_1^{BS}}}{\sqrt{(\hat{\mu}_2^{BS} + \hat{\lambda}_2^{BS} \hat{\delta}_2^{BS}) / (1 - \hat{\delta}_2^{BS}) \hat{\lambda}_2^{BS}}} \quad (2-45)$$

Therefore, the $100(1-\alpha)\%$ two-sided CI of θ_{diff} and θ_{ratio} for the delta-IG distribution based on the BCI method are given by

$$CI_{\theta_{diff}}^{BCI} = [\hat{\theta}_{diff}^{BS}(\alpha/2), \hat{\theta}_{diff}^{BS}(1 - \alpha/2)], \quad (2-46)$$

and

$$CI_{\theta_{ratio}}^{BCI} = [\hat{\theta}_{ratio}^{BS}(\alpha/2), \hat{\theta}_{ratio}^{BS}(1 - \alpha/2)], \quad (2-47)$$

where $\hat{\theta}_{diff}^{BS}(\alpha)$ and $\hat{\theta}_{ratio}^{BS}(\alpha)$ are the $100(\alpha)^{\text{th}}$ percentiles of the distribution of $\hat{\theta}_{diff}^{BS}$ and $\hat{\theta}_{ratio}^{BS}$, respectively.

2.3.4 Method of variance estimates recovery (MOVER)

The Method of Variance Estimates Recovery (MOVER), proposed by Zou and Donner (2009), is a flexible and efficient technique for constructing confidence intervals for a wide range of parameters, particularly the difference or ratio of two independent quantities. It is especially useful in situations where standard large-sample methods perform poorly, such as with small sample sizes or skewed distributions.

The key idea behind MOVER is to recover variance estimates from existing confidence limits. Specifically, given a parameter with lower and upper confidence bounds L and U , two variance estimates can be obtained: one based on the deviation between the point estimate and L , and the other based on the deviation from U . These recovered variances are then combined with the point estimates of individual

parameters to construct a confidence interval for a composite parameter (e.g., a difference or ratio). By allowing for asymmetry in the underlying distributions, MOVER often yields more accurate and reliable interval estimates.

The MOVER method provides the lower and upper confidence limits for the difference of two parameters or $\theta_{diff} = \theta_1 - \theta_2$, which are expressed as follows

$$L_{diff} = \hat{\theta}_1 - \hat{\theta}_2 - \sqrt{(\hat{\theta}_1 - l_1)^2 + (u_2 - \hat{\theta}_2)^2}, \quad (2-48)$$

and

$$U_{diff} = \hat{\theta}_1 - \hat{\theta}_2 + \sqrt{(u_1 - \hat{\theta}_1)^2 + (\hat{\theta}_2 - l_2)^2}, \quad (2-49)$$

where (l_1, u_1) and (l_2, u_2) are $100(1-\alpha)\%$ confidence limits for the parameters θ_1 and θ_2 .

The MOVER method provides the lower and upper confidence limits for the ratio of two parameters or $\theta_{ratio} = \theta_1/\theta_2$, which are expressed as follows

$$L_{ratio} = \frac{\hat{\theta}_1 \hat{\theta}_2 - \sqrt{(\hat{\theta}_1 \hat{\theta}_2)^2 - l_1 u_2 (2\hat{\theta}_1 - l_2)(2\hat{\theta}_2 - u_2)}}{u_2 (2\hat{\theta}_2 - u_2)}, \quad (2-50)$$

and

$$U_{ratio} = \frac{\hat{\theta}_1 \hat{\theta}_2 + \sqrt{(\hat{\theta}_1 \hat{\theta}_2)^2 - u_1 l_2 (2\hat{\theta}_1 - u_1)(2\hat{\theta}_2 - l_2)}}{l_2 (2\hat{\theta}_2 - l_2)}, \quad (2-51)$$

where (l_1, u_1) and (l_2, u_2) are $100(1-\alpha)\%$ confidence limits for the parameters θ_1 and θ_2 .

By utilizing CI computed from the AGCI method as shown in equations (2-34) and (2-35), we obtain the $100(1-\alpha)\%$ two-sided confidence intervals for θ_{diff} and θ_{ratio} as follows.

From the AGCI method, the GPQ for θ_{diff} can be defined as

$$R_{\tilde{\theta}_{diff}} = \sqrt{(R_{\tilde{\mu}_1} + R_{\lambda_1} R_{\delta_1}) / (1 - R_{\delta_1}) R_{\lambda_1}} - \sqrt{(R_{\tilde{\mu}_2} + R_{\lambda_2} R_{\delta_2}) / (1 - R_{\delta_2}) R_{\lambda_2}}, \quad (2-52)$$

and the GPQ for θ_{ratio} can be defined as

$$R_{\tilde{\theta}_{ratio}} = \frac{\sqrt{(R_{\tilde{\mu}_1} + R_{\lambda_1} R_{\delta_1}) / (1 - R_{\delta_1}) R_{\lambda_1}}}{\sqrt{(R_{\tilde{\mu}_2} + R_{\lambda_2} R_{\delta_2}) / (1 - R_{\delta_2}) R_{\lambda_2}}}. \quad (2-53)$$

The $100(1-\alpha)\%$ two-sided CI for θ_{diff} using the AGCI method is

$$CI_{\theta_{diff}}^{AGCI} = [L_{\theta_{diff}}^{AGCI}, U_{\theta_{diff}}^{AGCI}] = [R_{\tilde{\theta}_{diff}}(\alpha/2), R_{\tilde{\theta}_{diff}}(1 - \alpha/2)], \quad (2-54)$$

And the $100(1-\alpha)\%$ two-sided CI for θ_{ratio} using the AGCI method is

$$CI_{\theta_{ratio}}^{AGCI} = [L_{\theta_{ratio}}^{AGCI}, U_{\theta_{ratio}}^{AGCI}] = [R_{\tilde{\theta}_{ratio}}(\alpha/2), R_{\tilde{\theta}_{ratio}}(1 - \alpha/2)], \quad (2-55)$$

Therefore, the $100(1-\alpha)\%$ two-sided confidence intervals for θ_{diff} and θ_{ratio} based on the MOVER are

$$CI_{\theta_{diff}}^{MOVER} = [L_{\theta}^{MOVER}, U_{\theta}^{MOVER}] = [\hat{\theta}_1 - \hat{\theta}_2 - \sqrt{(\hat{\theta}_1 - l_1)^2 + (u_2 - \hat{\theta}_2)^2}, \hat{\theta}_1 - \hat{\theta}_2 + \sqrt{(u_1 - \hat{\theta}_1)^2 + (\hat{\theta}_2 - l_2)^2}], \quad (2-56)$$

and

$$\begin{aligned}
CI_{\theta_{ratio}}^{MOVER} &= [L_{\theta}^{MOVER}, U_{\theta}^{MOVER}] \\
&= \left[\frac{\hat{\theta}_1 \hat{\theta}_2 - \sqrt{(\hat{\theta}_1 \hat{\theta}_2)^2 - l_1 u_2 (2\hat{\theta}_1 - l_2)(2\hat{\theta}_2 - u_2)}}{u_2 (2\hat{\theta}_2 - u_2)}, \frac{\hat{\theta}_1 \hat{\theta}_2 + \sqrt{(\hat{\theta}_1 \hat{\theta}_2)^2 - u_1 l_2 (2\hat{\theta}_1 - u_1)(2\hat{\theta}_2 - l_2)}}{l_2 (2\hat{\theta}_2 - l_2)} \right].
\end{aligned} \tag{2-57}$$

where (l_1, u_1) and (l_2, u_2) are $100(1-\alpha)\%$ confidence limits computed from the AGCI method of θ_1 and θ_2 .

2.3.5 Normal approximation (NA)

In the normal approximation approach, the Delta method is applied to obtain the asymptotic normal distribution of the estimator θ . This method is grounded in large-sample theory and is commonly used as a benchmark due to its analytical simplicity and closed-form expression. The procedure can be outlined as follows:

Suppose that $f(w_1, w_2, w_3)$ is a differentiable scalar function of three variables. Let $F = f(w_1, w_2, w_3)$ be an estimator that depends on the parameters W_1, W_2 , and W_3 . By applying the Delta method, the asymptotic distribution of F can be derived through a stochastic approximation. In particular, the Delta method is used to establish the asymptotic normality of F as the sample size increases, as well as to obtain approximations for its mean and variance. Specifically, the function f is expanded using a first-order Taylor series around the points ϕ_1, ϕ_2, ϕ_3 , corresponding to W_1, W_2 , and W_3 , respectively, as follows:

$$\begin{aligned}
f(w_1, w_2, w_3) &\approx f(\phi_1, \phi_2, \phi_3) + \frac{\partial f(\phi_1, \phi_2, \phi_3)}{\partial w_1} (W_1 - \phi_1) + \frac{\partial f(\phi_1, \phi_2, \phi_3)}{\partial w_2} (W_2 - \phi_2) + \\
&\quad \frac{\partial f(\phi_1, \phi_2, \phi_3)}{\partial w_3} (W_3 - \phi_3).
\end{aligned} \tag{2-58}$$

The fundamental statistics are defined as $W_1 = \hat{\mu}$, $W_2 = \hat{\lambda}$, and $W_3 = \hat{\delta}$, where $\hat{\mu}$, $\hat{\lambda}$, and $\hat{\delta}$ denote the MLEs of μ , λ , and δ , respectively. Accordingly, the estimator of interest is expressed as $\hat{\theta} = f(\hat{\mu}, \hat{\lambda}, \hat{\delta})$, where the function $f(\mu, \lambda, \delta)$ is defined by

$$f(\mu, \lambda, \delta) = \sqrt{(\mu + \lambda\delta)/(1 - \delta)\lambda}.$$

The corresponding partial derivatives are given as follows:

$$\begin{aligned}
\frac{\partial f(\mu, \lambda, \delta)}{\partial \mu} &= \frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu + \lambda\delta} \right)^{\frac{1}{2}} \left(\frac{1}{(1-\delta)\lambda} \right), \\
\frac{\partial f(\mu, \lambda, \delta)}{\partial \lambda} &= \frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu + \lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1-\delta)\lambda\delta - (\mu + \lambda\delta)(1-\delta)}{((1-\delta)\lambda)^2} \right), \\
\frac{\partial f(\mu, \lambda, \delta)}{\partial \delta} &= \frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu + \lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1-\delta)\lambda^2 - (\mu + \lambda\delta)(-\lambda)}{((1-\delta)\lambda)^2} \right).
\end{aligned}$$

Therefore,

$$\begin{aligned}
 \hat{\theta} &\approx f(\mu, \lambda, \delta) + \frac{\partial f(\mu, \lambda, \delta)}{\partial \mu} (\hat{\mu} - \mu) + \frac{\partial f(\mu, \lambda, \delta)}{\partial \lambda} (\hat{\lambda} - \lambda) + \frac{\partial f(\mu, \lambda, \delta)}{\partial \delta} (\hat{\delta} - \delta) \\
 &= f(\mu, \lambda, \delta) + \left[\frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{1}{(1-\delta)\lambda} \right) \right] (\hat{\mu} - \mu) \\
 &\quad + \left[\frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1-\delta)\lambda\delta - (\mu+\lambda\delta)(1-\delta)}{((1-\delta)\lambda)^2} \right) \right] (\hat{\lambda} - \lambda) \\
 &\quad + \left[\frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1-\delta)\lambda^2 - (\mu+\lambda\delta)(-\lambda)}{((1-\delta)\lambda)^2} \right) \right] (\hat{\delta} - \delta).
 \end{aligned} \tag{2-59}$$

Taking the expectation and variance in Equation (2-59), the approximate mean and variance for the $\hat{\theta}$ is given by

$$E(\hat{\theta}) = \theta, \tag{2-60}$$

$$\begin{aligned}
 Var(\hat{\theta}) &\approx Var\left\{ f(\mu, \lambda, \delta) + \left[\frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{1}{(1-\delta)\lambda} \right) \right] (\hat{\mu} - \mu) \right. \\
 &\quad + \left[\frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1-\delta)\lambda\delta - (\mu+\lambda\delta)(1-\delta)}{((1-\delta)\lambda)^2} \right) \right] (\hat{\lambda} - \lambda) \\
 &\quad + \left. \left[\frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1-\delta)\lambda^2 - (\mu+\lambda\delta)(-\lambda)}{((1-\delta)\lambda)^2} \right) \right] (\hat{\delta} - \delta) \right\} \\
 &= \left[\frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{1}{(1-\delta)\lambda} \right) \right]^2 \left(\frac{\mu^3}{\lambda n_{i(1)}} \right) \\
 &\quad + \left[\frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1-\delta)\lambda\delta - (\mu+\lambda\delta)(1-\delta)}{((1-\delta)\lambda)^2} \right) \right]^2 \left(\frac{2\lambda^2}{n_{i(1)}} \right) \\
 &\quad + \left[\frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1-\delta)\lambda^2 - (\mu+\lambda\delta)(-\lambda)}{((1-\delta)\lambda)^2} \right) \right]^2 \left(\frac{\delta(1-\delta)}{n_i} \right).
 \end{aligned} \tag{2-61}$$

Thus, as $n_i \rightarrow \infty$, $\hat{\theta}$ is nearly normally distributed as $N(E(\hat{\theta}), Var(\hat{\theta}))$. In practice, the true values of μ , λ , and δ are unknown when estimating the parameter function θ from sample data. Therefore, plug-in estimators are used to approximate $E(\hat{\theta})$ and $Var(\hat{\theta})$, as given below.

$$\hat{E}(\hat{\theta}_i) = \sqrt{(\hat{\mu}_i + \hat{\lambda}_i \hat{\delta}_i) / (1 - \hat{\delta}_i) \hat{\lambda}_i}, \tag{2-62}$$

$$\begin{aligned}
 \hat{Var}(\hat{\theta}_i) &= \left[\frac{1}{2} \left(\frac{(1-\hat{\delta}_i)\hat{\lambda}_i}{\hat{\mu}_i + \hat{\lambda}_i \hat{\delta}_i} \right)^{\frac{1}{2}} \left(\frac{1}{(1-\hat{\delta}_i)\hat{\lambda}_i} \right) \right]^2 \left(\frac{\hat{\mu}_i^3}{\hat{\lambda}_i n_{i(1)}} \right) \\
 &\quad + \left[\frac{1}{2} \left(\frac{(1-\hat{\delta}_i)\hat{\lambda}_i}{\hat{\mu}_i + \hat{\lambda}_i \hat{\delta}_i} \right)^{\frac{1}{2}} \left(\frac{(1-\hat{\delta}_i)\hat{\lambda}_i \hat{\delta}_i - (\hat{\mu}_i + \hat{\lambda}_i \hat{\delta}_i)(1-\hat{\delta}_i)}{((1-\hat{\delta}_i)\hat{\lambda}_i)^2} \right) \right]^2 \left(\frac{2\hat{\lambda}_i^2}{n_{i(1)}} \right) \\
 &\quad + \left[\frac{1}{2} \left(\frac{(1-\hat{\delta}_i)\hat{\lambda}_i}{\hat{\mu}_i + \hat{\lambda}_i \hat{\delta}_i} \right)^{\frac{1}{2}} \left(\frac{(1-\hat{\delta}_i)\hat{\lambda}_i^2 - (\hat{\mu}_i + \hat{\lambda}_i \hat{\delta}_i)(-\hat{\lambda}_i)}{((1-\hat{\delta}_i)\hat{\lambda}_i)^2} \right) \right]^2 \left(\frac{\hat{\delta}_i(1-\hat{\delta}_i)}{n_i} \right).
 \end{aligned} \tag{2-63}$$

The estimated variance of the estimated difference between two coefficients of variation of delta-IG can be expressed as follows:

$$V\hat{ar}(\hat{\theta}_{diff}) = V\hat{ar}(\hat{\theta}_1) + V\hat{ar}(\hat{\theta}_2), \quad (2-64)$$

where $\text{cov}(\hat{\theta}_1, \hat{\theta}_2) = 0$.

The estimated variance of the estimated ratio between two coefficients of variation of delta-IG can be determined as follows:

Let

$$\theta_{ratio} = \frac{\theta_1}{\theta_2} = f(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2) = \sqrt{\frac{(\mu_1 + \lambda_1 \delta_1)}{(1 - \delta_1)\lambda_1}} / \sqrt{\frac{(\mu_2 + \lambda_2 \delta_2)}{(1 - \delta_2)\lambda_2}}. \quad (2-65)$$

Consider the estimator of the parameter θ_{ratio} , which is equal to

$$\hat{\theta}_{ratio} = \frac{\hat{\theta}_1}{\hat{\theta}_2} = f(\hat{\mu}_1, \hat{\lambda}_1, \hat{\delta}_1, \hat{\mu}_2, \hat{\lambda}_2, \hat{\delta}_2) = \sqrt{\frac{(\hat{\mu}_1 + \hat{\lambda}_1 \hat{\delta}_1)}{(1 - \hat{\delta}_1)\hat{\lambda}_1}} / \sqrt{\frac{(\hat{\mu}_2 + \hat{\lambda}_2 \hat{\delta}_2)}{(1 - \hat{\delta}_2)\hat{\lambda}_2}}. \quad (2-66)$$

Using the Delta method, the first-order Taylor series expansion can be written as follows:

$$\begin{aligned} f(\hat{\mu}_1, \hat{\lambda}_1, \hat{\delta}_1, \hat{\mu}_2, \hat{\lambda}_2, \hat{\delta}_2) &= f(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2) + \frac{\partial f(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \mu_1} (\hat{\mu}_1 - \mu_1) \\ &+ \frac{\partial f(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \lambda_1} (\hat{\lambda}_1 - \lambda_1) + \frac{\partial f(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \delta_1} (\hat{\delta}_1 - \delta_1) \\ &+ \frac{\partial f(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \mu_2} (\hat{\mu}_2 - \mu_2) + \frac{\partial f(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \lambda_2} (\hat{\lambda}_2 - \lambda_2) \\ &+ \frac{\partial f(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \delta_2} (\hat{\delta}_2 - \delta_2) + \text{Reminder}. \end{aligned} \quad (2-67)$$

The partial derivatives are as follows:

$$\begin{aligned} \frac{\partial f(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \mu_1} &= \frac{\sqrt{\frac{(\mu_2 + \lambda_2 \delta_2)}{(1 - \delta_2)\lambda_2}} \left(\frac{1}{2} \frac{(1 - \delta_1)\lambda_1}{\mu_1 + \lambda_1 \delta_1} \right)^{\frac{1}{2}} \left(\frac{1}{(1 - \delta_1)\lambda_1} \right)}{\frac{(\mu_2 + \lambda_2 \delta_2)}{(1 - \delta_2)\lambda_2}} = F_{\mu_1}. \\ \frac{\partial f(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \lambda_1} &= \frac{\sqrt{\frac{(\mu_2 + \lambda_2 \delta_2)}{(1 - \delta_2)\lambda_2}} \left(\frac{1}{2} \frac{(1 - \delta_1)\lambda_1}{\mu_1 + \lambda_1 \delta_1} \right)^{\frac{1}{2}} \left(\frac{(1 - \delta_1)\lambda_1 \delta_1 - (\mu_1 + \lambda_1 \delta_1)(1 - \delta_1)}{((1 - \delta_1)\lambda_1)^2} \right)}{\frac{(\mu_2 + \lambda_2 \delta_2)}{(1 - \delta_2)\lambda_2}} = F_{\lambda_1}. \\ \frac{\partial f(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \delta_1} &= \frac{\sqrt{\frac{(\mu_2 + \lambda_2 \delta_2)}{(1 - \delta_2)\lambda_2}} \left(\frac{1}{2} \frac{(1 - \delta_1)\lambda_1}{\mu_1 + \lambda_1 \delta_1} \right)^{\frac{1}{2}} \left(\frac{(1 - \delta_1)\lambda_1^2 - (\mu_1 + \lambda_1 \delta_1)(-\lambda_1)}{((1 - \delta_1)\lambda_1)^2} \right)}{\frac{(\mu_2 + \lambda_2 \delta_2)}{(1 - \delta_2)\lambda_2}} = F_{\delta_1}. \\ \frac{\partial f(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \mu_2} &= \frac{-\sqrt{\frac{(\mu_1 + \lambda_1 \delta_1)}{(1 - \delta_1)\lambda_1}} \left(\frac{1}{2} \frac{(1 - \delta_2)\lambda_2}{\mu_2 + \lambda_2 \delta_2} \right)^{\frac{1}{2}} \left(\frac{1}{(1 - \delta_2)\lambda_2} \right)}{\frac{(\mu_2 + \lambda_2 \delta_2)}{(1 - \delta_2)\lambda_2}} = F_{\mu_2}. \\ \frac{\partial f(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \lambda_2} &= \frac{-\sqrt{\frac{(\mu_1 + \lambda_1 \delta_1)}{(1 - \delta_1)\lambda_1}} \left[\frac{1}{2} \frac{(1 - \delta_2)\lambda_2}{\mu_2 + \lambda_2 \delta_2} \right]^{\frac{1}{2}} \left(\frac{(1 - \delta_2)\lambda_2 \delta_2 - (\mu_2 + \lambda_2 \delta_2)(1 - \delta_2)}{((1 - \delta_2)\lambda_2)^2} \right)}{\frac{(\mu_2 + \lambda_2 \delta_2)}{(1 - \delta_2)\lambda_2}} = F_{\lambda_2}. \\ \frac{\partial f(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \delta_2} &= \frac{-\sqrt{\frac{(\mu_1 + \lambda_1 \delta_1)}{(1 - \delta_1)\lambda_1}} \left[\frac{1}{2} \frac{(1 - \delta_2)\lambda_2}{\mu_2 + \lambda_2 \delta_2} \right]^{\frac{1}{2}} \left(\frac{(1 - \delta_2)\lambda_2^2 - (\mu_2 + \lambda_2 \delta_2)(-\lambda_2)}{((1 - \delta_2)\lambda_2)^2} \right)}{\frac{(\mu_2 + \lambda_2 \delta_2)}{(1 - \delta_2)\lambda_2}} = F_{\delta_2}. \end{aligned}$$

Therefore,

$$\begin{aligned}
 \hat{\theta}_{ratio} &\approx f(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2) + \frac{\partial f(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \mu_1} (\hat{\mu}_1 - \mu_1) \\
 &+ \frac{\partial f(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \lambda_1} (\hat{\lambda}_1 - \lambda_1) + \frac{\partial f(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \delta_1} (\hat{\delta}_1 - \delta_1) \\
 &+ \frac{\partial f(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \mu_2} (\hat{\mu}_2 - \mu_2) + \frac{\partial f(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \lambda_2} (\hat{\lambda}_2 - \lambda_2) \\
 &+ \frac{\partial f(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \delta_2} (\hat{\delta}_2 - \delta_2) \\
 &= f(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2) + F_{\mu_1} (\hat{\mu}_1 - \mu_1) + F_{\lambda_1} (\hat{\lambda}_1 - \lambda_1) \\
 &+ F_{\delta_1} (\hat{\delta}_1 - \delta_1) + F_{\mu_2} (\hat{\mu}_2 - \mu_2) + F_{\lambda_2} (\hat{\lambda}_2 - \lambda_2) + F_{\delta_2} (\hat{\delta}_2 - \delta_2).
 \end{aligned} \quad (2-68)$$

Then, taking the expectation and variance in the equation, the approximate mean and variance for the $\hat{\theta}_{ratio}$ is given by

$$E(\hat{\theta}_{ratio}) = \hat{\theta}_{ratio} \quad (2-69)$$

$$\begin{aligned}
 Var(\hat{\theta}_{ratio}) &= Var(f(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2) + F_{\mu_1} (\hat{\mu}_1 - \mu_1) + F_{\lambda_1} (\hat{\lambda}_1 - \lambda_1) \\
 &+ F_{\delta_1} (\hat{\delta}_1 - \delta_1) + F_{\mu_2} (\hat{\mu}_2 - \mu_2) + F_{\lambda_2} (\hat{\lambda}_2 - \lambda_2) + F_{\delta_2} (\hat{\delta}_2 - \delta_2)) \\
 &= F_{\mu_1}^2 Var(\hat{\mu}_1 - \mu_1) + F_{\lambda_1}^2 Var(\hat{\lambda}_1 - \lambda_1) + F_{\delta_1}^2 Var(\hat{\delta}_1 - \delta_1) \\
 &+ F_{\mu_2}^2 Var(\hat{\mu}_2 - \mu_2) + F_{\lambda_2}^2 Var(\hat{\lambda}_2 - \lambda_2) + F_{\delta_2}^2 Var(\hat{\delta}_2 - \delta_2) \\
 &= F_{\mu_1}^2 \left(\frac{\mu_1^3}{\lambda_1 n_{1(1)}} \right) + F_{\lambda_1}^2 \left(\frac{2\lambda_1^2}{n_{1(1)}} \right) + F_{\delta_1}^2 \left(\frac{\delta_1(1-\delta_1)}{n_1} \right) \\
 &+ F_{\mu_2}^2 \left(\frac{\mu_2^3}{\lambda_2 n_{2(1)}} \right) + F_{\lambda_2}^2 \left(\frac{2\lambda_2^2}{n_{2(1)}} \right) + F_{\delta_2}^2 \left(\frac{\delta_2(1-\delta_2)}{n_2} \right).
 \end{aligned} \quad (2-70)$$

The estimate of $Var(\hat{\theta}_{ratio})$ can be written as follows:

$$\begin{aligned}
 \hat{Var}(\hat{\theta}_{ratio}) &= F_{\mu_1}^2 \left(\frac{\hat{\mu}_1^3}{\hat{\lambda}_1 n_{1(1)}} \right) + F_{\lambda_1}^2 \left(\frac{2\hat{\lambda}_1^2}{n_{1(1)}} \right) + F_{\delta_1}^2 \left(\frac{\hat{\delta}_1(1-\hat{\delta}_1)}{n_1} \right) \\
 &+ F_{\mu_2}^2 \left(\frac{\hat{\mu}_2^3}{\hat{\lambda}_2 n_{2(1)}} \right) + F_{\lambda_2}^2 \left(\frac{2\hat{\lambda}_2^2}{n_{2(1)}} \right) + F_{\delta_2}^2 \left(\frac{\hat{\delta}_2(1-\hat{\delta}_2)}{n_2} \right).
 \end{aligned} \quad (2-71)$$

The $100(1 - \alpha)\%$ confidence interval for θ_{diff} using normal approximation is

$$CI_{\theta_{diff}}^{NA} = (\hat{\theta}_1 - \hat{\theta}_2) \pm z_{1-\frac{\alpha}{2}} \sqrt{\hat{Var}(\hat{\theta}_{diff})}, \quad (2-72)$$

where $z_{1-\frac{\alpha}{2}}$ is the $(1 - \frac{\alpha}{2})$ th percentile of the standard normal distribution.

And the $100(1 - \alpha)\%$ confidence interval for θ_{ratio} using normal approximation is

$$CI_{\theta_{ratio}}^{NA} = (\hat{\theta}_1 / \hat{\theta}_2) \pm z_{1-\frac{\alpha}{2}} \sqrt{\hat{Var}(\hat{\theta}_{ratio})}, \quad (2-73)$$

where $z_{1-\frac{\alpha}{2}}$ is the $(1 - \frac{\alpha}{2})$ th percentile of the standard normal distribution.

2.4 Simulation studies and results

This study assesses the performance of several methods for constructing confidence intervals for the coefficient of variation (CV), as well as for the difference and ratio of CVs under the delta-inverse Gaussian (delta-IG) distribution. For a single CV, the methods considered include GCI, AGCI, PBPCI, FCI, F-HPDCI, BCI, and B-HPDCI. For the difference and ratio of two CVs, the methods AGCI, FCI, BCI,

MOVER, and NA are employed. A Monte Carlo simulation study, implemented in R, is conducted to evaluate these methods in terms of coverage probabilities (CPs) and average widths (AWs). An effective confidence interval method is expected to achieve coverage probabilities close to the nominal confidence level. Among methods with comparable coverage performance, the preferred method is the one that produces the shortest interval width. The simulation is based on 5,000 replications. For the single CV, samples are generated from a delta-IG distribution, $X \sim \text{delta-IG}(\mu, \lambda, \delta)$, with sample sizes $n = 30, 50, 100$, and 200 . The mean μ is set to 0.5 or 1.0 , the shape parameter λ to $1, 5$, or 10 , and the zero-inflation probability δ to 0.3 or 0.5 .

For the difference and ratio of two CVs, data are generated from two independent delta-IG populations, $X_{ij} \sim \text{delta-IG}(\mu_i, \lambda_i, \delta_i)$, for $i = 1, 2$ and $j = 1, \dots, n_i$. The sample size combinations (n_1, n_2) include $(30, 30)$, $(50, 50)$, $(100, 100)$, $(200, 200)$, $(30, 50)$, $(50, 100)$, and $(100, 200)$. The mean parameters (μ_1, μ_2) are set to $(0.5, 0.5)$, $(0.5, 1.0)$, and $(1.0, 1.0)$, while the shape parameters (λ_1, λ_2) are $(1, 1)$, $(1, 5)$, $(5, 5)$, $(5, 10)$, and $(10, 10)$. The zero-inflation probabilities (δ_1, δ_2) are specified as $(0.3, 0.3)$, $(0.3, 0.5)$, and $(0.5, 0.5)$.

The performance of the competing methods for constructing confidence intervals of single coefficient of variation was evaluated based on coverage probabilities (CPs) and average widths (AWs), as summarized in Tables 2-1 and 2-2 and illustrated in Figure 2-1. Overall, AGCI, FCI, and F-HPDCI consistently achieved coverage probabilities close to the nominal level of 0.95 across most scenarios. Among these, AGCI showed superior performance in small-sample settings, maintaining stable coverage where other methods tended to deviate. As the sample size increased, FCI and F-HPDCI exhibited improved performance and generally outperformed the remaining approaches in terms of CPs. In terms of interval length, PBPCI produced the narrowest intervals, followed by AGCI and F-HPDCI. However, the superior compactness of PBPCI was often accompanied by undercoverage, indicating a trade-off between precision and reliability. The Bayesian approaches (BCI and B-HPDCI) yielded wider intervals compared to other methods. Between them, BCI provided coverage probabilities closer to the nominal level than B-HPDCI. When both criteria are considered jointly, AGCI and F-HPDCI emerged as the most reliable methods, offering a favorable balance between adequate coverage and reasonable interval width. FCI ranked next, followed by BCI, B-HPDCI, GCI, and PBPCI. From a computational perspective, AGCI required less computation time than F-HPDCI. However, F-HPDCI was simpler to implement due to the availability of built-in functions for highest density interval estimation in R.

The simulation results for the difference and ratio of two coefficients of variation (CVs) are summarized below. The results for the difference of two coefficients of variation were in Table 2-3, Table 2-4 and Figure 2-2. And the results for the ratio of two coefficients of variation were in Table 2-5, Table 2-6, and Figures 2-3. Overall, most methods achieved coverage probabilities (CPs) close to or exceeding the nominal level of 0.95 , indicating satisfactory reliability across a wide range of settings. Among the competing approaches, AGCI and MOVER consistently demonstrated strong performance in balancing coverage accuracy and interval efficiency. In terms of average

widths (AWs), AGCI and MOVER generally produced the narrowest intervals. Their relative performance, however, depended on the underlying parameter configurations. When the zero-inflation probabilities were equal ($\delta_1 = \delta_2$), MOVER typically yielded shorter intervals than AGCI. In contrast, when the probabilities differed ($\delta_1 \neq \delta_2$), AGCI tended to outperform MOVER in terms of interval width. As sample sizes increased, both AGCI and MOVER maintained stable coverage, with MOVER often achieving the smallest interval widths in larger balanced samples. For unbalanced sample size configurations, MOVER showed clear advantages, providing a more favorable trade-off between CPs and AWs compared to other methods. Taken together, AGCI and MOVER emerged as the most reliable approaches across the scenarios considered. While AGCI performed competitively across a broad range of conditions, MOVER showed particular strength in larger and unbalanced sample settings, making it a robust choice in practical applications.

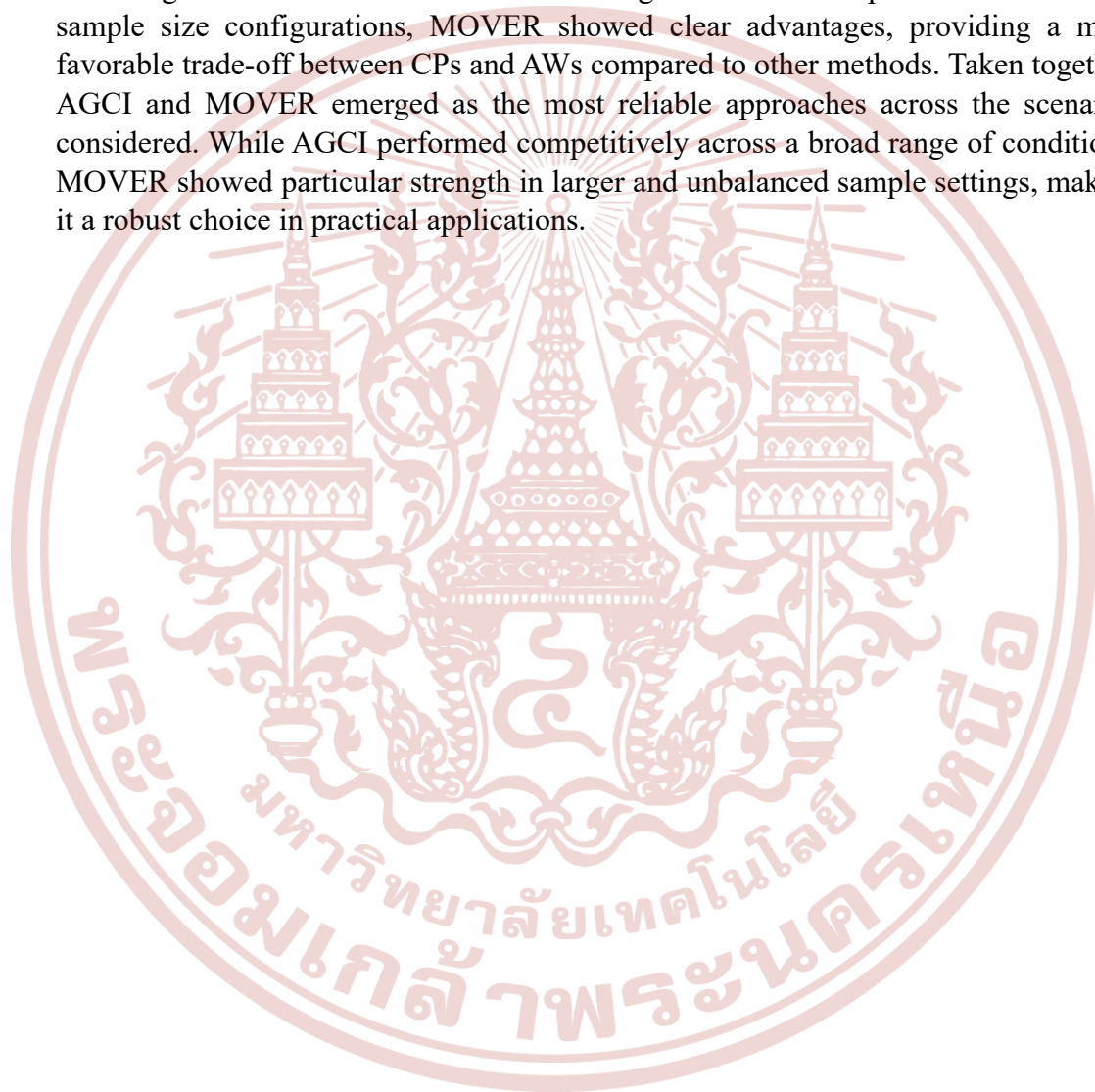


TABLE 2-1 The CPs of the 95% confidence intervals for the coefficient of variation of a delta-IG distribution.

n	μ	λ	δ	Coverage Probabilities						
				GCI	AGCI	PBPCI	FCI	F-HPDCI	BCI	B-HPDCI
30	0.5	1	0.3	0.9760	0.9440	0.8352	0.9630	0.9450	0.9480	0.9510
			0.5	0.9780	0.9580	0.7934	0.9640	0.9530	0.9360	0.9270
			5	0.3	0.9560	0.9480	0.8392	0.9520	0.9430	0.9500
				0.5	0.9640	0.9600	0.8052	0.9590	0.9510	0.9460
			10	0.3	0.9540	0.9490	0.8256	0.9530	0.9460	0.9520
				0.5	0.9700	0.9660	0.8048	0.9620	0.9550	0.9610
		1	1	0.3	0.9860	0.9540	0.8092	0.9540	0.9380	0.9420
				0.5	0.9890	0.9590	0.7754	0.9610	0.9420	0.9540
			5	0.3	0.9650	0.9480	0.8362	0.9570	0.9390	0.9470
				0.5	0.9710	0.9610	0.8012	0.9610	0.9610	0.9460
			10	0.3	0.9560	0.9480	0.8392	0.9520	0.9430	0.9500
				0.5	0.9640	0.9600	0.8052	0.9590	0.9510	0.9460
50	0.5	1	0.3	0.9822	0.9454	0.8712	0.9540	0.9484	0.9670	0.9620
			0.5	0.9792	0.9466	0.8422	0.9528	0.9414	0.9560	0.9370
			5	0.3	0.9570	0.9472	0.8796	0.9488	0.9480	0.9600
				0.5	0.9592	0.9470	0.8464	0.9488	0.9444	0.9560
			10	0.3	0.9540	0.9484	0.8794	0.9492	0.9494	0.9590
				0.5	0.9544	0.9478	0.8476	0.9482	0.9456	0.9650
		1	1	0.3	0.9940	0.9462	0.8628	0.9526	0.9398	0.9410
				0.5	0.9906	0.9462	0.8392	0.9542	0.9364	0.9570
			5	0.3	0.9654	0.9482	0.8758	0.9506	0.9486	0.9580
				0.5	0.9670	0.9474	0.8456	0.9514	0.9438	0.9610
			10	0.3	0.9590	0.9468	0.7724	0.9530	0.9450	0.9650
				0.5	0.9592	0.9470	0.8464	0.9488	0.9444	0.9210
100	0.5	1	0.3	0.9828	0.9514	0.9086	0.9538	0.9500	0.9570	0.9480
			0.5	0.9846	0.9554	0.9024	0.9590	0.9518	0.9440	0.9280
			5	0.3	0.9632	0.9522	0.9132	0.9504	0.9496	0.9610
				0.5	0.9656	0.9540	0.9008	0.9556	0.9508	0.9560
			10	0.3	0.9580	0.9536	0.9166	0.9514	0.9500	0.9570
				0.5	0.9590	0.9538	0.8980	0.9540	0.9508	0.9590
		1	1	0.3	0.9912	0.9534	0.9042	0.9546	0.9490	0.9420
				0.5	0.9926	0.9550	0.8948	0.9538	0.9450	0.9410
			5	0.3	0.9692	0.9530	0.9120	0.9500	0.9480	0.9590
				0.5	0.9724	0.9540	0.8998	0.9546	0.9500	0.9560
			10	0.3	0.9632	0.9522	0.9132	0.9504	0.9496	0.9390
				0.5	0.9656	0.9540	0.9008	0.9556	0.9508	0.9410
200	0.5	1	0.3	0.9814	0.9476	0.9302	0.9536	0.9518	0.9620	0.9380
			0.5	0.9826	0.9528	0.9246	0.9536	0.9494	0.9440	0.9330
			5	0.3	0.9542	0.9464	0.9350	0.9510	0.9502	0.9610
				0.5	0.9614	0.9516	0.9278	0.9502	0.9496	0.9450
			10	0.3	0.9506	0.9466	0.9356	0.9482	0.9500	0.9410
				0.5	0.9560	0.9516	0.9256	0.9498	0.9488	0.9590
		1	1	0.3	0.9942	0.9470	0.9272	0.9532	0.9476	0.9560
				0.5	0.9934	0.9538	0.9214	0.9516	0.9478	0.9620
			5	0.3	0.9630	0.9464	0.9336	0.9518	0.9504	0.9540

TABLE 2-1 (CONTINUED)

n	μ	λ	δ	Coverage Probabilities						
				GCI	AGCI	PBPCI	FCI	F-HPDCI	BCI	B-HPDCI
200	1	5	0.5	0.9690	0.9530	0.9284	0.9530	0.9490	0.9550	0.9420
		10	0.3	0.9542	0.9464	0.9350	0.9510	0.9502	0.9430	0.9360
			0.5	0.9614	0.9516	0.9278	0.9502	0.9496	0.9410	0.9300

Note: Bold values indicate coverage probabilities (CP) that satisfy the nominal confidence level of 0.95 ($CP \geq 0.945$, rounded to 0.950).



TABLE 2-2 The AWs of the 95% confidence intervals for the coefficient of variation of a delta-IG distribution.

n	μ	λ	δ	Average Widths						
				GCI	AGCI	PBPCI	FCI	F-HPDCI	BCI	B-HPDCI
30	0.5	1	0.3	0.6004	0.4636	0.3787	0.5074	0.4816	0.5263	0.5143
			0.5	0.8657	0.6313	0.4363	0.6856	0.6308	0.6393	0.6294
			5	0.3	0.1457	0.1375	0.0992	0.1405	0.1307	0.3560
				0.5	0.1878	0.1750	0.1045	0.1784	0.1599	0.3921
		10	0.3	0.0770	0.0748	0.0523	0.0755	0.0698	0.1343	0.1311
			0.5	0.0969	0.0933	0.0538	0.0942	0.0838	0.1567	0.1522
		1	1	0.3	1.1599	0.7191	0.6366	0.8461	0.8077	1.6274
				0.5	1.8319	1.0068	0.7640	1.1647	1.0809	2.5621
			5	0.3	0.2688	0.2403	0.1814	0.2501	0.2348	0.5529
				0.5	0.3590	0.3140	0.1969	0.3250	0.2944	0.6422
			10	0.3	0.1457	0.1375	0.0992	0.1405	0.1308	0.3811
				0.5	0.1878	0.1750	0.1045	0.1784	0.1599	0.4020
50	0.5	1	0.3	0.4260	0.3380	0.3129	0.3732	0.3616	0.5462	0.5264
			0.5	0.5717	0.4444	0.3737	0.4880	0.4642	0.5991	0.5794
			5	0.3	0.1037	0.0982	0.0815	0.1004	0.0961	0.4660
				0.5	0.1266	0.1193	0.0892	0.1217	0.1140	0.4867
		10	0.3	0.0545	0.0530	0.0431	0.0536	0.0511	0.6303	0.6173
			0.5	0.0650	0.0631	0.0461	0.0637	0.0594	0.4882	0.4817
		1	1	0.3	0.7912	0.5268	0.5303	0.6303	0.6128	0.4127
				0.5	1.1264	0.7166	0.6603	0.8495	0.8129	0.4621
			5	0.3	0.1923	0.1734	0.1490	0.1810	0.1742	0.5129
				0.5	0.2432	0.2174	0.1685	0.2261	0.2131	0.5322
			10	0.3	0.1037	0.0982	0.0815	0.1004	0.0961	0.3377
				0.5	0.1266	0.1193	0.0892	0.1217	0.1140	0.3620
100	0.5	1	0.3	0.2849	0.2297	0.2323	0.2552	0.2509	0.3584	0.3432
			0.5	0.3679	0.2944	0.2848	0.3262	0.3179	0.5864	0.5719
			5	0.3	0.0692	0.0658	0.0603	0.0673	0.0658	0.5968
				0.5	0.0816	0.0774	0.0678	0.0792	0.0766	0.6467
		10	0.3	0.0363	0.0354	0.0319	0.0358	0.0349	0.5176	0.5019
			0.5	0.0418	0.0407	0.0350	0.0411	0.0397	0.4638	0.4439
		1	1	0.3	0.5216	0.3594	0.3973	0.4353	0.4288	0.6388
				0.5	0.7041	0.4777	0.5055	0.5789	0.5629	0.4639
			5	0.3	0.1289	0.1170	0.1105	0.1225	0.1200	0.3420
				0.5	0.1572	0.1423	0.1281	0.1486	0.1441	0.3663
			10	0.3	0.0692	0.0658	0.0603	0.0673	0.0658	0.4280
				0.5	0.0816	0.0774	0.0678	0.0792	0.0766	0.3117
200	0.5	1	0.3	0.1968	0.1598	0.1697	0.1778	0.1761	0.5886	0.5706
			0.5	0.2498	0.2021	0.2103	0.2247	0.2217	0.6634	0.6491
			5	0.3	0.0478	0.0455	0.0442	0.0465	0.0460	0.4026
				0.5	0.0554	0.0527	0.0499	0.0539	0.0529	0.3471
		10	0.3	0.0250	0.0244	0.0233	0.0247	0.0243	0.4719	0.4543
			0.5	0.0283	0.0276	0.0258	0.0279	0.0274	0.5896	0.5765
		1	1	0.3	0.3582	0.2502	0.2901	0.3043	0.3017	0.5712
				0.5	0.4739	0.3289	0.3735	0.3990	0.3941	0.6705
			5	0.3	0.0891	0.0811	0.0808	0.0849	0.0839	0.3445
				0.5						0.3271

TABLE 2-2 (CONTINUED)

n	μ	λ	δ	Average Widths						
				GCI	AGCI	PBPCI	FCI	F-HPDCI	BCI	B-HPDCI
200	1	5	0.5	0.1069	<i>0.0972</i>	0.0946	0.1017	0.1000	0.5728	0.5565
		10	0.3	0.0478	<i>0.0455</i>	0.0442	0.0465	0.0460	0.3824	0.3717
			0.5	0.0554	<i>0.0527</i>	0.0499	0.0539	0.0529	0.3514	0.3382

Note: The shortest AW values are in italic.



TABLE 2-3 The CPs of the 95% confidence intervals for the difference between two coefficients of variation of delta-IG distributions.

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities				
				AGCI	FCI	BCI	MOVER	NA
(30,30)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9418	0.9456	0.9435	0.9476	0.9580
			(0.3,0.5)	0.9478	0.9552	0.9532	0.9490	0.9536
			(0.5,0.5)	0.9510	0.9522	0.9475	0.9500	0.9688
		(1,5)	(0.3,0.3)	0.9462	0.9516	0.9481	0.9494	0.9766
			(0.3,0.5)	0.9538	0.9542	0.9492	0.9546	0.9716
			(0.5,0.5)	0.9538	0.9572	0.9473	0.9570	0.9764
		(5,5)	(0.3,0.3)	0.9430	0.9456	0.9483	0.9474	0.9962
			(0.3,0.5)	0.9510	0.9526	0.9509	0.9530	0.9884
			(0.5,0.5)	0.9490	0.9504	0.9512	0.9506	0.9908
		(5,10)	(0.3,0.3)	0.9456	0.9468	0.9443	0.9490	0.9986
			(0.3,0.5)	0.9516	0.9556	0.9440	0.9534	0.9954
			(0.5,0.5)	0.9506	0.9524	0.9439	0.9540	0.9910
		(10,10)	(0.3,0.3)	0.9442	0.9460	0.9462	0.9474	0.9998
			(0.3,0.5)	0.9522	0.9518	0.9503	0.9530	0.9992
			(0.5,0.5)	0.9474	0.9488	0.9532	0.9528	0.9954
		(0.5,1)	(1,1)	(0.3,0.3)	0.9462	0.9484	0.9509	0.9480
			(1,1)	(0.3,0.5)	0.9510	0.9538	0.9516	0.9266
			(1,1)	(0.5,0.5)	0.9524	0.9536	0.9493	0.9546
		(1,5)	(1,1)	(0.3,0.3)	0.9456	0.9510	0.9529	0.9496
			(1,1)	(0.3,0.5)	0.9498	0.9534	0.9447	0.9544
			(1,1)	(0.5,0.5)	0.9516	0.9526	0.9430	0.9536
		(5,5)	(1,1)	(0.3,0.3)	0.9464	0.9482	0.9490	0.9504
			(1,1)	(0.3,0.5)	0.9518	0.9558	0.9424	0.9492
			(1,1)	(0.5,0.5)	0.9518	0.9552	0.9512	0.9560
		(5,10)	(1,1)	(0.3,0.3)	0.9430	0.9456	0.9434	0.9474
			(1,1)	(0.3,0.5)	0.9510	0.9526	0.9522	0.9530
			(1,1)	(0.5,0.5)	0.9490	0.9504	0.9449	0.9506
		(10,10)	(1,1)	(0.3,0.3)	0.9466	0.9458	0.9450	0.9496
			(1,1)	(0.3,0.5)	0.9532	0.9554	0.9422	0.9510
			(1,1)	(0.5,0.5)	0.9534	0.9536	0.9451	0.9544
		(1,1)	(1,1)	(0.3,0.3)	0.9430	0.9472	0.9485	0.9460
			(1,1)	(0.3,0.5)	0.9472	0.9562	0.9534	0.9500
			(1,1)	(0.5,0.5)	0.9494	0.9546	0.9439	0.9502
		(1,5)	(1,1)	(0.3,0.3)	0.9472	0.9468	0.9479	0.9470
			(1,1)	(0.3,0.5)	0.9528	0.9498	0.9483	0.9534
			(1,1)	(0.5,0.5)	0.9562	0.9558	0.9472	0.9578
		(5,5)	(1,1)	(0.3,0.3)	0.9432	0.9470	0.9447	0.9462
			(1,1)	(0.3,0.5)	0.9500	0.9540	0.9483	0.9514
			(1,1)	(0.5,0.5)	0.9508	0.9492	0.9440	0.9500
		(5,10)	(1,1)	(0.3,0.3)	0.9446	0.9482	0.9503	0.9472
			(1,1)	(0.3,0.5)	0.9522	0.9552	0.9431	0.9550
			(1,1)	(0.5,0.5)	0.9512	0.9520	0.9468	0.9528
		(10,10)	(1,1)	(0.3,0.3)	0.9430	0.9456	0.9448	0.9474
			(1,1)	(0.3,0.5)	0.9510	0.9526	0.9494	0.9530
			(1,1)	(0.5,0.5)	0.9490	0.9504	0.9462	0.9506

Note: Bold values indicate coverage probabilities (CP) that satisfy the nominal confidence level of 0.95 ($CP \geq 0.945$, rounded to 0.950).

TABLE 2-3 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities				
				AGCI	FCI	BCI	MOVER	NA
(50,50)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9452	0.9506	0.9508	0.9566	0.9524
			(0.3,0.5)	0.9478	0.9480	0.9526	0.9518	0.9484
			(0.5,0.5)	0.9536	0.9536	0.9512	0.9438	0.9586
			(1,5)	(0.3,0.3)	0.9528	0.9548	0.9423	0.9562
			(0.3,0.5)	0.9428	0.9504	0.9443	0.9528	0.9646
			(0.5,0.5)	0.9538	0.9556	0.9493	0.9470	0.9638
			(5,5)	(0.3,0.3)	0.9476	0.9482	0.9539	0.9542
			(0.3,0.5)	0.9474	0.9478	0.9452	0.9508	0.9844
			(0.5,0.5)	0.9566	0.9576	0.9534	0.9446	0.9784
		(5,10)	(0.3,0.3)	0.9516	0.9520	0.9526	0.9570	0.9974
			(0.3,0.5)	0.9450	0.9458	0.9462	0.9516	0.9900
			(0.5,0.5)	0.9558	0.9558	0.9462	0.9444	0.9882
		(10,10)	(0.3,0.3)	0.9464	0.9474	0.9472	0.9542	0.9998
			(0.3,0.5)	0.9468	0.9468	0.9486	0.9504	0.9970
			(0.5,0.5)	0.9562	0.9572	0.9445	0.9444	0.9942
		(0.5,1)	(1,1)	(0.3,0.3)	0.9494	0.9548	0.9483	0.9548
			(0.3,0.5)	0.9470	0.9476	0.9425	0.9514	0.9192
			(0.5,0.5)	0.9520	0.9540	0.9436	0.9442	0.9510
			(1,5)	(0.3,0.3)	0.9510	0.9528	0.9497	0.9562
			(0.3,0.5)	0.9440	0.9484	0.9471	0.9530	0.9886
			(0.5,0.5)	0.9556	0.9566	0.9498	0.9462	0.9848
			(5,5)	(0.3,0.3)	0.9456	0.9500	0.9532	0.9534
			(0.3,0.5)	0.9498	0.9488	0.9425	0.9508	0.9946
			(0.5,0.5)	0.9546	0.9552	0.9454	0.9450	0.9934
		(5,10)	(0.3,0.3)	0.9476	0.9482	0.9453	0.9542	1.0000
			(0.3,0.5)	0.9474	0.9478	0.9486	0.9508	0.9984
			(0.5,0.5)	0.9566	0.9576	0.9514	0.9446	0.9972
		(10,10)	(0.3,0.3)	0.9484	0.9494	0.9516	0.9544	1.0000
			(0.3,0.5)	0.9492	0.9472	0.9520	0.9510	1.0000
			(0.5,0.5)	0.9530	0.9560	0.9446	0.9450	0.9972
		(1,1)	(1,1)	(0.3,0.3)	0.9476	0.9536	0.9521	0.9556
			(0.3,0.5)	0.9482	0.9478	0.9427	0.9528	0.9204
			(0.5,0.5)	0.9528	0.9530	0.9476	0.9452	0.9384
			(1,5)	(0.3,0.3)	0.9516	0.9506	0.9463	0.9550
			(0.3,0.5)	0.9426	0.9440	0.9488	0.9518	0.9758
			(0.5,0.5)	0.9558	0.9540	0.9540	0.9464	0.9706
			(5,5)	(0.3,0.3)	0.9450	0.9504	0.9517	0.9556
			(0.3,0.5)	0.9466	0.9466	0.9466	0.9502	0.9994
			(0.5,0.5)	0.9556	0.9562	0.9528	0.9424	0.9978
		(5,10)	(0.3,0.3)	0.9510	0.9530	0.9469	0.9552	1.0000
			(0.3,0.5)	0.9452	0.9440	0.9503	0.9510	0.9998
			(0.5,0.5)	0.9578	0.9554	0.9459	0.9444	0.9986
		(10,10)	(0.3,0.3)	0.9476	0.9482	0.9445	0.9542	1.0000
			(0.3,0.5)	0.9474	0.9478	0.9470	0.9508	1.0000
			(0.5,0.5)	0.9566	0.9576	0.9509	0.9446	0.9998

TABLE 2-3 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities				
				AGCI	FCI	BCI	MOVER	NA
(100,100)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9462	0.9486	0.9522	0.9512	0.9488
			(0.3,0.5)	0.9524	0.9544	0.9429	0.9532	0.9344
			(0.5,0.5)	0.9508	0.9538	0.9451	0.9504	0.9492
		(1,5)	(0.3,0.3)	0.9492	0.9506	0.9477	0.9506	0.9706
			(0.3,0.5)	0.9528	0.9562	0.9500	0.9518	0.9628
			(0.5,0.5)	0.9498	0.9546	0.9456	0.9510	0.9688
		(5,5)	(0.3,0.3)	0.9454	0.9468	0.9422	0.9504	0.9928
			(0.3,0.5)	0.9508	0.9508	0.9457	0.9520	0.9824
			(0.5,0.5)	0.9504	0.9478	0.9473	0.9490	0.9718
		(5,10)	(0.3,0.3)	0.9478	0.9482	0.9458	0.9478	0.9974
			(0.3,0.5)	0.9532	0.9508	0.9538	0.9514	0.9890
			(0.5,0.5)	0.9526	0.9520	0.9458	0.9532	0.9830
		(10,10)	(0.3,0.3)	0.9460	0.9474	0.9527	0.9522	0.9998
			(0.3,0.5)	0.9502	0.9508	0.9504	0.9528	0.9936
			(0.5,0.5)	0.9504	0.9510	0.9430	0.9484	0.9896
	(0.5,1)	(1,1)	(0.3,0.3)	0.9442	0.9482	0.9511	0.9530	0.9282
			(0.3,0.5)	0.9516	0.9548	0.9433	0.9506	0.9118
			(0.5,0.5)	0.9496	0.9518	0.9478	0.9526	0.9400
		(1,5)	(0.3,0.3)	0.9474	0.9486	0.9456	0.9480	0.9956
			(0.3,0.5)	0.9516	0.9524	0.9461	0.9520	0.9836
			(0.5,0.5)	0.9516	0.9528	0.9526	0.9538	0.9808
		(5,5)	(0.3,0.3)	0.9476	0.9470	0.9462	0.9516	0.9986
			(0.3,0.5)	0.9528	0.9528	0.9460	0.9520	0.9936
			(0.5,0.5)	0.9500	0.9504	0.9507	0.9504	0.9906
		(5,10)	(0.3,0.3)	0.9454	0.9468	0.9535	0.9504	1.0000
			(0.3,0.5)	0.9508	0.9508	0.9496	0.9520	0.9988
			(0.5,0.5)	0.9504	0.9478	0.9496	0.9490	0.9982
		(10,10)	(0.3,0.3)	0.9476	0.9492	0.9472	0.9530	1.0000
			(0.3,0.5)	0.9524	0.9518	0.9510	0.9510	0.9996
			(0.5,0.5)	0.9498	0.9502	0.9421	0.9496	0.9992
	(1,1)	(1,1)	(0.3,0.3)	0.9468	0.9516	0.9513	0.9506	0.9270
			(0.3,0.5)	0.9500	0.9536	0.9433	0.9532	0.9132
			(0.5,0.5)	0.9502	0.9522	0.9485	0.9504	0.9378
		(1,5)	(0.3,0.3)	0.9474	0.9518	0.9528	0.9500	0.9862
			(0.3,0.5)	0.9528	0.9502	0.9453	0.9514	0.9700
			(0.5,0.5)	0.9508	0.9530	0.9488	0.9514	0.9740
		(5,5)	(0.3,0.3)	0.9464	0.9458	0.9450	0.9500	0.9998
			(0.3,0.5)	0.9510	0.9496	0.9468	0.9520	0.9984
			(0.5,0.5)	0.9500	0.9508	0.9437	0.9500	0.9972
		(5,10)	(0.3,0.3)	0.9478	0.9476	0.9510	0.9480	1.0000
			(0.3,0.5)	0.9506	0.9486	0.9504	0.9520	0.9996
			(0.5,0.5)	0.9516	0.9526	0.9473	0.9544	0.9990
		(10,10)	(0.3,0.3)	0.9454	0.9468	0.9497	0.9504	1.0000
			(0.3,0.5)	0.9508	0.9508	0.9480	0.9520	1.0000
			(0.5,0.5)	0.9504	0.9478	0.9510	0.9490	0.9998

TABLE 2-3 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities				
				AGCI	FCI	BCI	MOVER	NA
(200,200)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9462	0.9486	0.9522	0.9512	0.9400
			(0.3,0.5)	0.9524	0.9544	0.9429	0.9532	0.9400
			(0.5,0.5)	0.9508	0.9538	0.9451	0.9504	0.9466
		(1,5)	(0.3,0.3)	0.9492	0.9506	0.9477	0.9506	0.9682
			(0.3,0.5)	0.9528	0.9562	0.9500	0.9518	0.9542
			(0.5,0.5)	0.9498	0.9546	0.9456	0.9510	0.9560
		(5,5)	(0.3,0.3)	0.9454	0.9468	0.9422	0.9504	0.9902
			(0.3,0.5)	0.9508	0.9508	0.9457	0.9520	0.9784
			(0.5,0.5)	0.9504	0.9478	0.9473	0.9490	0.9704
		(5,10)	(0.3,0.3)	0.9478	0.9482	0.9458	0.9478	0.9962
			(0.3,0.5)	0.9532	0.9508	0.9538	0.9514	0.9842
			(0.5,0.5)	0.9526	0.9520	0.9458	0.9532	0.9808
		(10,10)	(0.3,0.3)	0.9460	0.9474	0.9527	0.9522	0.9996
			(0.3,0.5)	0.9502	0.9508	0.9504	0.9528	0.9932
			(0.5,0.5)	0.9504	0.9510	0.9430	0.9484	0.9854
		(0.5,1)	(1,1)	0.9442	0.9482	0.9511	0.9530	0.9292
			(0.3,0.5)	0.9516	0.9548	0.9433	0.9506	0.9132
			(0.5,0.5)	0.9496	0.9518	0.9478	0.9526	0.9380
		(1,5)	(0.3,0.3)	0.9474	0.9486	0.9456	0.9480	0.9956
			(0.3,0.5)	0.9516	0.9524	0.9461	0.9520	0.9822
			(0.5,0.5)	0.9516	0.9528	0.9526	0.9538	0.9782
		(5,5)	(0.3,0.3)	0.9476	0.9470	0.9462	0.9516	0.9992
			(0.3,0.5)	0.9528	0.9528	0.9460	0.9520	0.9904
			(0.5,0.5)	0.9500	0.9504	0.9507	0.9504	0.9896
		(5,10)	(0.3,0.3)	0.9454	0.9468	0.9535	0.9504	1.0000
			(0.3,0.5)	0.9508	0.9508	0.9496	0.9520	0.9986
			(0.5,0.5)	0.9504	0.9478	0.9496	0.9490	0.9980
		(10,10)	(0.3,0.3)	0.9476	0.9492	0.9472	0.9530	1.0000
			(0.3,0.5)	0.9524	0.9518	0.9510	0.9510	0.9990
			(0.5,0.5)	0.9498	0.9502	0.9421	0.9496	0.9994
		(1,1)	(0.3,0.3)	0.9468	0.9516	0.9513	0.9506	0.9244
			(0.3,0.5)	0.9500	0.9536	0.9433	0.9532	0.9100
			(0.5,0.5)	0.9502	0.9522	0.9485	0.9504	0.9302
		(1,5)	(0.3,0.3)	0.9474	0.9518	0.9528	0.9500	0.9886
			(0.3,0.5)	0.9528	0.9502	0.9453	0.9514	0.9694
			(0.5,0.5)	0.9508	0.9530	0.9488	0.9514	0.9704
		(5,5)	(0.3,0.3)	0.9464	0.9458	0.9450	0.9500	1.0000
			(0.3,0.5)	0.9510	0.9496	0.9468	0.9520	0.9992
			(0.5,0.5)	0.9500	0.9508	0.9437	0.9500	0.9956
		(5,10)	(0.3,0.3)	0.9478	0.9476	0.9510	0.9480	1.0000
			(0.3,0.5)	0.9506	0.9486	0.9504	0.9520	1.0000
			(0.5,0.5)	0.9516	0.9526	0.9473	0.9544	0.9994
		(10,10)	(0.3,0.3)	0.9454	0.9468	0.9497	0.9504	1.0000
			(0.3,0.5)	0.9508	0.9508	0.9480	0.9520	1.0000
			(0.5,0.5)	0.9504	0.9478	0.9510	0.9490	1.0000

TABLE 2-3 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities				
				AGCI	FCI	BCI	MOVER	NA
(30,50)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9470	0.9470	0.9527	0.9488	0.9578
			(0.3,0.5)	0.9526	0.9514	0.9497	0.9462	0.9622
			(0.5,0.5)	0.9482	0.9492	0.9512	0.9538	0.9630
		(1,5)	(0.3,0.3)	0.9460	0.9510	0.9474	0.9532	0.9742
			(0.3,0.5)	0.9554	0.9522	0.9441	0.9458	0.9682
			(0.5,0.5)	0.9476	0.9564	0.9430	0.9550	0.9590
		(5,5)	(0.3,0.3)	0.9496	0.9498	0.9481	0.9498	0.9980
			(0.3,0.5)	0.9504	0.9498	0.9458	0.9454	0.9918
			(0.5,0.5)	0.9510	0.9518	0.9466	0.9550	0.9848
		(5,10)	(0.3,0.3)	0.9496	0.9502	0.9483	0.9520	0.9998
			(0.3,0.5)	0.9532	0.9550	0.9533	0.9452	0.9946
			(0.5,0.5)	0.9514	0.9554	0.9496	0.9582	0.9906
		(10,10)	(0.3,0.3)	0.9484	0.9482	0.9455	0.9514	0.9998
			(0.3,0.5)	0.9532	0.9510	0.9498	0.9444	0.9984
			(0.5,0.5)	0.9490	0.9500	0.9467	0.9556	0.9958
		(0.5,1)	(1,1)	0.9494	0.9494	0.9449	0.9478	0.9646
			(0.3,0.5)	0.9546	0.9524	0.9477	0.9482	0.9502
			(0.5,0.5)	0.9498	0.9514	0.9493	0.9508	0.9690
		(1,5)	(0.3,0.3)	0.9496	0.9504	0.9430	0.9520	0.9946
			(0.3,0.5)	0.9538	0.9498	0.9484	0.9452	0.9888
			(0.5,0.5)	0.9490	0.9566	0.9467	0.9554	0.9846
		(5,5)	(0.3,0.3)	0.9482	0.9494	0.9450	0.9468	0.9998
			(0.3,0.5)	0.9532	0.9532	0.9444	0.9488	0.9978
			(0.5,0.5)	0.9510	0.9510	0.9430	0.9530	0.9946
		(5,10)	(0.3,0.3)	0.9496	0.9498	0.9446	0.9498	1.0000
			(0.3,0.5)	0.9504	0.9498	0.9426	0.9454	0.9990
			(0.5,0.5)	0.9510	0.9518	0.9507	0.9550	0.9984
		(10,10)	(0.3,0.3)	0.9480	0.9480	0.9496	0.9496	1.0000
			(0.3,0.5)	0.9540	0.9536	0.9497	0.9478	1.0000
			(0.5,0.5)	0.9520	0.9524	0.9436	0.9504	0.9986
		(1,1)	(0.3,0.3)	0.9496	0.9524	0.9423	0.9470	0.9582
			(0.3,0.5)	0.9510	0.9536	0.9442	0.9480	0.9568
			(0.5,0.5)	0.9492	0.9518	0.9483	0.9546	0.9608
		(1,5)	(0.3,0.3)	0.9466	0.9504	0.9526	0.9520	0.9866
			(0.3,0.5)	0.9544	0.9516	0.9436	0.9450	0.9790
			(0.5,0.5)	0.9470	0.9546	0.9447	0.9554	0.9718
		(5,5)	(0.3,0.3)	0.9510	0.9496	0.9520	0.9470	1.0000
			(0.3,0.5)	0.9534	0.9520	0.9446	0.9444	0.9998
			(0.5,0.5)	0.9482	0.9528	0.9493	0.9558	0.9996
		(5,10)	(0.3,0.3)	0.9498	0.9518	0.9499	0.9530	1.0000
			(0.3,0.5)	0.9548	0.9536	0.9534	0.9426	1.0000
			(0.5,0.5)	0.9496	0.9562	0.9479	0.9572	1.0000
		(10,10)	(0.3,0.3)	0.9496	0.9498	0.9447	0.9498	1.0000
			(0.3,0.5)	0.9504	0.9498	0.9471	0.9454	1.0000
			(0.5,0.5)	0.9510	0.9518	0.9535	0.9550	1.0000

TABLE 2-3 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities				
				AGCI	FCI	BCI	MOVER	NA
(50,100)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9558	0.9546	0.9501	0.9536	0.9614
			(0.3,0.5)	0.9476	0.9454	0.9484	0.9564	0.9560
			(0.5,0.5)	0.9450	0.9468	0.9529	0.9518	0.9642
		(1,5)	(0.3,0.3)	0.9526	0.9564	0.9424	0.9554	0.9744
			(0.3,0.5)	0.9488	0.9492	0.9468	0.9462	0.9654
			(0.5,0.5)	0.9490	0.9564	0.9427	0.9500	0.9626
		(5,5)	(0.3,0.3)	0.9534	0.9564	0.9459	0.9528	0.9970
			(0.3,0.5)	0.9466	0.9486	0.9462	0.9510	0.9914
			(0.5,0.5)	0.9464	0.9450	0.9528	0.9518	0.9808
		(5,10)	(0.3,0.3)	0.9544	0.9574	0.9481	0.9550	0.9992
			(0.3,0.5)	0.9484	0.9504	0.9507	0.9466	0.9914
			(0.5,0.5)	0.9474	0.9498	0.9470	0.9522	0.9856
		(10,10)	(0.3,0.3)	0.9546	0.9558	0.9473	0.9530	1.0000
			(0.3,0.5)	0.9460	0.9482	0.9457	0.9518	0.9994
			(0.5,0.5)	0.9458	0.9454	0.9503	0.9508	0.9956
		(0.5,1)	(1,1)	0.9564	0.9588	0.9487	0.9524	0.9670
			(0.3,0.5)	0.9438	0.9464	0.9525	0.9520	0.9594
			(0.5,0.5)	0.9448	0.9500	0.9537	0.9544	0.9678
		(1,5)	(0.3,0.3)	0.9538	0.9576	0.9430	0.9568	0.9954
			(0.3,0.5)	0.9478	0.9490	0.9485	0.9478	0.9896
			(0.5,0.5)	0.9474	0.9542	0.9473	0.9498	0.9816
		(5,5)	(0.3,0.3)	0.9568	0.9568	0.9484	0.9526	0.9992
			(0.3,0.5)	0.9476	0.9496	0.9518	0.9526	0.9972
			(0.5,0.5)	0.9460	0.9462	0.9499	0.9542	0.9944
		(5,10)	(0.3,0.3)	0.9534	0.9564	0.9430	0.9528	1.0000
			(0.3,0.5)	0.9466	0.9486	0.9512	0.9510	0.9996
			(0.5,0.5)	0.9464	0.9450	0.9501	0.9518	0.9990
		(10,10)	(0.3,0.3)	0.9578	0.9580	0.9491	0.9526	1.0000
			(0.3,0.5)	0.9480	0.9508	0.9454	0.9530	1.0000
			(0.5,0.5)	0.9436	0.9462	0.9493	0.9528	0.9998
		(1,1)	(1,1)	0.9570	0.9582	0.9476	0.9532	0.9576
			(0.3,0.5)	0.9468	0.9482	0.9534	0.9528	0.9502
			(0.5,0.5)	0.9452	0.9474	0.9426	0.9514	0.9576
		(1,5)	(0.3,0.3)	0.9542	0.9590	0.9507	0.9540	0.9890
			(0.3,0.5)	0.9480	0.9492	0.9472	0.9472	0.9798
			(0.5,0.5)	0.9466	0.9504	0.9484	0.9484	0.9732
		(5,5)	(0.3,0.3)	0.9542	0.9554	0.9508	0.9526	1.0000
			(0.3,0.5)	0.9458	0.9494	0.9460	0.9536	1.0000
			(0.5,0.5)	0.9452	0.9456	0.9472	0.9536	0.9992
		(5,10)	(0.3,0.3)	0.9536	0.9568	0.9427	0.9536	1.0000
			(0.3,0.5)	0.9478	0.9496	0.9515	0.9462	1.0000
			(0.5,0.5)	0.9462	0.9508	0.9479	0.9516	0.9998
		(10,10)	(0.3,0.3)	0.9534	0.9564	0.9522	0.9528	1.0000
			(0.3,0.5)	0.9466	0.9486	0.9478	0.9510	1.0000
			(0.5,0.5)	0.9464	0.9450	0.9511	0.9518	1.0000

TABLE 2-3 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities				
				AGCI	FCI	BCI	MOVER	NA
(100,200)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9490	0.9482	0.9477	0.9490	0.9634
			(0.3,0.5)	0.9480	0.9522	0.9479	0.9490	0.9580
			(0.5,0.5)	0.9548	0.9538	0.9432	0.9554	0.9584
		(1,5)	(0.3,0.3)	0.9476	0.9508	0.9503	0.9530	0.9768
			(0.3,0.5)	0.9458	0.9500	0.9494	0.9490	0.9576
			(0.5,0.5)	0.9542	0.9526	0.9487	0.9526	0.9590
		(5,5)	(0.3,0.3)	0.9504	0.9474	0.9472	0.9498	0.9952
			(0.3,0.5)	0.9486	0.9502	0.9493	0.9466	0.9866
			(0.5,0.5)	0.9528	0.9542	0.9526	0.9550	0.9766
		(5,10)	(0.3,0.3)	0.9480	0.9474	0.9425	0.9502	0.9980
			(0.3,0.5)	0.9506	0.9484	0.9528	0.9494	0.9932
			(0.5,0.5)	0.9532	0.9546	0.9518	0.9530	0.9836
		(10,10)	(0.3,0.3)	0.9490	0.9456	0.9499	0.9488	1.0000
			(0.3,0.5)	0.9488	0.9496	0.9494	0.9470	0.9980
			(0.5,0.5)	0.9544	0.9532	0.9520	0.9562	0.9906
	(0.5,1)	(1,1)	(0.3,0.3)	0.9494	0.9524	0.9495	0.9454	0.9646
			(0.3,0.5)	0.9492	0.9522	0.9438	0.9494	0.9546
			(0.5,0.5)	0.9516	0.9552	0.9465	0.9548	0.9638
		(1,5)	(0.3,0.3)	0.9484	0.9488	0.9422	0.9500	0.9968
			(0.3,0.5)	0.9488	0.9496	0.9478	0.9478	0.9860
			(0.5,0.5)	0.9528	0.9536	0.9501	0.9522	0.9850
		(5,5)	(0.3,0.3)	0.9492	0.9498	0.9507	0.9448	0.9996
			(0.3,0.5)	0.9514	0.9524	0.9498	0.9482	0.9972
			(0.5,0.5)	0.9554	0.9554	0.9485	0.9526	0.9938
		(5,10)	(0.3,0.3)	0.9504	0.9474	0.9495	0.9498	1.0000
			(0.3,0.5)	0.9486	0.9502	0.9466	0.9466	0.9998
			(0.5,0.5)	0.9528	0.9542	0.9443	0.9550	0.9992
		(10,10)	(0.3,0.3)	0.9486	0.9502	0.9528	0.9448	1.0000
			(0.3,0.5)	0.9502	0.9506	0.9460	0.9496	1.0000
			(0.5,0.5)	0.9564	0.9538	0.9434	0.9534	0.9996
	(1,1)	(1,1)	(0.3,0.3)	0.9476	0.9482	0.9445	0.9478	0.9462
			(0.3,0.5)	0.9472	0.9518	0.9429	0.9486	0.9378
			(0.5,0.5)	0.9528	0.9584	0.9479	0.9550	0.9482
		(1,5)	(0.3,0.3)	0.9472	0.9462	0.9446	0.9540	0.9886
			(0.3,0.5)	0.9468	0.9462	0.9475	0.9490	0.9716
			(0.5,0.5)	0.9550	0.9530	0.9444	0.9522	0.9684
		(5,5)	(0.3,0.3)	0.9492	0.9470	0.9496	0.9488	1.0000
			(0.3,0.5)	0.9506	0.9470	0.9501	0.9468	1.0000
			(0.5,0.5)	0.9540	0.9550	0.9526	0.9562	0.9988
		(5,10)	(0.3,0.3)	0.9458	0.9486	0.9474	0.9498	1.0000
			(0.3,0.5)	0.9502	0.9508	0.9463	0.9488	0.9998
			(0.5,0.5)	0.9542	0.9568	0.9536	0.9532	0.9996
		(10,10)	(0.3,0.3)	0.9504	0.9474	0.9446	0.9498	1.0000
			(0.3,0.5)	0.9486	0.9502	0.9523	0.9466	1.0000
			(0.5,0.5)	0.9528	0.9542	0.9523	0.9550	1.0000

TABLE 2-4 The AWs of the 95% confidence intervals for the difference between two coefficients of variation of delta-IG distributions.

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths				
				AGCI	FCI	BCI	MOVER	NA
(30,30)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.6819	0.7496	0.8476	<i>0.6813</i>	0.9005
			(0.3,0.5)	<i>0.8259</i>	0.9016	0.9725	0.8283	1.1040
			(0.5,0.5)	0.9617	1.0505	1.1205	<i>0.9550</i>	1.3440
		(1,5)	(0.3,0.3)	<i>0.4878</i>	0.5315	0.6034	0.4883	0.9658
			(0.3,0.5)	<i>0.5070</i>	0.5495	0.6367	0.5101	1.0540
			(0.5,0.5)	0.6748	0.7322	0.7980	<i>0.6688</i>	1.2910
		(5,5)	(0.3,0.3)	0.2049	0.2094	0.3275	<i>0.2047</i>	1.0220
			(0.3,0.5)	<i>0.2387</i>	0.2432	0.4082	0.2394	1.1020
			(0.5,0.5)	0.2715	0.2770	0.3729	<i>0.2696</i>	1.2300
		(5,10)	(0.3,0.3)	<i>0.1626</i>	0.1656	0.3046	0.1628	1.2060
			(0.3,0.5)	<i>0.1767</i>	0.1797	0.3143	0.1778	1.1680
			(0.5,0.5)	0.2135	0.2175	0.3455	<i>0.2120</i>	1.3090
		(10,10)	(0.3,0.3)	0.1122	0.1134	0.2473	<i>0.1120</i>	1.3650
			(0.3,0.5)	<i>0.1291</i>	0.1303	0.3155	0.1295	1.3260
			(0.5,0.5)	0.1457	0.1473	0.2502	<i>0.1448</i>	1.3780
	(0.5,1)	(1,1)	(0.3,0.3)	0.8809	1.0195	1.0078	<i>0.8798</i>	1.0470
			(0.3,0.5)	<i>1.1474</i>	1.3181	1.3227	1.1483	1.2830
			(0.5,0.5)	1.2594	1.4426	1.4530	<i>1.2534</i>	1.5400
		(1,5)	(0.3,0.3)	<i>0.5365</i>	0.5819	0.7067	0.5366	1.4130
			(0.3,0.5)	<i>0.5887</i>	0.6335	0.7405	0.5913	1.3260
			(0.5,0.5)	0.7463	0.8063	0.9382	<i>0.7401</i>	1.5930
		(5,5)	(0.3,0.3)	0.2872	0.2975	0.4823	<i>0.2867</i>	1.4490
			(0.3,0.5)	<i>0.3585</i>	0.3701	0.5038	0.3590	1.3650
			(0.5,0.5)	0.3852	0.3976	0.5352	<i>0.3830</i>	1.5570
		(5,10)	(0.3,0.3)	0.2049	0.2094	0.3292	<i>0.2047</i>	2.1870
			(0.3,0.5)	<i>0.2387</i>	0.2432	0.4368	0.2394	1.7110
			(0.5,0.5)	0.2715	0.2770	0.3894	<i>0.2696</i>	1.9710
		(10,10)	(0.3,0.3)	0.1627	0.1658	0.2870	<i>0.1622</i>	2.2770
			(0.3,0.5)	<i>0.1996</i>	0.2028	0.3807	0.1998	1.8270
			(0.5,0.5)	0.2131	0.2165	0.4024	<i>0.2117</i>	2.0290
	(1,1)	(1,1)	(0.3,0.3)	1.0521	1.2505	1.1906	<i>1.0512</i>	1.1760
			(0.3,0.5)	<i>1.2963</i>	1.5199	1.4631	1.2984	1.3970
			(0.5,0.5)	1.5216	1.7907	1.7174	<i>1.5126</i>	1.7050
		(1,5)	(0.3,0.3)	<i>0.7659</i>	0.8947	0.9256	0.7664	1.5120
			(0.3,0.5)	<i>0.8058</i>	0.9283	0.9395	0.8098	1.4240
			(0.5,0.5)	1.0883	1.2601	1.1974	<i>1.0792</i>	1.7750
		(5,5)	(0.3,0.3)	0.3563	0.3710	0.5345	<i>0.3562</i>	1.7980
			(0.3,0.5)	<i>0.4216</i>	0.4375	0.5553	0.4227	1.7120
			(0.5,0.5)	0.4842	0.5033	0.6629	<i>0.4810</i>	1.8220
		(5,10)	(0.3,0.3)	0.2870	0.2972	0.4569	<i>0.2873</i>	2.4290
			(0.3,0.5)	<i>0.3157</i>	0.3258	0.4407	0.3172	2.0050
			(0.5,0.5)	0.3860	0.3997	0.4958	<i>0.3832</i>	2.1860
		(10,10)	(0.3,0.3)	0.2049	0.2094	0.3409	<i>0.2047</i>	2.9450
			(0.3,0.5)	<i>0.2387</i>	0.2432	0.3438	0.2394	2.5720
			(0.5,0.5)	0.2715	0.2770	0.4573	<i>0.2696</i>	2.5080

Note: The shortest AW values are in italic.

TABLE 2-4 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths				
				AGCI	FCI	BCI	MOVER	NA
(50,50)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.4920	0.5448	0.8001	0.4916	0.6925
			(0.3,0.5)	0.5771	0.6375	0.7640	0.5793	0.8477
			(0.5,0.5)	0.6553	0.7230	0.8340	0.6536	1.0210
		(1,5)	(0.3,0.3)	0.3561	0.3908	0.4612	0.3550	0.7279
			(0.3,0.5)	0.3649	0.3990	0.4891	0.3650	0.7974
			(0.5,0.5)	0.4636	0.5065	0.5643	0.4643	0.9740
		(5,5)	(0.3,0.3)	0.1444	0.1478	0.3223	0.1443	0.7581
			(0.3,0.5)	0.1618	0.1654	0.2832	0.1624	0.8250
			(0.5,0.5)	0.1783	0.1823	0.3209	0.1778	0.9202
		(5,10)	(0.3,0.3)	0.1151	0.1174	0.2568	0.1147	0.8894
			(0.3,0.5)	0.1217	0.1239	0.2630	0.1219	0.8689
			(0.5,0.5)	0.1402	0.1428	0.2824	0.1402	0.9758
		(10,10)	(0.3,0.3)	0.0783	0.0792	0.2421	0.0782	0.9978
			(0.3,0.5)	0.0866	0.0876	0.2237	0.0870	0.9818
			(0.5,0.5)	0.0946	0.0956	0.2540	0.0943	1.0240
		(0.5,1)	(1,1)	0.6394	0.7491	0.7683	0.6394	0.8023
			(0.3,0.5)	0.8066	0.9481	0.9759	0.8098	0.9880
			(0.5,0.5)	0.8700	1.0136	1.0474	0.8668	1.1740
		(1,5)	(0.3,0.3)	0.3884	0.4242	0.5118	0.3876	1.0410
			(0.3,0.5)	0.4153	0.4511	0.5326	0.4161	0.9780
			(0.5,0.5)	0.5080	0.5527	0.6550	0.5079	1.1870
		(5,5)	(0.3,0.3)	0.2046	0.2127	0.3872	0.2047	1.0620
			(0.3,0.5)	0.2444	0.2541	0.3767	0.2458	1.0050
			(0.5,0.5)	0.2581	0.2679	0.4139	0.2567	1.1390
		(5,10)	(0.3,0.3)	0.1444	0.1478	0.2635	0.1443	1.5830
			(0.3,0.5)	0.1618	0.1654	0.3072	0.1624	1.2500
			(0.5,0.5)	0.1783	0.1823	0.3705	0.1778	1.4430
		(10,10)	(0.3,0.3)	0.1147	0.1171	0.2317	0.1148	1.6550
			(0.3,0.5)	0.1341	0.1369	0.2528	0.1350	1.3330
			(0.5,0.5)	0.1408	0.1435	0.2926	0.1400	1.4780
		(1,1)	(0.3,0.3)	0.7640	0.9194	1.3128	0.7635	0.9034
			(0.3,0.5)	0.9145	1.0959	1.0370	0.9172	1.0670
			(0.5,0.5)	1.0500	1.2542	1.1740	1.0475	1.3130
		(1,5)	(0.3,0.3)	0.5612	0.6641	0.7009	0.5599	1.1220
			(0.3,0.5)	0.5809	0.6815	0.7427	0.5812	1.0650
			(0.5,0.5)	0.7554	0.8859	0.8632	0.7559	1.3240
		(5,5)	(0.3,0.3)	0.2540	0.2656	0.4247	0.2538	1.2980
			(0.3,0.5)	0.2896	0.3023	0.4235	0.2908	1.2510
			(0.5,0.5)	0.3232	0.3373	0.5004	0.3223	1.3270
		(5,10)	(0.3,0.3)	0.2051	0.2132	0.3956	0.2047	1.7550
			(0.3,0.5)	0.2191	0.2270	0.3458	0.2196	1.4570
			(0.5,0.5)	0.2571	0.2667	0.4509	0.2570	1.5930
		(10,10)	(0.3,0.3)	0.1444	0.1478	0.2840	0.1443	2.1190
			(0.3,0.5)	0.1618	0.1654	0.2638	0.1624	1.8730
			(0.5,0.5)	0.1783	0.1823	0.3638	0.1778	1.8200

TABLE 2-4 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths				
				AGCI	FCI	BCI	MOVER	NA
(100,100)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.2271	0.2536	0.3336	0.2271	0.4877
			(0.3,0.5)	0.2599	0.2896	0.3710	0.2593	0.5945
			(0.5,0.5)	0.2894	0.3226	0.4441	0.2883	0.7172
		(1,5)	(0.3,0.3)	0.1660	0.1841	0.2897	0.1660	0.5051
			(0.3,0.5)	0.1685	0.1865	0.2991	0.1680	0.5556
			(0.5,0.5)	0.2097	0.2323	0.3209	0.2089	0.6772
		(5,5)	(0.3,0.3)	0.0648	0.0664	0.2284	0.0649	0.5206
			(0.3,0.5)	0.0705	0.0721	0.2136	0.0703	0.5696
			(0.5,0.5)	0.0757	0.0775	0.1762	0.0754	0.6356
		(5,10)	(0.3,0.3)	0.0518	0.0530	0.2435	0.0519	0.6066
			(0.3,0.5)	0.0537	0.0548	0.1646	0.0535	0.5982
			(0.5,0.5)	0.0602	0.0614	0.1775	0.0599	0.6709
		(10,10)	(0.3,0.3)	0.0348	0.0352	0.2263	0.0348	0.6800
			(0.3,0.5)	0.0373	0.0377	0.1701	0.0372	0.6734
			(0.5,0.5)	0.0397	0.0402	0.2365	0.0395	0.7032
		(0.5,1)	(1,1)	0.2979	0.3547	0.4761	0.2978	0.5655
			(0.3,0.5)	0.3676	0.4395	0.5646	0.3669	0.6928
			(0.5,0.5)	0.3895	0.4630	0.5018	0.3880	0.8267
		(1,5)	(0.3,0.3)	0.1796	0.1980	0.3196	0.1796	0.7125
			(0.3,0.5)	0.1882	0.2065	0.3275	0.1877	0.6725
			(0.5,0.5)	0.2262	0.2491	0.4015	0.2253	0.8124
		(5,5)	(0.3,0.3)	0.0934	0.0974	0.2889	0.0935	0.7221
			(0.3,0.5)	0.1081	0.1126	0.2167	0.1078	0.6859
			(0.5,0.5)	0.1117	0.1164	0.3027	0.1112	0.7768
		(5,10)	(0.3,0.3)	0.0648	0.0664	0.2157	0.0649	1.0720
			(0.3,0.5)	0.0705	0.0721	0.2226	0.0703	0.8503
			(0.5,0.5)	0.0757	0.0775	0.2434	0.0754	0.9788
		(10,10)	(0.3,0.3)	0.0519	0.0530	0.1667	0.0519	1.1140
			(0.3,0.5)	0.0585	0.0597	0.1780	0.0584	0.9048
			(0.5,0.5)	0.0601	0.0614	0.1826	0.0599	1.0030
		(1,1)	(0.3,0.3)	0.3554	0.4341	0.4821	0.3553	0.6371
			(0.3,0.5)	0.4163	0.5070	0.5887	0.4154	0.7508
			(0.5,0.5)	0.4702	0.5725	0.5892	0.4686	0.9200
		(1,5)	(0.3,0.3)	0.2630	0.3169	0.4397	0.2630	0.7677
			(0.3,0.5)	0.2690	0.3222	0.3850	0.2683	0.7318
			(0.5,0.5)	0.3444	0.4138	0.4582	0.3433	0.9096
		(5,5)	(0.3,0.3)	0.1155	0.1211	0.3063	0.1155	0.8812
			(0.3,0.5)	0.1280	0.1340	0.2811	0.1277	0.8479
			(0.5,0.5)	0.1395	0.1461	0.2618	0.1389	0.8977
		(5,10)	(0.3,0.3)	0.0934	0.0974	0.2895	0.0934	1.1850
			(0.3,0.5)	0.0976	0.1015	0.2293	0.0973	0.9855
			(0.5,0.5)	0.1118	0.1165	0.2233	0.1113	1.0760
		(10,10)	(0.3,0.3)	0.0648	0.0664	0.2243	0.0649	1.4270
			(0.3,0.5)	0.0705	0.0721	0.2186	0.0703	1.2670
			(0.5,0.5)	0.0757	0.0775	0.2222	0.0754	1.2330

TABLE 2-4 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths				
				AGCI	FCI	BCI	MOVER	NA
(200,200)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.2271	0.2536	0.3336	0.2271	0.3440
			(0.3,0.5)	0.2599	0.2896	0.3710	0.2593	0.4186
			(0.5,0.5)	0.2894	0.3226	0.4441	0.2883	0.5037
		(1,5)	(0.3,0.3)	0.1660	0.1841	0.2897	0.1660	0.3540
			(0.3,0.5)	0.1685	0.1865	0.2991	0.1680	0.3900
			(0.5,0.5)	0.2097	0.2323	0.3209	0.2089	0.4757
		(5,5)	(0.3,0.3)	0.0648	0.0664	0.2284	0.0649	0.3635
			(0.3,0.5)	0.0705	0.0721	0.2136	0.0703	0.3987
			(0.5,0.5)	0.0757	0.0775	0.1762	0.0754	0.4444
		(5,10)	(0.3,0.3)	0.0518	0.0530	0.2435	0.0519	0.4217
			(0.3,0.5)	0.0537	0.0548	0.1646	0.0535	0.4177
			(0.5,0.5)	0.0602	0.0614	0.1775	0.0599	0.4683
		(10,10)	(0.3,0.3)	0.0348	0.0352	0.2263	0.0348	0.4721
			(0.3,0.5)	0.0373	0.0377	0.1701	0.0372	0.4688
			(0.5,0.5)	0.0397	0.0402	0.2365	0.0395	0.4907
	(0.5,1)	(1,1)	(0.3,0.3)	0.2979	0.3547	0.4761	0.2978	0.3998
			(0.3,0.5)	0.3676	0.4395	0.5646	0.3669	0.4894
			(0.5,0.5)	0.3895	0.4630	0.5018	0.3880	0.5815
		(1,5)	(0.3,0.3)	0.1796	0.1980	0.3196	0.1796	0.4944
			(0.3,0.5)	0.1882	0.2065	0.3275	0.1877	0.4691
			(0.5,0.5)	0.2262	0.2491	0.4015	0.2253	0.5661
		(5,5)	(0.3,0.3)	0.0934	0.0974	0.2889	0.0935	0.5016
			(0.3,0.5)	0.1081	0.1126	0.2167	0.1078	0.4761
			(0.5,0.5)	0.1117	0.1164	0.3027	0.1112	0.5408
		(5,10)	(0.3,0.3)	0.0648	0.0664	0.2157	0.0649	0.7435
			(0.3,0.5)	0.0705	0.0721	0.2226	0.0703	0.5890
			(0.5,0.5)	0.0757	0.0775	0.2434	0.0754	0.6787
		(10,10)	(0.3,0.3)	0.0519	0.0530	0.1667	0.0519	0.7745
			(0.3,0.5)	0.0585	0.0597	0.1780	0.0584	0.6265
			(0.5,0.5)	0.0601	0.0614	0.1826	0.0599	0.6949
	(1,1)	(1,1)	(0.3,0.3)	0.3554	0.4341	0.4821	0.3553	0.4490
			(0.3,0.5)	0.4163	0.5070	0.5887	0.4154	0.5306
			(0.5,0.5)	0.4702	0.5725	0.5892	0.4686	0.6482
		(1,5)	(0.3,0.3)	0.2630	0.3169	0.4397	0.2630	0.5350
			(0.3,0.5)	0.2690	0.3222	0.3850	0.2683	0.5121
			(0.5,0.5)	0.3444	0.4138	0.4582	0.3433	0.6351
		(5,5)	(0.3,0.3)	0.1155	0.1211	0.3063	0.1155	0.6091
			(0.3,0.5)	0.1280	0.1340	0.2811	0.1277	0.5879
			(0.5,0.5)	0.1395	0.1461	0.2618	0.1389	0.6217
		(5,10)	(0.3,0.3)	0.0934	0.0974	0.2895	0.0934	0.8212
			(0.3,0.5)	0.0976	0.1015	0.2293	0.0973	0.6819
			(0.5,0.5)	0.1118	0.1165	0.2233	0.1113	0.7448
		(10,10)	(0.3,0.3)	0.0648	0.0664	0.2243	0.0649	0.9864
			(0.3,0.5)	0.0705	0.0721	0.2186	0.0703	0.8762
			(0.5,0.5)	0.0757	0.0775	0.2222	0.0754	0.8514

TABLE 2-4 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths				
				AGCI	FCI	BCI	MOVER	NA
(30,50)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.5952	0.6553	0.7435	0.5913	0.8437
			(0.3,0.5)	0.6713	0.7396	0.7841	0.6727	0.9776
			(0.5,0.5)	0.8206	0.8973	0.9882	0.8138	1.2320
		(1,5)	(0.3,0.3)	0.4781	0.5216	0.5851	0.4744	0.8883
			(0.3,0.5)	0.4821	0.5254	0.6717	0.4860	0.9213
			(0.5,0.5)	0.6561	0.7091	0.7719	0.6471	1.1750
		(5,5)	(0.3,0.3)	0.1772	0.1810	0.3405	0.1759	0.9486
			(0.3,0.5)	0.1927	0.1970	0.2951	0.1935	0.9801
			(0.5,0.5)	0.2285	0.2331	0.3752	0.2264	1.1130
		(5,10)	(0.3,0.3)	0.1514	0.1543	0.2736	0.1501	1.1190
			(0.3,0.5)	0.1564	0.1594	0.3260	0.1576	1.0380
			(0.5,0.5)	0.1952	0.1986	0.3071	0.1927	1.1870
		(10,10)	(0.3,0.3)	0.0967	0.0977	0.2939	0.0959	1.2880
			(0.3,0.5)	0.1040	0.1052	0.2069	0.1045	1.2150
			(0.5,0.5)	0.1221	0.1234	0.2246	0.1211	1.2640
		(0.5,1)	(1,1)	0.7275	0.8425	0.8827	0.7234	0.9902
			(0.3,0.5)	0.8848	1.0287	1.0709	0.8840	1.1580
			(0.5,0.5)	1.0123	1.1614	1.1866	1.0070	1.4360
		(1,5)	(0.3,0.3)	0.5051	0.5497	0.6347	0.5015	1.3110
			(0.3,0.5)	0.5256	0.5707	0.6543	0.5287	1.1750
			(0.5,0.5)	0.6931	0.7483	0.8520	0.6846	1.4690
		(5,5)	(0.3,0.3)	0.2318	0.2399	0.3546	0.2304	1.3600
			(0.3,0.5)	0.2700	0.2798	0.3952	0.2699	1.2240
			(0.5,0.5)	0.3002	0.3103	0.4264	0.2987	1.4120
		(5,10)	(0.3,0.3)	0.1772	0.1810	0.3214	0.1759	2.0300
			(0.3,0.5)	0.1927	0.1970	0.3613	0.1935	1.5560
			(0.5,0.5)	0.2285	0.2331	0.4243	0.2264	1.8240
		(10,10)	(0.3,0.3)	0.1298	0.1322	0.2465	0.1290	2.1440
			(0.3,0.5)	0.1485	0.1512	0.3385	0.1485	1.6850
			(0.5,0.5)	0.1638	0.1666	0.2938	0.1631	1.8700
		(1,1)	(1,1)	0.9206	1.0980	1.0617	0.9147	1.1240
			(0.3,0.5)	1.0555	1.2557	1.1623	1.0572	1.2760
			(0.5,0.5)	1.3034	1.5349	1.4795	1.2936	1.6220
		(1,5)	(0.3,0.3)	0.7468	0.8738	0.8997	0.7409	1.4200
			(0.3,0.5)	0.7587	0.8840	0.9243	0.7642	1.2920
			(0.5,0.5)	1.0511	1.2110	1.1774	1.0386	1.6430
		(5,5)	(0.3,0.3)	0.3096	0.3226	0.4828	0.3074	1.7040
			(0.3,0.5)	0.3415	0.3560	0.4907	0.3426	1.6040
			(0.5,0.5)	0.4100	0.4262	0.5199	0.4066	1.7000
		(5,10)	(0.3,0.3)	0.2667	0.2766	0.3928	0.2645	2.3000
			(0.3,0.5)	0.2776	0.2878	0.3880	0.2796	1.8690
			(0.5,0.5)	0.3524	0.3643	0.5113	0.3484	2.0500
		(10,10)	(0.3,0.3)	0.1772	0.1810	0.3258	0.1759	2.8080
			(0.3,0.5)	0.1927	0.1970	0.3023	0.1935	2.4650
			(0.5,0.5)	0.2285	0.2331	0.3421	0.2264	2.3880

TABLE 2-4 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths				
				AGCI	FCI	BCI	MOVER	NA
(50,100)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.4174	0.4635	0.5380	0.4159	0.6390
			(0.3,0.5)	0.4603	0.5115	0.6184	0.4593	0.7290
			(0.5,0.5)	0.5502	0.6075	0.6520	0.5483	0.9311
		(1,5)	(0.3,0.3)	0.3461	0.3820	0.4601	0.3449	0.6625
			(0.3,0.5)	0.3492	0.3856	0.5442	0.3478	0.6793
			(0.5,0.5)	0.4541	0.4969	0.6285	0.4521	0.8774
		(5,5)	(0.3,0.3)	0.1215	0.1244	0.2839	0.1211	0.6972
			(0.3,0.5)	0.1295	0.1326	0.2499	0.1291	0.7119
			(0.5,0.5)	0.1484	0.1516	0.3457	0.1477	0.8162
		(5,10)	(0.3,0.3)	0.1058	0.1082	0.2966	0.1056	0.8233
			(0.3,0.5)	0.1085	0.1108	0.2418	0.1081	0.7566
			(0.5,0.5)	0.1290	0.1315	0.2924	0.1284	0.8707
		(10,10)	(0.3,0.3)	0.0657	0.0665	0.2322	0.0655	0.9424
			(0.3,0.5)	0.0694	0.0702	0.2424	0.0692	0.8823
			(0.5,0.5)	0.0785	0.0794	0.2162	0.0781	0.9270
		(0.5,1)	(1,1)	0.5042	0.5879	0.6752	0.5027	0.7556
			(0.3,0.5)	0.5979	0.7027	0.7667	0.5973	0.8678
			(0.5,0.5)	0.6740	0.7829	0.8365	0.6720	1.0770
		(1,5)	(0.3,0.3)	0.3616	0.3980	0.4784	0.3603	0.9726
			(0.3,0.5)	0.3726	0.4097	0.5551	0.3713	0.8637
			(0.5,0.5)	0.4738	0.5175	0.6036	0.4719	1.0840
		(5,5)	(0.3,0.3)	0.1572	0.1629	0.2599	0.1567	0.9939
			(0.3,0.5)	0.1781	0.1849	0.3412	0.1779	0.8899
			(0.5,0.5)	0.1939	0.2008	0.3126	0.1932	1.0330
		(5,10)	(0.3,0.3)	0.1215	0.1244	0.2443	0.1211	1.5010
			(0.3,0.5)	0.1295	0.1326	0.2385	0.1291	1.1420
			(0.5,0.5)	0.1484	0.1516	0.2784	0.1477	1.3350
		(10,10)	(0.3,0.3)	0.0872	0.0888	0.2634	0.0869	1.5730
			(0.3,0.5)	0.0969	0.0988	0.2968	0.0969	1.2270
			(0.5,0.5)	0.1046	0.1065	0.2900	0.1042	1.3750
		(1,1)	(0.3,0.3)	0.6496	0.7845	0.7786	0.6474	0.8613
			(0.3,0.5)	0.7279	0.8809	0.8749	0.7266	0.9592
			(0.5,0.5)	0.8847	1.0599	1.0495	0.8819	1.2230
		(1,5)	(0.3,0.3)	0.5426	0.6469	0.6729	0.5408	1.0560
			(0.3,0.5)	0.5501	0.6567	0.6962	0.5482	0.9560
			(0.5,0.5)	0.7352	0.8656	0.9175	0.7321	1.2290
		(5,5)	(0.3,0.3)	0.2145	0.2243	0.3988	0.2138	1.2480
			(0.3,0.5)	0.2316	0.2424	0.3996	0.2310	1.1600
			(0.5,0.5)	0.2699	0.2815	0.4572	0.2689	1.2330
		(5,10)	(0.3,0.3)	0.1881	0.1961	0.3157	0.1876	1.6760
			(0.3,0.5)	0.1939	0.2020	0.3401	0.1932	1.3630
			(0.5,0.5)	0.2358	0.2448	0.4217	0.2347	1.4970
		(10,10)	(0.3,0.3)	0.1215	0.1244	0.2445	0.1211	2.0530
			(0.3,0.5)	0.1295	0.1326	0.2521	0.1291	1.8010
			(0.5,0.5)	0.1484	0.1516	0.3071	0.1477	1.7330

TABLE 2-4 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths				
				AGCI	FCI	BCI	MOVER	NA
(100,200)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.2825	0.3146	0.4128	0.2824	0.4509
			(0.3,0.5)	0.3099	0.3452	0.4929	0.3100	0.5132
			(0.5,0.5)	0.3628	0.4035	0.4697	0.3611	0.6524
		(1,5)	(0.3,0.3)	0.2347	0.2602	0.3494	0.2346	0.4635
			(0.3,0.5)	0.2362	0.2614	0.3502	0.2366	0.4757
			(0.5,0.5)	0.3002	0.3323	0.4562	0.2985	0.6133
		(5,5)	(0.3,0.3)	0.0811	0.0831	0.1890	0.0811	0.4812
			(0.3,0.5)	0.0858	0.0878	0.2391	0.0859	0.4925
			(0.5,0.5)	0.0957	0.0980	0.2515	0.0952	0.5659
		(5,10)	(0.3,0.3)	0.0707	0.0723	0.1953	0.0707	0.5668
			(0.3,0.5)	0.0720	0.0736	0.2111	0.0722	0.5243
			(0.5,0.5)	0.0832	0.0851	0.2395	0.0827	0.6043
		(10,10)	(0.3,0.3)	0.0436	0.0441	0.1445	0.0436	0.6450
			(0.3,0.5)	0.0457	0.0462	0.1952	0.0457	0.6076
			(0.5,0.5)	0.0503	0.0509	0.1801	0.0500	0.6408
		(0.5,1)	(1,1)	0.3427	0.4016	0.4529	0.3427	0.5335
			(0.3,0.5)	0.4055	0.4797	0.5945	0.4052	0.6127
			(0.5,0.5)	0.4482	0.5246	0.6158	0.4466	0.7580
		(1,5)	(0.3,0.3)	0.2449	0.2705	0.4157	0.2447	0.6761
			(0.3,0.5)	0.2511	0.2766	0.4490	0.2515	0.6011
			(0.5,0.5)	0.3125	0.3448	0.4149	0.3108	0.7505
		(5,5)	(0.3,0.3)	0.1057	0.1098	0.2536	0.1058	0.6876
			(0.3,0.5)	0.1191	0.1239	0.2432	0.1191	0.6138
			(0.5,0.5)	0.1267	0.1316	0.2790	0.1262	0.7140
		(5,10)	(0.3,0.3)	0.0811	0.0831	0.2396	0.0811	1.0380
			(0.3,0.5)	0.0858	0.0878	0.2251	0.0859	0.7862
			(0.5,0.5)	0.0957	0.0980	0.2402	0.0952	0.9210
		(10,10)	(0.3,0.3)	0.0584	0.0595	0.2375	0.0584	1.0820
			(0.3,0.5)	0.0645	0.0658	0.2092	0.0645	0.8439
			(0.5,0.5)	0.0679	0.0693	0.2072	0.0676	0.9457
		(1,1)	(0.3,0.3)	0.4414	0.5373	0.6351	0.4412	0.6060
			(0.3,0.5)	0.4925	0.5991	0.5984	0.4926	0.6774
			(0.5,0.5)	0.5878	0.7127	0.7870	0.5854	0.8606
		(1,5)	(0.3,0.3)	0.3692	0.4451	0.5683	0.3690	0.7326
			(0.3,0.5)	0.3731	0.4478	0.5068	0.3737	0.6661
			(0.5,0.5)	0.4892	0.5871	0.6251	0.4868	0.8531
		(5,5)	(0.3,0.3)	0.1441	0.1509	0.3381	0.1441	0.8508
			(0.3,0.5)	0.1545	0.1618	0.2795	0.1546	0.7926
			(0.5,0.5)	0.1757	0.1839	0.2911	0.1748	0.8417
		(5,10)	(0.3,0.3)	0.1265	0.1320	0.3217	0.1264	1.1520
			(0.3,0.5)	0.1295	0.1350	0.2528	0.1297	0.9315
			(0.5,0.5)	0.1534	0.1601	0.2786	0.1526	1.0250
		(10,10)	(0.3,0.3)	0.0811	0.0831	0.2028	0.0811	1.3980
			(0.3,0.5)	0.0858	0.0878	0.2053	0.0859	1.2230
			(0.5,0.5)	0.0957	0.0980	0.2593	0.0952	1.1840

TABLE 2-5 The CPs of the 95% confidence intervals for the ratio between two coefficients of variation of delta-IG distributions.

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities				
				AGCI	FCI	BCI	MOVER	NA
(30,30)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9418	0.9456	0.9451	0.9476	0.9510
			(0.3,0.5)	0.9490	0.9520	0.9527	0.9520	0.9876
			(0.5,0.5)	0.9510	0.9522	0.9428	0.9500	0.9906
		(1,5)	(0.3,0.3)	0.9472	0.9500	0.9535	0.9498	0.8604
			(0.3,0.5)	0.9542	0.9544	0.9509	0.9552	0.9710
			(0.5,0.5)	0.9544	0.9550	0.9470	0.9572	0.9696
		(5,5)	(0.3,0.3)	0.9430	0.9456	0.9510	0.9474	0.9690
			(0.3,0.5)	0.9518	0.9538	0.9489	0.9544	0.9904
			(0.5,0.5)	0.9490	0.9504	0.9474	0.9506	0.9840
		(5,10)	(0.3,0.3)	0.9464	0.9474	0.9532	0.9494	0.9806
			(0.3,0.5)	0.9538	0.9580	0.9442	0.9540	0.9946
			(0.5,0.5)	0.9504	0.9522	0.9501	0.9548	0.9858
		(10,10)	(0.3,0.3)	0.9442	0.9460	0.9515	0.9474	0.9864
			(0.3,0.5)	0.9516	0.9528	0.9523	0.9550	0.9964
			(0.5,0.5)	0.9474	0.9488	0.9424	0.9528	0.9888
	(0.5,1)	(1,1)	(0.3,0.3)	0.9446	0.9482	0.9514	0.9476	0.9832
			(0.3,0.5)	0.9468	0.9546	0.9465	0.9506	0.9952
			(0.5,0.5)	0.9518	0.9522	0.9523	0.9536	0.9972
		(1,5)	(0.3,0.3)	0.9456	0.9490	0.9526	0.9494	0.9852
			(0.3,0.5)	0.9516	0.9538	0.9526	0.9542	0.9970
			(0.5,0.5)	0.9520	0.9532	0.9506	0.9532	0.9974
		(5,5)	(0.3,0.3)	0.9452	0.9486	0.9446	0.9502	0.9992
			(0.3,0.5)	0.9510	0.9546	0.9498	0.9490	1.0000
			(0.5,0.5)	0.9520	0.9558	0.9478	0.9556	0.9996
		(5,10)	(0.3,0.3)	0.9430	0.9456	0.9520	0.9474	1.0000
			(0.3,0.5)	0.9518	0.9538	0.9520	0.9544	1.0000
			(0.5,0.5)	0.9490	0.9504	0.9514	0.9506	1.0000
		(10,10)	(0.3,0.3)	0.9462	0.9446	0.9492	0.9496	1.0000
			(0.3,0.5)	0.9506	0.9534	0.9513	0.9516	1.0000
			(0.5,0.5)	0.9522	0.9536	0.9532	0.9548	1.0000
		(1,1)	(0.3,0.3)	0.9430	0.9472	0.9494	0.9460	0.9730
			(0.3,0.5)	0.9494	0.9550	0.9503	0.9494	0.9962
			(0.5,0.5)	0.9494	0.9546	0.9512	0.9502	0.9956
		(1,5)	(0.3,0.3)	0.9450	0.9474	0.9540	0.9484	0.9544
			(0.3,0.5)	0.9528	0.9508	0.9431	0.9528	0.9914
			(0.5,0.5)	0.9554	0.9534	0.9497	0.9576	0.9878
		(5,5)	(0.3,0.3)	0.9432	0.9470	0.9522	0.9462	0.9996
			(0.3,0.5)	0.9488	0.9520	0.9529	0.9522	1.0000
			(0.5,0.5)	0.9508	0.9492	0.9489	0.9500	0.9994
		(5,10)	(0.3,0.3)	0.9466	0.9480	0.9471	0.9470	0.9998
			(0.3,0.5)	0.9536	0.9570	0.9433	0.9562	1.0000
			(0.5,0.5)	0.9514	0.9518	0.9506	0.9520	0.9996
		(10,10)	(0.3,0.3)	0.9430	0.9456	0.9444	0.9474	1.0000
			(0.3,0.5)	0.9518	0.9538	0.9509	0.9544	1.0000
			(0.5,0.5)	0.9490	0.9504	0.9505	0.9506	1.0000

Note: Bold values indicate coverage probabilities (CP) that satisfy the nominal confidence level of 0.95 ($CP \geq 0.945$, rounded to 0.950).

TABLE 2-5 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities				
				AGCI	FCI	BCI	MOVER	NA
(50,50)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9452	0.9506	0.9538	0.9566	0.9570
			(0.3,0.5)	0.9462	0.9454	0.9486	0.9524	0.9906
			(0.5,0.5)	0.9536	0.9536	0.9476	0.9438	0.9938
		(1,5)	(0.3,0.3)	0.9536	0.9554	0.9510	0.9570	0.8476
			(0.3,0.5)	0.9426	0.9504	0.9530	0.9526	0.9762
			(0.5,0.5)	0.9544	0.9560	0.9496	0.9468	0.9652
		(5,5)	(0.3,0.3)	0.9476	0.9482	0.9421	0.9542	0.9622
			(0.3,0.5)	0.9446	0.9448	0.9519	0.9498	0.9928
			(0.5,0.5)	0.9566	0.9576	0.9463	0.9446	0.9820
		(5,10)	(0.3,0.3)	0.9504	0.9518	0.9503	0.9568	0.9712
			(0.3,0.5)	0.9452	0.9468	0.9462	0.9520	0.9954
			(0.5,0.5)	0.9562	0.9574	0.9497	0.9444	0.9810
		(10,10)	(0.3,0.3)	0.9464	0.9474	0.9499	0.9542	0.9860
			(0.3,0.5)	0.9456	0.9460	0.9462	0.9508	0.9978
			(0.5,0.5)	0.9562	0.9572	0.9421	0.9444	0.9880
		(0.5,1)	(1,1)	0.9496	0.9540	0.9487	0.9558	0.9850
			(0.3,0.5)	0.9484	0.9456	0.9512	0.9542	0.9988
			(0.5,0.5)	0.9504	0.9526	0.9515	0.9450	0.9982
		(1,5)	(0.3,0.3)	0.9506	0.9506	0.9524	0.9542	0.9810
			(0.3,0.5)	0.9450	0.9492	0.9531	0.9526	0.9976
			(0.5,0.5)	0.9546	0.9562	0.9461	0.9458	0.9946
		(5,5)	(0.3,0.3)	0.9470	0.9496	0.9477	0.9552	0.9994
			(0.3,0.5)	0.9486	0.9472	0.9450	0.9500	1.0000
			(0.5,0.5)	0.9544	0.9550	0.9505	0.9442	0.9992
		(5,10)	(0.3,0.3)	0.9476	0.9482	0.9476	0.9542	1.0000
			(0.3,0.5)	0.9446	0.9448	0.9494	0.9498	1.0000
			(0.5,0.5)	0.9566	0.9576	0.9441	0.9446	0.9994
		(10,10)	(0.3,0.3)	0.9478	0.9484	0.9534	0.9540	1.0000
			(0.3,0.5)	0.9484	0.9474	0.9510	0.9522	1.0000
			(0.5,0.5)	0.9532	0.9552	0.9533	0.9450	1.0000
		(1,1)	(0.3,0.3)	0.9476	0.9536	0.9481	0.9556	0.9798
			(0.3,0.5)	0.9464	0.9462	0.9539	0.9510	0.9950
			(0.5,0.5)	0.9528	0.9530	0.9505	0.9452	0.9970
		(1,5)	(0.3,0.3)	0.9498	0.9500	0.9467	0.9548	0.9456
			(0.3,0.5)	0.9416	0.9428	0.9494	0.9534	0.9896
			(0.5,0.5)	0.9556	0.9548	0.9456	0.9466	0.9828
		(5,5)	(0.3,0.3)	0.9450	0.9504	0.9430	0.9556	0.9996
			(0.3,0.5)	0.9458	0.9448	0.9478	0.9498	1.0000
			(0.5,0.5)	0.9556	0.9562	0.9536	0.9424	0.9980
		(5,10)	(0.3,0.3)	0.9500	0.9532	0.9420	0.9550	1.0000
			(0.3,0.5)	0.9454	0.9472	0.9509	0.9512	1.0000
			(0.5,0.5)	0.9570	0.9552	0.9465	0.9438	0.9998
		(10,10)	(0.3,0.3)	0.9476	0.9482	0.9445	0.9542	1.0000
			(0.3,0.5)	0.9446	0.9448	0.9422	0.9498	1.0000
			(0.5,0.5)	0.9566	0.9576	0.9447	0.9446	0.9996

TABLE 2-5 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities				
				AGCI	FCI	BCI	MOVER	NA
(100,100)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9462	0.9480	0.9467	0.9488	0.9562
			(0.3,0.5)	0.9476	0.9510	0.9527	0.9478	0.9948
			(0.5,0.5)	0.9470	0.9418	0.9514	0.9486	0.9928
		(1,5)	(0.3,0.3)	0.9474	0.9508	0.9496	0.9470	0.8392
			(0.3,0.5)	0.9534	0.9566	0.9443	0.9516	0.9804
			(0.5,0.5)	0.9494	0.9512	0.9534	0.9510	0.9636
		(5,5)	(0.3,0.3)	0.9472	0.9450	0.9511	0.9482	0.9542
			(0.3,0.5)	0.9480	0.9482	0.9526	0.9508	0.9966
			(0.5,0.5)	0.9450	0.9436	0.9469	0.9450	0.9816
		(5,10)	(0.3,0.3)	0.9484	0.9474	0.9514	0.9488	0.9652
			(0.3,0.5)	0.9538	0.9546	0.9524	0.9506	0.9960
			(0.5,0.5)	0.9482	0.9486	0.9455	0.9478	0.9822
		(10,10)	(0.3,0.3)	0.9464	0.9456	0.9503	0.9488	0.9838
			(0.3,0.5)	0.9476	0.9476	0.9441	0.9502	0.9982
			(0.5,0.5)	0.9464	0.9450	0.9482	0.9468	0.9924
		(0.5,1)	(1,1)	(0.3,0.3)	0.9476	0.9450	0.9492	0.9486
			(1,1)	(0.3,0.5)	0.9490	0.9472	0.9489	0.9488
			(1,1)	(0.5,0.5)	0.9460	0.9476	0.9469	0.9470
		(1,5)	(1,5)	(0.3,0.3)	0.9468	0.9502	0.9423	0.9502
			(1,5)	(0.3,0.5)	0.9518	0.9554	0.9525	0.9510
			(1,5)	(0.5,0.5)	0.9470	0.9476	0.9539	0.9478
		(5,5)	(5,5)	(0.3,0.3)	0.9464	0.9464	0.9498	0.9504
			(5,5)	(0.3,0.5)	0.9472	0.9464	0.9472	0.9488
			(5,5)	(0.5,0.5)	0.9440	0.9442	0.9502	0.9490
		(5,10)	(5,10)	(0.3,0.3)	0.9472	0.9450	0.9529	0.9482
			(5,10)	(0.3,0.5)	0.9480	0.9482	0.9481	0.9508
			(5,10)	(0.5,0.5)	0.9450	0.9436	0.9525	0.9450
		(10,10)	(10,10)	(0.3,0.3)	0.9456	0.9478	0.9496	0.9510
			(10,10)	(0.3,0.5)	0.9462	0.9456	0.9488	0.9488
			(10,10)	(0.5,0.5)	0.9444	0.9450	0.9493	0.9512
		(1,1)	(1,1)	(0.3,0.3)	0.9480	0.9470	0.9444	0.9486
			(1,1)	(0.3,0.5)	0.9484	0.9506	0.9423	0.9470
			(1,1)	(0.5,0.5)	0.9452	0.9476	0.9485	0.9476
		(1,5)	(1,5)	(0.3,0.3)	0.9480	0.9508	0.9532	0.9498
			(1,5)	(0.3,0.5)	0.9534	0.9548	0.9466	0.9522
			(1,5)	(0.5,0.5)	0.9496	0.9474	0.9494	0.9498
		(5,5)	(5,5)	(0.3,0.3)	0.9452	0.9468	0.9484	0.9472
			(5,5)	(0.3,0.5)	0.9502	0.9512	0.9463	0.9478
			(5,5)	(0.5,0.5)	0.9466	0.9424	0.9526	0.9474
		(5,10)	(5,10)	(0.3,0.3)	0.9474	0.9486	0.9522	0.9486
			(5,10)	(0.3,0.5)	0.9520	0.9544	0.9428	0.9480
			(5,10)	(0.5,0.5)	0.9468	0.9460	0.9448	0.9486
		(10,10)	(10,10)	(0.3,0.3)	0.9472	0.9450	0.9478	0.9482
			(10,10)	(0.3,0.5)	0.9480	0.9482	0.9463	0.9508
			(10,10)	(0.5,0.5)	0.9450	0.9436	0.9481	0.9450

TABLE 2-5 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities				
				AGCI	FCI	BCI	MOVER	NA
(200,200)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9462	0.9486	0.9459	0.9512	0.9518
			(0.3,0.5)	0.9514	0.9538	0.9431	0.9534	0.9962
			(0.5,0.5)	0.9508	0.9538	0.9482	0.9504	0.9918
		(1,5)	(0.3,0.3)	0.9474	0.9476	0.9420	0.9488	0.8336
			(0.3,0.5)	0.9528	0.9558	0.9474	0.9516	0.9714
			(0.5,0.5)	0.9486	0.9554	0.9505	0.9522	0.9514
		(5,5)	(0.3,0.3)	0.9454	0.9468	0.9535	0.9504	0.9490
			(0.3,0.5)	0.9498	0.9504	0.9516	0.9528	0.9966
			(0.5,0.5)	0.9504	0.9478	0.9494	0.9490	0.9808
		(5,10)	(0.3,0.3)	0.9474	0.9480	0.9424	0.9480	0.9580
			(0.3,0.5)	0.9536	0.9540	0.9470	0.9540	0.9964
			(0.5,0.5)	0.9522	0.9522	0.9435	0.9540	0.9806
		(10,10)	(0.3,0.3)	0.9460	0.9474	0.9496	0.9522	0.9816
			(0.3,0.5)	0.9504	0.9506	0.9479	0.9538	0.9988
			(0.5,0.5)	0.9504	0.9510	0.9525	0.9484	0.9888
		(0.5,1)	(1,1)	(0.3,0.3)	0.9456	0.9480	0.9521	0.9534
			(1,1)	(0.3,0.5)	0.9512	0.9534	0.9443	0.9530
			(1,1)	(0.5,0.5)	0.9500	0.9504	0.9429	0.9496
			(1,5)	(0.3,0.3)	0.9460	0.9492	0.9487	0.9490
			(1,5)	(0.3,0.5)	0.9534	0.9540	0.9458	0.9516
		(5,5)	(1,5)	(0.5,0.5)	0.9526	0.9518	0.9500	0.9540
			(5,5)	(0.3,0.3)	0.9460	0.9458	0.9466	0.9522
			(5,5)	(0.3,0.5)	0.9516	0.9532	0.9496	0.9534
			(5,5)	(0.5,0.5)	0.9500	0.9488	0.9535	0.9504
		(5,10)	(5,10)	(0.3,0.3)	0.9454	0.9468	0.9426	0.9504
			(5,10)	(0.3,0.5)	0.9498	0.9504	0.9442	0.9528
			(5,10)	(0.5,0.5)	0.9504	0.9478	0.9430	0.9490
		(10,10)	(10,10)	(0.3,0.3)	0.9470	0.9486	0.9467	0.9534
			(10,10)	(0.3,0.5)	0.9504	0.9528	0.9510	0.9534
			(10,10)	(0.5,0.5)	0.9494	0.9508	0.9506	0.9492
		(1,1)	(1,1)	(0.3,0.3)	0.9468	0.9516	0.9515	0.9506
			(1,1)	(0.3,0.5)	0.9502	0.9530	0.9449	0.9518
			(1,1)	(0.5,0.5)	0.9502	0.9522	0.9476	0.9504
		(1,5)	(1,5)	(0.3,0.3)	0.9466	0.9512	0.9456	0.9486
			(1,5)	(0.3,0.5)	0.9538	0.9516	0.9447	0.9516
			(1,5)	(0.5,0.5)	0.9510	0.9542	0.9460	0.9514
		(5,5)	(5,5)	(0.3,0.3)	0.9464	0.9458	0.9499	0.9500
			(5,5)	(0.3,0.5)	0.9504	0.9466	0.9523	0.9522
			(5,5)	(0.5,0.5)	0.9500	0.9508	0.9458	0.9500
		(5,10)	(5,10)	(0.3,0.3)	0.9476	0.9470	0.9490	0.9502
			(5,10)	(0.3,0.5)	0.9536	0.9526	0.9449	0.9520
			(5,10)	(0.5,0.5)	0.9520	0.9540	0.9489	0.9530
		(10,10)	(10,10)	(0.3,0.3)	0.9454	0.9468	0.9480	0.9504
			(10,10)	(0.3,0.5)	0.9498	0.9504	0.9462	0.9528
			(10,10)	(0.5,0.5)	0.9504	0.9478	0.9514	0.9490

TABLE 2-5 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities						
				AGCI	FCI	BCI	MOVER	NA		
(30,50)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9470	0.9470	0.9523	0.9488	0.9702		
			(0.3,0.5)	0.9534	0.9528	0.9481	0.9462	0.9980		
			(0.5,0.5)	0.9482	0.9492	0.9495	0.9538	0.9968		
			(1,5)	(0.3,0.3)	0.9472	0.9506	0.9521	0.9522	0.8758	
				(0.3,0.5)	0.9556	0.9524	0.9536	0.9460	0.9856	
				(0.5,0.5)	0.9486	0.9554	0.9495	0.9548	0.9690	
			(5,5)	(0.3,0.3)	0.9496	0.9498	0.9466	0.9498	0.9728	
				(0.3,0.5)	0.9540	0.9524	0.9514	0.9460	0.9962	
				(0.5,0.5)	0.9510	0.9518	0.9508	0.9550	0.9892	
		(5,10)	(0.3,0.3)	0.9490	0.9496	0.9437	0.9520	0.9868		
			(0.3,0.5)	0.9548	0.9550	0.9434	0.9432	0.9966		
			(0.5,0.5)	0.9514	0.9554	0.9514	0.9582	0.9928		
		(10,10)	(0.3,0.3)	0.9484	0.9482	0.9445	0.9514	0.9914		
			(0.3,0.5)	0.9542	0.9534	0.9526	0.9454	0.9986		
			(0.5,0.5)	0.9490	0.9500	0.9521	0.9556	0.9960		
		(0.5,1)	(1,1)	(0.3,0.3)	0.9498	0.9504	0.9488	0.9466	0.9950	
				(0.3,0.5)	0.9532	0.9506	0.9421	0.9494	0.9990	
				(0.5,0.5)	0.9466	0.9502	0.9478	0.9512	0.9994	
			(1,5)	(0.3,0.3)	0.9498	0.9492	0.9441	0.9502	0.9886	
				(0.3,0.5)	0.9534	0.9508	0.9501	0.9446	0.9984	
				(0.5,0.5)	0.9508	0.9554	0.9458	0.9550	0.9962	
			(5,5)	(0.3,0.3)	0.9480	0.9498	0.9540	0.9464	0.9998	
				(0.3,0.5)	0.9504	0.9500	0.9446	0.9444	1.0000	
				(0.5,0.5)	0.9508	0.9514	0.9455	0.9536	0.9998	
			(5,10)	(0.3,0.3)	0.9496	0.9498	0.9496	0.9498	1.0000	
				(0.3,0.5)	0.9540	0.9524	0.9427	0.9460	1.0000	
				(0.5,0.5)	0.9510	0.9518	0.9520	0.9550	1.0000	
			(10,10)	(0.3,0.3)	0.9482	0.9486	0.9441	0.9488	1.0000	
				(0.3,0.5)	0.9506	0.9494	0.9489	0.9432	1.0000	
				(0.5,0.5)	0.9520	0.9526	0.9456	0.9508	1.0000	
			(1,1)	(1,1)	(0.3,0.3)	0.9496	0.9524	0.9495	0.9470	0.9916
					(0.3,0.5)	0.9518	0.9546	0.9445	0.9462	0.9984
					(0.5,0.5)	0.9492	0.9518	0.9426	0.9546	0.9992
		(1,5)		(0.3,0.3)	0.9454	0.9496	0.9535	0.9510	0.9608	
				(0.3,0.5)	0.9556	0.9520	0.9443	0.9458	0.9928	
				(0.5,0.5)	0.9490	0.9532	0.9522	0.9570	0.9874	
		(5,5)		(0.3,0.3)	0.9510	0.9496	0.9527	0.9470	0.9994	
				(0.3,0.5)	0.9530	0.9528	0.9515	0.9448	1.0000	
				(0.5,0.5)	0.9482	0.9528	0.9497	0.9558	0.9998	
		(5,10)		(0.3,0.3)	0.9506	0.9508	0.9492	0.9518	1.0000	
				(0.3,0.5)	0.9546	0.9546	0.9423	0.9432	1.0000	
				(0.5,0.5)	0.9494	0.9548	0.9483	0.9580	1.0000	
		(10,10)		(0.3,0.3)	0.9496	0.9498	0.9497	0.9498	1.0000	
				(0.3,0.5)	0.9540	0.9524	0.9533	0.9460	1.0000	
				(0.5,0.5)	0.9510	0.9518	0.9511	0.9550	1.0000	

TABLE 2-5 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities				
				AGCI	FCI	BCI	MOVER	NA
(50,100)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9558	0.9546	0.9472	0.9536	0.9728
			(0.3,0.5)	0.9472	0.9448	0.9531	0.9522	0.9974
			(0.5,0.5)	0.9450	0.9468	0.9484	0.9518	0.9972
		(1,5)	(0.3,0.3)	0.9546	0.9568	0.9423	0.9564	0.8724
			(0.3,0.5)	0.9488	0.9492	0.9524	0.9466	0.9814
			(0.5,0.5)	0.9464	0.9554	0.9468	0.9482	0.9684
		(5,5)	(0.3,0.3)	0.9534	0.9564	0.9480	0.9528	0.9704
			(0.3,0.5)	0.9458	0.9490	0.9461	0.9490	0.9970
			(0.5,0.5)	0.9464	0.9450	0.9539	0.9518	0.9900
		(5,10)	(0.3,0.3)	0.9542	0.9568	0.9519	0.9548	0.9862
			(0.3,0.5)	0.9488	0.9512	0.9442	0.9450	0.9956
			(0.5,0.5)	0.9480	0.9492	0.9494	0.9522	0.9898
		(10,10)	(0.3,0.3)	0.9546	0.9558	0.9466	0.9530	0.9916
			(0.3,0.5)	0.9478	0.9494	0.9484	0.9478	0.9996
			(0.5,0.5)	0.9458	0.9454	0.9439	0.9508	0.9948
		(0.5,1)	(1,1)	(0.3,0.3)	0.9552	0.9562	0.9496	0.9524
			(1,1)	(0.3,0.5)	0.9470	0.9478	0.9439	0.9546
			(1,1)	(0.5,0.5)	0.9428	0.9456	0.9468	0.9550
		(1,5)	(1,5)	(0.3,0.3)	0.9528	0.9574	0.9540	0.9564
			(1,5)	(0.3,0.5)	0.9478	0.9494	0.9455	0.9482
			(1,5)	(0.5,0.5)	0.9470	0.9520	0.9539	0.9500
		(5,5)	(5,5)	(0.3,0.3)	0.9576	0.9562	0.9422	0.9536
			(5,5)	(0.3,0.5)	0.9460	0.9490	0.9450	0.9540
			(5,5)	(0.5,0.5)	0.9458	0.9454	0.9529	0.9540
		(5,10)	(5,10)	(0.3,0.3)	0.9534	0.9564	0.9537	0.9528
			(5,10)	(0.3,0.5)	0.9458	0.9490	0.9493	0.9490
			(5,10)	(0.5,0.5)	0.9464	0.9450	0.9430	0.9518
		(10,10)	(10,10)	(0.3,0.3)	0.9578	0.9576	0.9446	0.9528
			(10,10)	(0.3,0.5)	0.9464	0.9480	0.9426	0.9548
			(10,10)	(0.5,0.5)	0.9446	0.9458	0.9531	0.9538
		(1,1)	(1,1)	(0.3,0.3)	0.9570	0.9582	0.9425	0.9532
			(1,1)	(0.3,0.5)	0.9450	0.9494	0.9421	0.9518
			(1,1)	(0.5,0.5)	0.9452	0.9474	0.9426	0.9514
		(1,5)	(1,5)	(0.3,0.3)	0.9546	0.9580	0.9439	0.9554
			(1,5)	(0.3,0.5)	0.9478	0.9492	0.9478	0.9468
			(1,5)	(0.5,0.5)	0.9454	0.9488	0.9445	0.9496
		(5,5)	(5,5)	(0.3,0.3)	0.9542	0.9554	0.9440	0.9526
			(5,5)	(0.3,0.5)	0.9444	0.9476	0.9522	0.9494
			(5,5)	(0.5,0.5)	0.9452	0.9456	0.9490	0.9536
		(5,10)	(5,10)	(0.3,0.3)	0.9534	0.9574	0.9528	0.9528
			(5,10)	(0.3,0.5)	0.9480	0.9488	0.9446	0.9462
			(5,10)	(0.5,0.5)	0.9458	0.9496	0.9453	0.9514
		(10,10)	(10,10)	(0.3,0.3)	0.9534	0.9564	0.9503	0.9528
			(10,10)	(0.3,0.5)	0.9458	0.9490	0.9465	0.9490
			(10,10)	(0.5,0.5)	0.9464	0.9450	0.9481	0.9518

TABLE 2-5 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities				
				AGCI	FCI	BCI	MOVER	NA
(100,200)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9490	0.9482	0.9469	0.9490	0.9728
			(0.3,0.5)	0.9492	0.9510	0.9433	0.9478	0.9976
			(0.5,0.5)	0.9548	0.9538	0.9516	0.9554	0.9964
		(1,5)	(0.3,0.3)	0.9470	0.9500	0.9527	0.9522	0.8806
			(0.3,0.5)	0.9460	0.9498	0.9424	0.9490	0.9764
			(0.5,0.5)	0.9540	0.9526	0.9465	0.9520	0.9648
		(5,5)	(0.3,0.3)	0.9504	0.9474	0.9456	0.9498	0.9618
			(0.3,0.5)	0.9508	0.9492	0.9442	0.9478	0.9972
			(0.5,0.5)	0.9528	0.9542	0.9492	0.9550	0.9876
		(5,10)	(0.3,0.3)	0.9462	0.9478	0.9511	0.9506	0.9770
			(0.3,0.5)	0.9478	0.9484	0.9505	0.9496	0.9984
			(0.5,0.5)	0.9538	0.9548	0.9448	0.9536	0.9904
		(10,10)	(0.3,0.3)	0.9490	0.9456	0.9516	0.9488	0.9922
			(0.3,0.5)	0.9500	0.9494	0.9454	0.9464	0.9992
			(0.5,0.5)	0.9544	0.9532	0.9431	0.9562	0.9944
	(0.5,1)	(1,1)	(0.3,0.3)	0.9484	0.9520	0.9527	0.9476	0.9974
			(0.3,0.5)	0.9486	0.9500	0.9538	0.9492	1.0000
			(0.5,0.5)	0.9530	0.9540	0.9540	0.9534	0.9998
		(1,5)	(0.3,0.3)	0.9480	0.9496	0.9485	0.9512	0.9868
			(0.3,0.5)	0.9480	0.9492	0.9450	0.9480	0.9982
			(0.5,0.5)	0.9526	0.9538	0.9534	0.9534	0.9948
		(5,5)	(0.3,0.3)	0.9476	0.9496	0.9420	0.9452	0.9994
			(0.3,0.5)	0.9480	0.9490	0.9428	0.9486	1.0000
			(0.5,0.5)	0.9552	0.9550	0.9523	0.9522	0.9992
		(5,10)	(0.3,0.3)	0.9504	0.9474	0.9484	0.9498	1.0000
			(0.3,0.5)	0.9508	0.9492	0.9510	0.9478	1.0000
			(0.5,0.5)	0.9528	0.9542	0.9477	0.9550	1.0000
		(10,10)	(0.3,0.3)	0.9486	0.9508	0.9475	0.9444	1.0000
			(0.3,0.5)	0.9484	0.9512	0.9500	0.9474	1.0000
			(0.5,0.5)	0.9562	0.9544	0.9512	0.9538	1.0000
	(1,1)	(1,1)	(0.3,0.3)	0.9476	0.9482	0.9521	0.9478	0.9922
			(0.3,0.5)	0.9486	0.9506	0.9421	0.9472	0.9990
			(0.5,0.5)	0.9528	0.9584	0.9504	0.9550	0.9990
		(1,5)	(0.3,0.3)	0.9476	0.9476	0.9464	0.9506	0.9558
			(0.3,0.5)	0.9474	0.9470	0.9431	0.9476	0.9904
			(0.5,0.5)	0.9552	0.9538	0.9495	0.9530	0.9844
		(5,5)	(0.3,0.3)	0.9492	0.9470	0.9538	0.9488	1.0000
			(0.3,0.5)	0.9490	0.9516	0.9438	0.9472	1.0000
			(0.5,0.5)	0.9540	0.9550	0.9535	0.9562	0.9998
		(5,10)	(0.3,0.3)	0.9462	0.9478	0.9528	0.9498	0.9998
			(0.3,0.5)	0.9474	0.9492	0.9436	0.9494	1.0000
			(0.5,0.5)	0.9540	0.9562	0.9459	0.9526	1.0000
		(10,10)	(0.3,0.3)	0.9504	0.9474	0.9484	0.9498	1.0000
			(0.3,0.5)	0.9508	0.9492	0.9490	0.9478	1.0000
			(0.5,0.5)	0.9528	0.9542	0.9529	0.9550	1.0000

TABLE 2-6 The AWs of the 95% confidence intervals for the ratio between two coefficients of variation of delta-IG distributions.

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths				
				AGCI	FCI	BCI	MOVER	NA
(30,30)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.5971	0.6552	0.7104	<i>0.5979</i>	0.9005
			(0.3,0.5)	<i>0.4490</i>	0.4897	0.6338	0.4511	1.1040
			(0.5,0.5)	0.6183	0.6738	0.7460	<i>0.6130</i>	1.3440
		(1,5)	(0.3,0.3)	<i>0.6602</i>	0.7159	0.8222	0.6613	0.9658
			(0.3,0.5)	<i>0.4474</i>	0.4854	0.5594	0.4502	1.0540
			(0.5,0.5)	0.6196	0.6705	0.7964	<i>0.6139</i>	1.2910
		(5,5)	(0.3,0.3)	0.2578	0.2631	0.3670	<i>0.2580</i>	1.0220
			(0.3,0.5)	<i>0.1693</i>	0.1724	0.3375	0.1701	1.1020
			(0.5,0.5)	0.2341	0.2387	0.3457	<i>0.2324</i>	1.2300
		(5,10)	(0.3,0.3)	<i>0.2286</i>	0.2328	0.4144	0.2291	1.2060
			(0.3,0.5)	<i>0.1487</i>	0.1514	0.3052	0.1498	1.1680
			(0.5,0.5)	0.2009	0.2047	0.3885	<i>0.1994</i>	1.3090
		(10,10)	(0.3,0.3)	0.1533	0.1549	0.3284	<i>0.1533</i>	1.3650
			(0.3,0.5)	<i>0.0969</i>	0.0979	0.1992	0.0974	1.3260
			(0.5,0.5)	0.1340	0.1354	0.2757	<i>0.1331</i>	1.3780
	(0.5,1)	(1,1)	(0.3,0.3)	0.5206	0.6005	0.6824	<i>0.5214</i>	1.0470
			(0.3,0.5)	<i>0.4198</i>	0.4783	0.5813	0.4215	1.2830
			(0.5,0.5)	0.5756	0.6545	0.6961	<i>0.5704</i>	1.5400
		(1,5)	(0.3,0.3)	<i>0.6477</i>	0.6993	0.8071	0.6487	1.4130
			(0.3,0.5)	<i>0.4525</i>	0.4882	0.5534	0.4549	1.3260
			(0.5,0.5)	0.6262	0.6747	0.7349	<i>0.6206</i>	1.5930
		(5,5)	(0.3,0.3)	<i>0.2908</i>	0.3002	0.4594	0.2911	1.4490
			(0.3,0.5)	<i>0.1984</i>	0.2038	0.3540	0.1991	1.3650
			(0.5,0.5)	0.2793	0.2873	0.4727	<i>0.2775</i>	1.5570
		(5,10)	(0.3,0.3)	0.2578	0.2631	0.4089	<i>0.2580</i>	2.1870
			(0.3,0.5)	<i>0.1693</i>	0.1724	0.3564	0.1701	1.7110
			(0.5,0.5)	0.2341	0.2387	0.3748	<i>0.2324</i>	1.9710
		(10,10)	(0.3,0.3)	<i>0.1926</i>	0.1958	0.3357	0.1924	2.2770
			(0.3,0.5)	<i>0.1232</i>	0.1249	0.3081	0.1235	1.8270
			(0.5,0.5)	0.1752	0.1778	0.2870	<i>0.1740</i>	2.0290
	(1,1)	(1,1)	(0.3,0.3)	0.7298	0.8713	0.8530	<i>0.7308</i>	1.1760
			(0.3,0.5)	<i>0.5859</i>	0.6897	0.7729	0.5883	1.3970
			(0.5,0.5)	0.8033	0.9459	0.9164	<i>0.7955</i>	1.7050
		(1,5)	(0.3,0.3)	<i>0.9525</i>	1.0978	1.1259	0.9540	1.5120
			(0.3,0.5)	<i>0.6682</i>	0.7677	0.8370	0.6718	1.4240
			(0.5,0.5)	0.9379	1.0771	1.1029	<i>0.9295</i>	1.7750
		(5,5)	(0.3,0.3)	0.3956	0.4111	0.5490	<i>0.3963</i>	1.7980
			(0.3,0.5)	<i>0.2736</i>	0.2837	0.4532	0.2749	1.7120
			(0.5,0.5)	0.3781	0.3924	0.5531	<i>0.3753</i>	1.8220
		(5,10)	(0.3,0.3)	<i>0.3781</i>	0.3909	0.5617	0.3790	2.4290
			(0.3,0.5)	<i>0.2529</i>	0.2616	0.3894	0.2545	2.0050
			(0.5,0.5)	0.3458	0.3578	0.4746	<i>0.3432</i>	2.1860
		(10,10)	(0.3,0.3)	0.2578	0.2631	0.4548	<i>0.2580</i>	2.9450
			(0.3,0.5)	<i>0.1693</i>	0.1724	0.3394	0.1701	2.5720
			(0.5,0.5)	0.2341	0.2387	0.4095	<i>0.2324</i>	2.5080

Note: The shortest AW values are in italic.

TABLE 2-6 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths					
				AGCI	FCI	BCI	MOVER	NA	
(50,50)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.4428	0.4895	0.8900	0.4416	0.6925	
			(0.3,0.5)	0.3305	0.3634	0.4313	0.3303	0.8477	
			(0.5,0.5)	0.4366	0.4799	0.5741	0.4371	1.0210	
		(1,5)	(0.3,0.3)	0.4850	0.5297	0.6121	0.4834	0.7279	
			(0.3,0.5)	0.3274	0.3581	0.5172	0.3271	0.7974	
			(0.5,0.5)	0.4289	0.4672	0.5759	0.4296	0.9740	
		(5,5)	(0.3,0.3)	0.1857	0.1898	0.3740	0.1852	0.7581	
			(0.3,0.5)	0.1191	0.1216	0.2903	0.1193	0.8250	
			(0.5,0.5)	0.1575	0.1608	0.3027	0.1573	0.9202	
		(5,10)	(0.3,0.3)	0.1634	0.1667	0.3420	0.1629	0.8894	
			(0.3,0.5)	0.1052	0.1072	0.2310	0.1052	0.8689	
			(0.5,0.5)	0.1333	0.1358	0.2926	0.1334	0.9758	
		(10,10)	(0.3,0.3)	0.1086	0.1099	0.2348	0.1084	0.9978	
			(0.3,0.5)	0.0669	0.0676	0.1720	0.0671	0.9818	
			(0.5,0.5)	0.0883	0.0893	0.2799	0.0882	1.0240	
		(0.5,1)	(1,1)	(0.3,0.3)	0.3871	0.4511	0.5163	0.3861	0.8023
				(0.3,0.5)	0.3113	0.3591	0.4897	0.3109	0.9880
				(0.5,0.5)	0.4115	0.4751	0.5139	0.4122	1.1740
		(1,5)	(0.3,0.3)	0.4767	0.5181	0.6648	0.4753	1.0410	
			(0.3,0.5)	0.3302	0.3591	0.4451	0.3301	0.9780	
			(0.5,0.5)	0.4348	0.4714	0.5438	0.4353	1.1870	
		(5,5)	(0.3,0.3)	0.2138	0.2215	0.4072	0.2135	1.0620	
			(0.3,0.5)	0.1418	0.1465	0.2909	0.1421	1.0050	
			(0.5,0.5)	0.1945	0.2012	0.3767	0.1940	1.1390	
		(5,10)	(0.3,0.3)	0.1857	0.1898	0.3185	0.1852	1.5830	
			(0.3,0.5)	0.1191	0.1216	0.2718	0.1193	1.2500	
			(0.5,0.5)	0.1575	0.1608	0.2799	0.1573	1.4430	
		(10,10)	(0.3,0.3)	0.1394	0.1420	0.2717	0.1393	1.6550	
			(0.3,0.5)	0.0859	0.0874	0.2037	0.0861	1.3330	
			(0.5,0.5)	0.1193	0.1215	0.2998	0.1189	1.4780	
		(1,1)	(1,1)	(0.3,0.3)	0.5424	0.6543	0.5985	0.5410	0.9034
				(0.3,0.5)	0.4347	0.5187	0.6179	0.4340	1.0670
				(0.5,0.5)	0.5725	0.6826	0.7435	0.5738	1.3130
		(1,5)	(0.3,0.3)	0.7045	0.8217	0.8543	0.7027	1.1220	
			(0.3,0.5)	0.4917	0.5742	0.6656	0.4913	1.0650	
			(0.5,0.5)	0.6584	0.7650	0.8553	0.6594	1.3240	
		(5,5)	(0.3,0.3)	0.2897	0.3023	0.4469	0.2890	1.2980	
			(0.3,0.5)	0.1968	0.2048	0.3843	0.1969	1.2510	
			(0.5,0.5)	0.2605	0.2713	0.4282	0.2603	1.3270	
		(5,10)	(0.3,0.3)	0.2743	0.2847	0.4655	0.2735	1.7550	
			(0.3,0.5)	0.1814	0.1880	0.2823	0.1814	1.4570	
			(0.5,0.5)	0.2342	0.2427	0.3594	0.2344	1.5930	
		(10,10)	(0.3,0.3)	0.1857	0.1898	0.3341	0.1852	2.1190	
			(0.3,0.5)	0.1191	0.1216	0.2875	0.1193	1.8730	
			(0.5,0.5)	0.1575	0.1608	0.3033	0.1573	1.8200	

TABLE 2-6 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths					
				AGCI	FCI	BCI	MOVER	NA	
(100,100)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.3024	0.3360	0.4718	0.3024	0.4877	
			(0.3,0.5)	0.2245	0.2488	0.3755	0.2239	0.5945	
			(0.5,0.5)	0.2933	0.3256	0.4547	0.2924	0.7172	
		(1,5)	(0.3,0.3)	0.3282	0.3604	0.4335	0.3284	0.5051	
			(0.3,0.5)	0.2214	0.2436	0.4030	0.2208	0.5556	
			(0.5,0.5)	0.2855	0.3144	0.4822	0.2842	0.6772	
		(5,5)	(0.3,0.3)	0.1239	0.1268	0.3153	0.1240	0.5206	
			(0.3,0.5)	0.0784	0.0802	0.1893	0.0783	0.5696	
			(0.5,0.5)	0.1018	0.1043	0.2993	0.1013	0.6356	
		(5,10)	(0.3,0.3)	0.1084	0.1106	0.2512	0.1085	0.6066	
			(0.3,0.5)	0.0695	0.0709	0.2171	0.0693	0.5982	
			(0.5,0.5)	0.0859	0.0878	0.2297	0.0854	0.6709	
		(10,10)	(0.3,0.3)	0.0717	0.0725	0.1931	0.0717	0.6800	
			(0.3,0.5)	0.0435	0.0440	0.2248	0.0435	0.6734	
			(0.5,0.5)	0.0563	0.0571	0.1834	0.0560	0.7032	
		(0.5,1)	(1,1)	(0.3,0.3)	0.2653	0.3112	0.3755	0.2652	0.5655
				(0.3,0.5)	0.2129	0.2487	0.3434	0.2122	0.6928
				(0.5,0.5)	0.2789	0.3266	0.4603	0.2783	0.8267
		(1,5)	(1,1)	(0.3,0.3)	0.3232	0.3530	0.4701	0.3235	0.7125
				(0.3,0.5)	0.2229	0.2438	0.3741	0.2223	0.6725
				(0.5,0.5)	0.2891	0.3165	0.4746	0.2879	0.8124
		(5,5)	(1,1)	(0.3,0.3)	0.1452	0.1508	0.3095	0.1453	0.7221
				(0.3,0.5)	0.0947	0.0982	0.2569	0.0945	0.6859
				(0.5,0.5)	0.1287	0.1337	0.2310	0.1280	0.7768
		(5,10)	(1,1)	(0.3,0.3)	0.1239	0.1268	0.3208	0.1240	1.0720
				(0.3,0.5)	0.0784	0.0802	0.2198	0.0783	0.8503
				(0.5,0.5)	0.1018	0.1043	0.2136	0.1013	0.9788
		(10,10)	(1,1)	(0.3,0.3)	0.0938	0.0956	0.2166	0.0938	1.1140
				(0.3,0.5)	0.0564	0.0574	0.1917	0.0563	0.9048
				(0.5,0.5)	0.0776	0.0793	0.2134	0.0772	1.0030
		(1,1)	(1,1)	(0.3,0.3)	0.3715	0.4509	0.5247	0.3714	0.6371
				(0.3,0.5)	0.2971	0.3591	0.4936	0.2960	0.7508
				(0.5,0.5)	0.3874	0.4693	0.5418	0.3867	0.9200
		(1,5)	(1,1)	(0.3,0.3)	0.4796	0.5645	0.6403	0.4798	0.7677
				(0.3,0.5)	0.3338	0.3946	0.4538	0.3329	0.7318
				(0.5,0.5)	0.4417	0.5232	0.5857	0.4400	0.9096
		(5,5)	(1,1)	(0.3,0.3)	0.1958	0.2047	0.3852	0.1958	0.8812
				(0.3,0.5)	0.1315	0.1374	0.2621	0.1312	0.8479
				(0.5,0.5)	0.1714	0.1794	0.2958	0.1707	0.8977
		(5,10)	(1,1)	(0.3,0.3)	0.1838	0.1911	0.3037	0.1840	1.1850
				(0.3,0.5)	0.1209	0.1257	0.3097	0.1206	0.9855
				(0.5,0.5)	0.1529	0.1592	0.2669	0.1522	1.0760
		(10,10)	(1,1)	(0.3,0.3)	0.1239	0.1268	0.2808	0.1240	1.4270
				(0.3,0.5)	0.0784	0.0802	0.2341	0.0783	1.2670
				(0.5,0.5)	0.1018	0.1043	0.2788	0.1013	1.2330

TABLE 2-6 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths				
				AGCI	FCI	BCI	MOVER	NA
(200,200)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.2103	0.2347	0.4042	0.2101	0.3440
			(0.3,0.5)	0.1554	0.1732	0.2932	0.1549	0.4186
			(0.5,0.5)	0.2017	0.2246	0.3331	0.2011	0.5037
		(1,5)	(0.3,0.3)	0.2273	0.2508	0.3933	0.2273	0.3540
			(0.3,0.5)	0.1529	0.1693	0.3402	0.1524	0.3900
			(0.5,0.5)	0.1951	0.2155	0.3450	0.1944	0.4757
		(5,5)	(0.3,0.3)	0.0852	0.0872	0.2225	0.0852	0.3635
			(0.3,0.5)	0.0534	0.0547	0.2491	0.0532	0.3987
			(0.5,0.5)	0.0686	0.0702	0.2203	0.0684	0.4444
		(5,10)	(0.3,0.3)	0.0743	0.0759	0.1891	0.0743	0.4217
			(0.3,0.5)	0.0474	0.0485	0.1754	0.0473	0.4177
			(0.5,0.5)	0.0578	0.0590	0.2563	0.0576	0.4683
		(10,10)	(0.3,0.3)	0.0490	0.0496	0.1975	0.0490	0.4721
			(0.3,0.5)	0.0295	0.0298	0.1895	0.0294	0.4688
			(0.5,0.5)	0.0377	0.0381	0.1760	0.0375	0.4907
	(0.5,1)	(1,1)	(0.3,0.3)	0.1848	0.2179	0.3073	0.1845	0.3998
			(0.3,0.5)	0.1478	0.1741	0.2524	0.1474	0.4894
			(0.5,0.5)	0.1929	0.2274	0.3437	0.1924	0.5815
		(1,5)	(0.3,0.3)	0.2241	0.2457	0.3944	0.2240	0.4944
			(0.3,0.5)	0.1538	0.1692	0.3445	0.1533	0.4691
			(0.5,0.5)	0.1975	0.2168	0.3187	0.1968	0.5661
		(5,5)	(0.3,0.3)	0.1007	0.1048	0.2359	0.1007	0.5016
			(0.3,0.5)	0.0649	0.0674	0.2447	0.0647	0.4761
			(0.5,0.5)	0.0879	0.0914	0.2070	0.0876	0.5408
		(5,10)	(0.3,0.3)	0.0852	0.0872	0.2810	0.0852	0.7435
			(0.3,0.5)	0.0534	0.0547	0.2080	0.0532	0.5890
			(0.5,0.5)	0.0686	0.0702	0.1781	0.0684	0.6787
		(10,10)	(0.3,0.3)	0.0647	0.0660	0.2368	0.0646	0.7745
			(0.3,0.5)	0.0383	0.0390	0.1870	0.0382	0.6265
			(0.5,0.5)	0.0526	0.0537	0.1632	0.0524	0.6949
	(1,1)	(1,1)	(0.3,0.3)	0.2586	0.3158	0.4009	0.2583	0.4490
			(0.3,0.5)	0.2063	0.2514	0.3922	0.2057	0.5306
			(0.5,0.5)	0.2677	0.3255	0.3905	0.2670	0.6482
		(1,5)	(0.3,0.3)	0.3331	0.3948	0.5173	0.3329	0.5350
			(0.3,0.5)	0.2310	0.2755	0.3818	0.2304	0.5121
			(0.5,0.5)	0.3032	0.3606	0.4753	0.3023	0.6351
		(5,5)	(0.3,0.3)	0.1354	0.1419	0.2603	0.1353	0.6091
			(0.3,0.5)	0.0902	0.0945	0.2313	0.0899	0.5879
			(0.5,0.5)	0.1166	0.1221	0.2236	0.1162	0.6217
		(5,10)	(0.3,0.3)	0.1266	0.1319	0.3181	0.1266	0.8212
			(0.3,0.5)	0.0829	0.0865	0.1831	0.0826	0.6819
			(0.5,0.5)	0.1035	0.1078	0.2113	0.1031	0.7448
		(10,10)	(0.3,0.3)	0.0852	0.0872	0.2758	0.0852	0.9864
			(0.3,0.5)	0.0534	0.0547	0.2376	0.0532	0.8762
			(0.5,0.5)	0.0686	0.0702	0.1919	0.0684	0.8514

TABLE 2-6 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths				
				AGCI	FCI	BCI	MOVER	NA
(30,50)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.5385	0.5919	0.6830	0.5355	0.8437
			(0.3,0.5)	0.4014	0.4402	0.5172	0.4039	0.9776
			(0.5,0.5)	0.5549	0.6044	0.6956	0.5484	1.2320
		(1,5)	(0.3,0.3)	0.6417	0.6982	0.7549	0.6372	0.8883
			(0.3,0.5)	0.4338	0.4729	0.6014	0.4376	0.9213
			(0.5,0.5)	0.6018	0.6494	0.7783	0.5934	1.1750
		(5,5)	(0.3,0.3)	0.2287	0.2335	0.3315	0.2270	0.9486
			(0.3,0.5)	0.1493	0.1525	0.3041	0.1504	0.9801
			(0.5,0.5)	0.2034	0.2074	0.3933	0.2013	1.1130
		(5,10)	(0.3,0.3)	0.2142	0.2182	0.4017	0.2123	1.1190
			(0.3,0.5)	0.1392	0.1420	0.3189	0.1407	1.0380
			(0.5,0.5)	0.1855	0.1887	0.3631	0.1831	1.1870
		(10,10)	(0.3,0.3)	0.1345	0.1360	0.2673	0.1334	1.2880
			(0.3,0.5)	0.0847	0.0857	0.2333	0.0854	1.2150
			(0.5,0.5)	0.1147	0.1158	0.2681	0.1136	1.2640
		(0.5,1)	(1,1)	0.4575	0.5246	0.6011	0.4552	0.9902
			(0.3,0.5)	0.3646	0.4161	0.5534	0.3665	1.1580
			(0.5,0.5)	0.5019	0.5686	0.6518	0.4964	1.4360
		(1,5)	(0.3,0.3)	0.6094	0.6611	0.7975	0.6055	1.3110
			(0.3,0.5)	0.4239	0.4606	0.5668	0.4272	1.1750
			(0.5,0.5)	0.5882	0.6333	0.7417	0.5804	1.4690
		(5,5)	(0.3,0.3)	0.2471	0.2550	0.4290	0.2456	1.3600
			(0.3,0.5)	0.1666	0.1717	0.3417	0.1674	1.2240
			(0.5,0.5)	0.2316	0.2386	0.3850	0.2298	1.4120
		(5,10)	(0.3,0.3)	0.2287	0.2335	0.3658	0.2270	2.0300
			(0.3,0.5)	0.1493	0.1525	0.2699	0.1504	1.5560
			(0.5,0.5)	0.2034	0.2074	0.3180	0.2013	1.8240
		(10,10)	(0.3,0.3)	0.1598	0.1625	0.3247	0.1587	2.1440
			(0.3,0.5)	0.1008	0.1024	0.2806	0.1013	1.6850
			(0.5,0.5)	0.1409	0.1431	0.2544	0.1401	1.8700
		(1,1)	(0.3,0.3)	0.6567	0.7828	0.7829	0.6530	1.1240
			(0.3,0.5)	0.5212	0.6179	0.6545	0.5243	1.2760
			(0.5,0.5)	0.7213	0.8461	0.9125	0.7131	1.6220
		(1,5)	(0.3,0.3)	0.9132	1.0593	1.1125	0.9069	1.4200
			(0.3,0.5)	0.6387	0.7420	0.7661	0.6437	1.2920
			(0.5,0.5)	0.9015	1.0328	1.0875	0.8901	1.6430
		(5,5)	(0.3,0.3)	0.3548	0.3694	0.5010	0.3524	1.7040
			(0.3,0.5)	0.2436	0.2535	0.3908	0.2454	1.6040
			(0.5,0.5)	0.3342	0.3467	0.5295	0.3307	1.7000
		(5,10)	(0.3,0.3)	0.3539	0.3669	0.4841	0.3512	2.3000
			(0.3,0.5)	0.2361	0.2450	0.4144	0.2383	1.8690
			(0.5,0.5)	0.3203	0.3309	0.4417	0.3164	2.0500
		(10,10)	(0.3,0.3)	0.2287	0.2335	0.4213	0.2270	2.8080
			(0.3,0.5)	0.1493	0.1525	0.3093	0.1504	2.4650
			(0.5,0.5)	0.2034	0.2074	0.4007	0.2013	2.3880

TABLE 2-6 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths				
				AGCI	FCI	BCI	MOVER	NA
(50,100)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.3838	0.4259	0.5269	0.3831	0.6390
			(0.3,0.5)	0.2868	0.3183	0.4071	0.2854	0.7290
			(0.5,0.5)	0.3803	0.4189	0.4901	0.3792	0.9311
		(1,5)	(0.3,0.3)	0.4647	0.5117	0.6623	0.4634	0.6625
			(0.3,0.5)	0.3167	0.3499	0.4583	0.3153	0.6793
			(0.5,0.5)	0.4173	0.4562	0.5579	0.4156	0.8774
		(5,5)	(0.3,0.3)	0.1588	0.1625	0.3539	0.1584	0.6972
			(0.3,0.5)	0.1037	0.1062	0.2646	0.1033	0.7119
			(0.5,0.5)	0.1338	0.1367	0.2927	0.1333	0.8162
		(5,10)	(0.3,0.3)	0.1503	0.1536	0.3139	0.1499	0.8233
			(0.3,0.5)	0.0986	0.1008	0.2081	0.0981	0.7566
			(0.5,0.5)	0.1232	0.1255	0.2600	0.1225	0.8707
		(10,10)	(0.3,0.3)	0.0922	0.0933	0.2889	0.0919	0.9424
			(0.3,0.5)	0.0582	0.0589	0.2444	0.0580	0.8823
			(0.5,0.5)	0.0743	0.0751	0.1965	0.0740	0.9270
		(0.5,1)	(1,1)	0.3243	0.3746	0.5166	0.3240	0.7556
			(0.3,0.5)	0.2590	0.2988	0.3856	0.2577	0.8678
			(0.5,0.5)	0.3437	0.3942	0.4529	0.3428	1.0770
		(1,5)	(0.3,0.3)	0.4381	0.4809	0.5557	0.4370	0.9726
			(0.3,0.5)	0.3066	0.3375	0.5053	0.3052	0.8637
			(0.5,0.5)	0.4051	0.4417	0.5864	0.4036	1.0840
		(5,5)	(0.3,0.3)	0.1713	0.1773	0.3438	0.1710	0.9939
			(0.3,0.5)	0.1146	0.1184	0.2655	0.1142	0.8899
			(0.5,0.5)	0.1533	0.1584	0.3099	0.1528	1.0330
		(5,10)	(0.3,0.3)	0.1588	0.1625	0.2947	0.1584	1.5010
			(0.3,0.5)	0.1037	0.1062	0.2634	0.1033	1.1420
			(0.5,0.5)	0.1338	0.1367	0.3094	0.1333	1.3350
		(10,10)	(0.3,0.3)	0.1092	0.1112	0.2332	0.1089	1.5730
			(0.3,0.5)	0.0681	0.0693	0.2223	0.0679	1.2270
			(0.5,0.5)	0.0916	0.0932	0.2477	0.0912	1.3750
		(1,1)	(0.3,0.3)	0.4697	0.5672	0.6070	0.4690	0.8613
			(0.3,0.5)	0.3739	0.4522	0.5563	0.3721	0.9592
			(0.5,0.5)	0.4999	0.5967	0.6489	0.4985	1.2230
		(1,5)	(0.3,0.3)	0.6631	0.7840	0.8601	0.6612	1.0560
			(0.3,0.5)	0.4668	0.5559	0.6630	0.4648	0.9560
			(0.5,0.5)	0.6319	0.7406	0.7370	0.6295	1.2290
		(5,5)	(0.3,0.3)	0.2498	0.2611	0.3605	0.2492	1.2480
			(0.3,0.5)	0.1716	0.1795	0.3243	0.1709	1.1600
			(0.5,0.5)	0.2241	0.2334	0.3370	0.2232	1.2330
		(5,10)	(0.3,0.3)	0.2511	0.2615	0.4277	0.2505	1.6760
			(0.3,0.5)	0.1687	0.1760	0.3570	0.1679	1.3630
			(0.5,0.5)	0.2159	0.2240	0.3353	0.2150	1.4970
		(10,10)	(0.3,0.3)	0.1588	0.1625	0.2617	0.1584	2.0530
			(0.3,0.5)	0.1037	0.1062	0.2504	0.1033	1.8010
			(0.5,0.5)	0.1338	0.1367	0.3088	0.1333	1.7330

TABLE 2-6 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths				
				AGCI	FCI	BCI	MOVER	NA
(100,200)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.2624	0.2920	0.3789	0.2619	0.4509
			(0.3,0.5)	0.1949	0.2168	0.3247	0.1952	0.5132
			(0.5,0.5)	0.2537	0.2820	0.4344	0.2526	0.6524
		(1,5)	(0.3,0.3)	0.3161	0.3493	0.4487	0.3157	0.4635
			(0.3,0.5)	0.2147	0.2377	0.4098	0.2152	0.4757
			(0.5,0.5)	0.2765	0.3056	0.4066	0.2750	0.6133
		(5,5)	(0.3,0.3)	0.1067	0.1093	0.2460	0.1066	0.4812
			(0.3,0.5)	0.0691	0.0708	0.2113	0.0693	0.4925
			(0.5,0.5)	0.0868	0.0889	0.1869	0.0863	0.5659
		(5,10)	(0.3,0.3)	0.1007	0.1030	0.2908	0.1007	0.5668
			(0.3,0.5)	0.0658	0.0672	0.2443	0.0659	0.5243
			(0.5,0.5)	0.0796	0.0814	0.2587	0.0791	0.6043
		(10,10)	(0.3,0.3)	0.0614	0.0622	0.2011	0.0614	0.6450
			(0.3,0.5)	0.0385	0.0389	0.2243	0.0386	0.6076
			(0.5,0.5)	0.0478	0.0483	0.2434	0.0475	0.6408
		(0.5,1)	(1,1)	0.2223	0.2576	0.4217	0.2219	0.5335
			(0.3,0.5)	0.1766	0.2046	0.2830	0.1769	0.6127
			(0.5,0.5)	0.2308	0.2677	0.3733	0.2299	0.7580
		(1,5)	(0.3,0.3)	0.2983	0.3285	0.4734	0.2980	0.6761
			(0.3,0.5)	0.2078	0.2292	0.3218	0.2082	0.6011
			(0.5,0.5)	0.2686	0.2958	0.3976	0.2673	0.7505
		(5,5)	(0.3,0.3)	0.1163	0.1206	0.2612	0.1163	0.6876
			(0.3,0.5)	0.0769	0.0796	0.2398	0.0770	0.6138
			(0.5,0.5)	0.1012	0.1050	0.2423	0.1008	0.7140
		(5,10)	(0.3,0.3)	0.1067	0.1093	0.2229	0.1066	1.0380
			(0.3,0.5)	0.0691	0.0708	0.2093	0.0693	0.7862
			(0.5,0.5)	0.0868	0.0889	0.2679	0.0863	0.9210
		(10,10)	(0.3,0.3)	0.0736	0.0750	0.2001	0.0736	1.0820
			(0.3,0.5)	0.0453	0.0461	0.2330	0.0453	0.8439
			(0.5,0.5)	0.0599	0.0611	0.1856	0.0597	0.9457
		(1,1)	(0.3,0.3)	0.3221	0.3919	0.4679	0.3215	0.6060
			(0.3,0.5)	0.2551	0.3097	0.3931	0.2555	0.6774
			(0.5,0.5)	0.3358	0.4069	0.5039	0.3345	0.8606
		(1,5)	(0.3,0.3)	0.4529	0.5410	0.6019	0.4524	0.7326
			(0.3,0.5)	0.3176	0.3802	0.5009	0.3182	0.6661
			(0.5,0.5)	0.4219	0.5036	0.5221	0.4200	0.8531
		(5,5)	(0.3,0.3)	0.1693	0.1773	0.2741	0.1691	0.8508
			(0.3,0.5)	0.1154	0.1208	0.2770	0.1156	0.7926
			(0.5,0.5)	0.1471	0.1540	0.2778	0.1464	0.8417
		(5,10)	(0.3,0.3)	0.1695	0.1769	0.3416	0.1694	1.1520
			(0.3,0.5)	0.1133	0.1183	0.2465	0.1136	0.9315
			(0.5,0.5)	0.1410	0.1471	0.3139	0.1403	1.0250
		(10,10)	(0.3,0.3)	0.1067	0.1093	0.2230	0.1066	1.3980
			(0.3,0.5)	0.0691	0.0708	0.1908	0.0693	1.2230
			(0.5,0.5)	0.0868	0.0889	0.1902	0.0863	1.1840

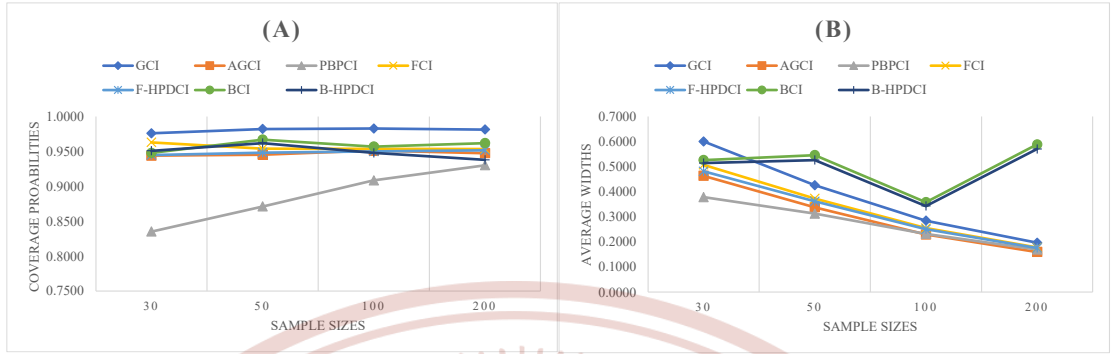


FIGURE 2-1 Comparison of the CPs (A) and AWs (B) for estimating the 95% confidence interval for the single coefficient of variation of a delta-IG distribution with various sample sizes at $(\mu, \lambda, \delta) = (0.5, 1.0, 0.3)$.

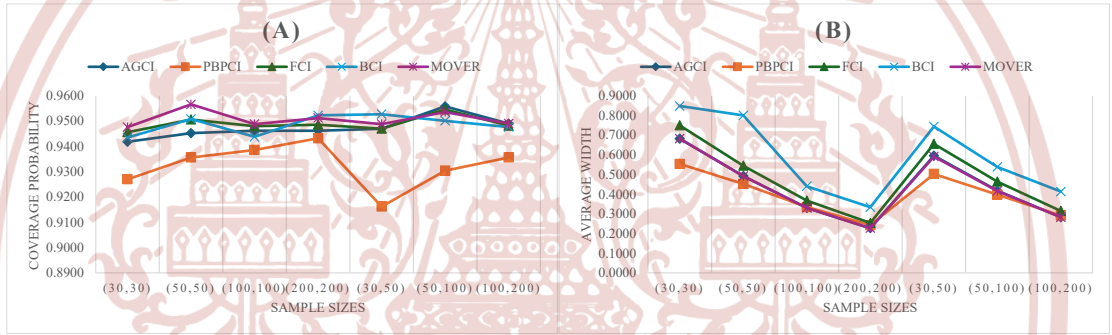


FIGURE 2-2 Comparison of the CPs (A) and AWs (B) for estimating the 95% confidence interval for the difference of two coefficients of variation of delta-IG distributions with various sample sizes at $(\mu_1, \mu_2, \lambda_1, \lambda_2, \delta_1, \delta_2) = (0.5, 0.5, 1.0, 1.0, 0.3, 0.3)$.

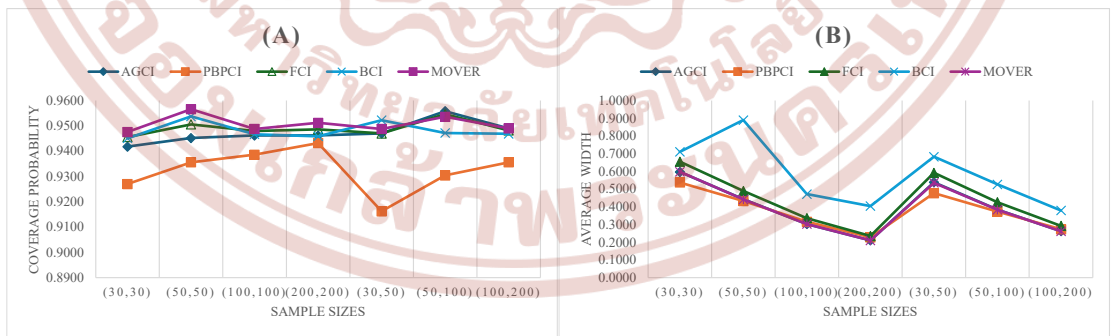


FIGURE 2-3 Comparison of the CPs (A) and AWs (B) for estimating the 95% confidence interval for the ratio of two coefficients of variation of delta-IG distributions with various sample sizes at $(\mu_1, \mu_2, \lambda_1, \lambda_2, \delta_1, \delta_2) = (0.5, 0.5, 1.0, 1.0, 0.3, 0.3)$.

2.5 An empirical application

To assess the proposed methods for estimating the confidence interval for a single coefficient of variation, the difference, and the ratio of coefficients of variation for the delta-IG distributions, traffic fatality data are employed to evaluate their performance.

2.5.1 Application of the single coefficient of variation of a delta-IG distribution

To demonstrate the effectiveness of the confidence intervals for the coefficient of variation of a delta-IG distribution constructed using GCI, AGCI, PBPCI, FCI, F-HPDCI, BCI, and B-HPDCI, traffic fatality data are analyzed. The dataset is obtained from a statistical report on road accident casualties published by the Road Accident Victims Protection Company Limited and is available on the Thai Road Safety Center (<https://www.thairsc.com/bangkok-in-depth-statistics>). It consists of death counts from traffic accidents recorded in December 2024 across 232 districts in the central region of Thailand. The aim is to evaluate the performance of the proposed confidence intervals for the coefficient of variation of the delta-inverse Gaussian distribution based on real data. Table 2-7 shows that among the 232 observations, there are 169 observations were greater than zero. To compare traffic fatality estimates in the central region of Thailand, the normal Q–Q plot and histogram of the death count data are presented in Figure 2-4. By partitioning the death count data into zero and non-zero observations and focusing on the positive values (Maneerat et al., 2021), the most appropriate distribution for the positive component was identified. To evaluate the underlying distribution of the data, the positive death count datasets were assessed using the Akaike information criterion (AIC) and the Bayesian information criterion (BIC). The results, computed based on Equations (2-68) and (2-69), are presented in Table 2-8. Based on the AIC and BIC values, the inverse Gaussian (IG) distribution provided the best fit to the positive death count data.

$$AIC = -2 \ln(L) + 2m, \quad (2-74)$$

and

$$BIC = -2 \ln(L) + m \ln(n), \quad (2-75)$$

where L is the likelihood function, m is the number of parameters, and n is the amount of data.

The descriptive statistics for the death count data from the central region of Thailand are as follows: approximate coefficient of variation is $\hat{\theta} = 1.0700$, with parameters $\hat{\mu} = 2.6900$, $\hat{\lambda} = 4.8300$, and $\hat{\delta} = 0.2716$.

Table 2-9 presents the confidence intervals for the coefficient of variation of the death count data in the central region of Thailand using the considered methods. The results show that PBPCI produced the narrowest interval width, followed by AGCI, F-HPDCI, FCI, GCI, B-HPDCI, and BCI. However, when CP was considered from the simulation results, it was observed that the CP values of the PBPCI were below the nominal confidence level. Overall, AGCI and PBPCI can be regarded as the most effective methods for this dataset.

TABLE 2-7 The traffic death count data from the Central region of Thailand in December 2024.

Death count from each district of the Central region of Thailand in December 2024																													
5	1	2	1	4	2	2	1	2	2	0	0	1	3	0	4	5	0	1	1	0	4	5	2	1	3	1	2	0	0
0	0	2	6	2	0	1	4	1	1	0	0	0	0	0	5	2	4	0	0	0	2	4	3	9	2	9	5	5	6
6	1	5	3	9	6	5	4	7	0	2	1	0	0	0	0	2	2	8	0	0	2	2	1	1	0	1	1	2	0
1	2	3	1	3	2	2	5	9	1	0	1	1	1	2	1	4	5	1	4	0	0	1	2	0	1	0	2	0	1
6	0	2	3	1	0	1	3	1	4	1	0	1	1	0	2	2	0	1	0	0	0	0	4	1	3	3	3	0	2
2	1	1	2	2	4	4	3	2	2	1	6	0	0	1	0	0	2	0	5	1	0	2	2	1	0	4	3	1	3
0	2	2	2	2	4	2	0	1	6	1	0	2	1	0	0	4	1	1	1	0	1	3	0	0	2	1	1	2	2
2	9	1	1	1	2	4	1	0	3	2	1	0	0	0	0	3	2	1	1	10	12								

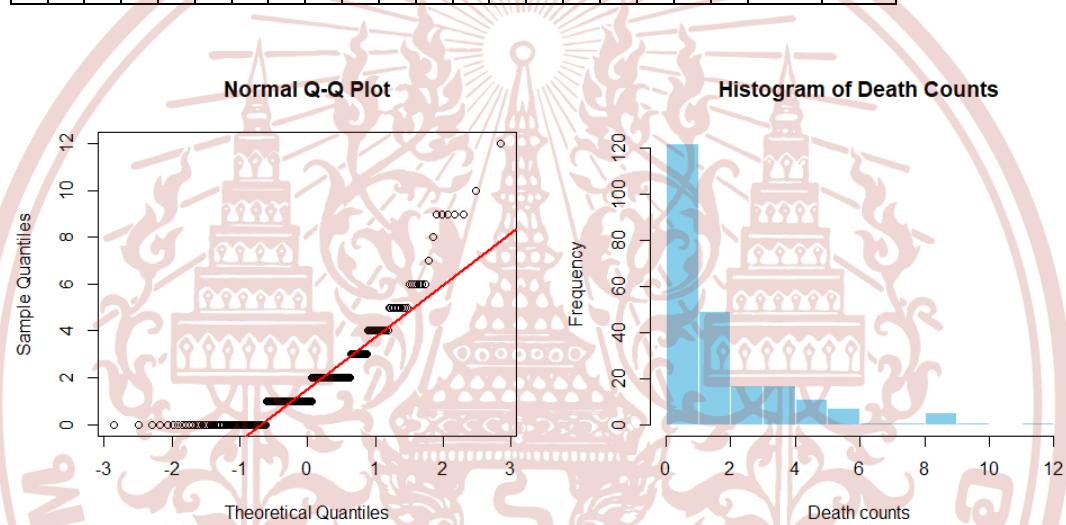


FIGURE 2-4 (left) Normal Q-Q plot and (right) Histogram of death count data from Central region.

TABLE 2-8 AIC and BIC values of positive observation of death count data for the Central region data set.

Distributions	Criterion	
	AIC	BIC
Normal	734.0833	740.3431
Lognormal	603.4880	609.7478
Weibull	642.3002	648.5600
Gamma	627.5243	633.7841
Exponential	674.7548	677.8847
Logistic	709.4140	715.6738
Cauchy	704.9006	711.1604
Inverse Gaussian	596.3504	602.6102

TABLE 2-9 Confidence intervals for the coefficient of variation of death count data for the Central region data set.

Methods	CIs for CV (θ)	
	Interval	Width
GCI	0.9869–1.1772	0.1902
AGCI	1.0016–1.1557	0.1541
PBPCI	0.9961–1.1353	0.1392
FCI	0.9904–1.1687	0.1784
F-HPDCI	0.9860–1.1634	0.1774
BCI	0.7539–1.2226	0.4687
B-HPDCI	0.7583–1.2024	0.4441

2.5.2 Application of the difference and ratio of coefficients of variation in delta-IG distributions

To illustrate the effectiveness of confidence intervals for the difference and ratio of two coefficients of variation under the delta-inverse Gaussian distribution, methods including AGCI, FCI, BCI, MOVER, and NA are applied to traffic fatality data. The dataset is sourced from a statistical report on road accident casualties published by the Road Accident Victims Protection Company Limited and is available via the Thai Road Safety Center (<https://www.thairsc.com/bangkok-in-depth-statistics>). It comprises death counts from traffic accidents recorded in December 2024 and December 2023 across 232 districts in the central region of Thailand which shown in Table 2-10. The normal Q–Q plots and histograms of the death count data are shown in Figures 2-5 and 2-6 for the December 2024 and December 2023 datasets, respectively. After separating the data into zero and non-zero observations and focusing on the positive values, the most suitable distribution for the positive component was identified. To evaluate the underlying distribution of the data, the positive death count datasets were assessed using AIC and BIC. Based on the AIC and BIC values reported in Table 2-11, the inverse Gaussian (IG) distribution provided the best fit. The descriptive statistics for the death count data from the central region of Thailand are as follows: estimated coefficients of variation are $\hat{\theta}_1 = 1.0671$ for December 2024 and $\hat{\theta}_2 = 1.2063$ for December 2023. The corresponding parameter estimates are $(\hat{\mu}_1, \hat{\mu}_2) = (2.6900, 2.8600)$, $(\hat{\lambda}_1, \hat{\lambda}_2) = (4.8300, 4.2600)$ and $(\hat{\delta}_1, \hat{\delta}_2) = (0.2716, 0.3190)$. Table 2-12 reports the confidence intervals for the difference and ratio of the two coefficients of variation using the methods under consideration. The results indicate that AGCI produced the narrowest interval width, followed by MOVER, FCI, BCI, and NA. Combining the evidence from both the simulation study and the real-data application, AGCI and MOVER are identified as the most appropriate methods for constructing confidence intervals for the difference and ratio of coefficients of variation in delta-IG populations.

TABLE 2-10 The traffic death count data from the Central region of Thailand in December 2024 and December 2023.

Death count from each district of the Central region of Thailand in December 2024																								
5	1	2	1	4	2	2	1	2	2	0	0	1	3	0	4	5	0	1	1	0	4	5	2	1
0	0	2	6	2	0	1	4	1	1	0	0	0	0	0	5	2	4	0	0	0	2	4	3	9
6	1	5	3	9	6	5	4	7	0	2	1	0	0	0	0	2	2	8	0	0	2	2	1	1
1	2	3	1	3	2	2	5	9	1	0	1	1	1	2	1	4	5	1	4	0	0	1	2	0
6	0	2	3	1	0	1	3	1	4	1	0	1	1	0	2	2	0	1	0	0	0	0	4	1
2	1	1	2	2	4	4	3	2	2	1	6	0	0	1	0	0	2	0	5	1	0	2	2	1
0	2	2	2	2	4	2	0	1	6	1	0	2	1	0	0	4	1	1	1	0	1	3	0	0
2	9	1	1	1	2	4	1	0	3	2	1	0	0	0	0	3	2	1	1	10	12			
Death count from each district of the Central region of Thailand in December 2023																								
0	1	2	1	5	2	1	1	2	1	3	2	1	2	1	0	2	3	1	9	1	4	0	0	0
0	0	2	5	3	2	1	5	9	4	0	1	0	0	0	2	4	3	0	0	3	4	4	2	2
2	2	1	1	1	2	3	0	4	1	1	0	0	0	2	1	2	1	2	0	0	2	0	2	0
1	0	2	1	2	0	4	4	4	1	0	0	0	2	1	0	0	1	2	3	0	0	0	0	0
0	0	1	1	1	0	0	2	0	0	0	2	2	1	0	2	1	2	0	0	0	0	1	2	4
0	2	1	0	1	5	0	2	7	4	1	3	0	1	7	0	1	0	3	6	0	0	0	2	2
2	0	0	3	1	6	0	1	0	6	0	0	1	0	0	0	3	2	3	0	1	1	4	0	2
0	4	1	4	1	2	1	0	2	2	3	3	1	3	0	5	14	18	10	10	10	10	10	10	10

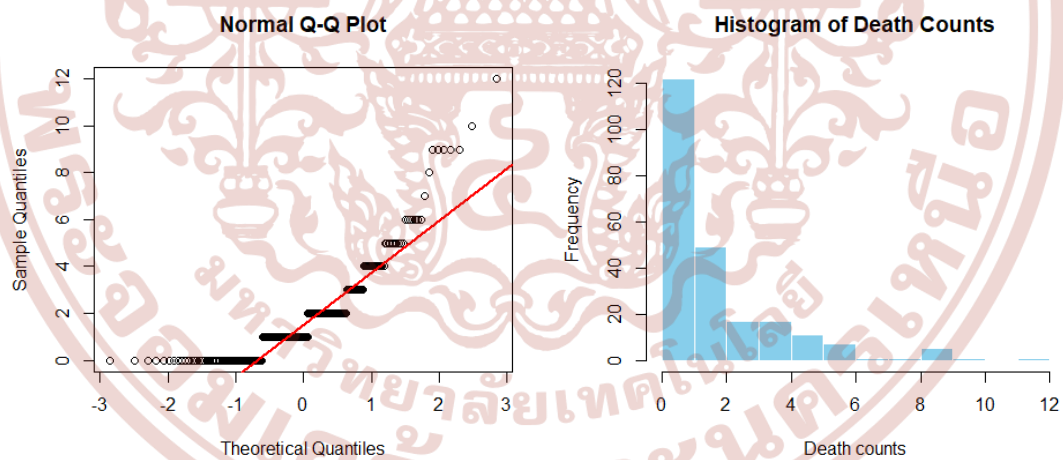


FIGURE 2-5 (left) Normal Q-Q plot and (right) Histogram of death count data from Central region in December 2024.

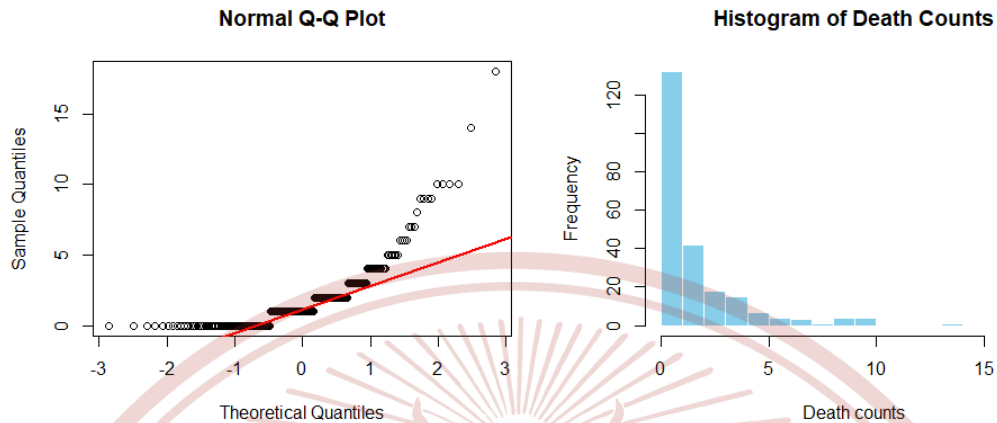


FIGURE 2-6 (left) Normal Q-Q plot and (right) Histogram of death count data from Central region in December 2023.

TABLE 2-11 AIC and BIC values of positive observation of death count data for the Central region data set.

Distributions	December 2024		December 2023	
	Criterion		Criterion	
	AIC	BIC	AIC	BIC
Normal	734.0833	740.3431	760.2574	766.3825
Lognormal	603.4880	609.7478	589.3224	595.4475
Weibull	642.3002	648.5600	636.0289	642.1540
Gamma	627.5243	633.7841	622.5030	628.6282
Exponential	674.7548	677.8847	650.1435	653.2061
Logistic	709.4140	715.6738	715.6138	721.7390
Cauchy	704.9006	711.1604	679.8915	686.0167
Inverse Gaussian	596.3504	602.6102	582.1770	588.3022

TABLE 2-12 Confidence intervals for the difference and ratio of two coefficients of variation of death count data for the Central region data set between December 2024 and December 2023.

Methods	$\hat{\theta}_{diff} = \hat{\theta}_1 - \hat{\theta}_2 = -0.1392$		Methods	$\hat{\theta}_{ratio} = \hat{\theta}_1 / \hat{\theta}_2 = 0.8846$	
	Interval	Width		Interval	Width
AGCI	(-0.4608, 0.0835)	0.5443	AGCI	(0.6721, 1.0758)	0.4037
FCI	(-0.4256, 0.1232)	0.5488	FCI	(0.6841, 1.1173)	0.4333
BCI	(-0.5212, 0.1269)	0.6481	BCI	(0.5789, 1.1673)	0.5884
MOVER	(-0.4516, 0.0947)	0.5463	MOVER	(0.6773, 1.0844)	0.4070
NA	(-0.7531, 0.4747)	1.2278	NA	(0.2707, 1.4985)	1.2278

2.6 Conclusions

This study investigated several methods for constructing confidence intervals for the CV, as well as for the difference and ratio of CVs under the delta-inverse Gaussian distribution, using both simulation and real-data analyses.

For a single CV, the simulation results showed that AGCI, FCI, and F-HPDCI consistently achieved coverage probabilities close to the nominal level. Among these methods, AGCI performed particularly well in small-sample settings, while FCI and F-HPDCI became more competitive as the sample size increased. Although PBPCI produced the narrowest intervals, it often suffered from undercoverage, indicating a lack of reliability. Bayesian methods (BCI and B-HPDCI) generally resulted in wider intervals. Overall, AGCI and F-HPDCI provided the best balance between coverage accuracy and interval width, with AGCI offering the additional advantage of lower computational cost.

For the difference and ratio of two CVs, both AGCI and MOVER demonstrated strong and consistent performance across a wide range of scenarios. Their effectiveness depended on the underlying parameter configurations: MOVER tended to produce narrower intervals when the zero-inflation probabilities were equal or when sample sizes were large or unbalanced, whereas AGCI performed better when the zero-inflation probabilities differed. In general, these two methods outperformed the alternatives in terms of both coverage and efficiency.

The real-data application using traffic fatality data supported the simulation findings. For a single CV, PBPCI yielded the narrowest intervals, but its tendency toward undercoverage limits its practical use, reinforcing AGCI as a more reliable choice. For the difference and ratio of CVs, AGCI produced the narrowest intervals, followed closely by MOVER, confirming their suitability in applied settings.

In Chapter 2 focuses on the construction of confidence intervals for different forms of the CV in the delta-IG distribution, including the single CV, as well as the difference and ratio of CVs. Extending this framework, Chapter 3 addresses the development of confidence intervals for the variance of the delta-IG distribution, covering the variance of a single population, along with the difference and ratio of variances. Variance is a fundamental measure of dispersion, reflecting how data vary around the mean and providing insight into the variability and stability of traffic fatality observations. Constructing confidence intervals for variance allows for a more comprehensive assessment of the uncertainty associated with such variability.

TABLE 2-13 Summary of the chapter.

Aspect	Key Finding	Recommended Method
Single CV	Reliable coverage achieved by several methods	AGCI (best overall)
	Good performance improves with larger samples	FCI, F-HPDCI
	Narrowest intervals but undercoverage issue	PBPCI (not recommended)
Difference and Ratio of CVs	Consistent and accurate across scenarios	AGCI, MOVER

CHAPTER 3

CONFIDENCE INTERVALS FOR A SINGLE VARIANCE, THE DIFFERENCE BETWEEN VARIANCES, AND THE RATIO BETWEEN VARIANCES OF THE DELTA-INVERSE GAUSSIAN DISTRIBUTIONS

This chapter introduces methods for constructing confidence intervals for the variance of the delta-IG distribution, including the variance of a single population, as well as the difference and ratio of variances. These approaches are suitable for data consisting of positive values following an inverse Gaussian distribution and zero values governed by a binomial distribution. For a single variance, the methods considered include adjusted generalized confidence intervals (AGCI), parametric bootstrap percentile confidence intervals (PBPCI), fiducial confidence intervals (FCI), and Bayesian credible intervals (BCI). In particular, AGCI, PBPCI, and FCI incorporate estimation of the zero proportion using variance-stabilizing transformation (VST), Wilson's score, and Hannig's methods. The performance of these approaches is illustrated using traffic fatality data from Bangkok. For the difference and ratio of variances, the methods AGCI, FCI, BCI, MOVER, and normal approximation (NA) are applied. These methods are evaluated using traffic fatality data from the northern and southern regions of Thailand.

3.1 Introduction

Estimating population parameters is a fundamental task in statistical inference, where point estimation and interval estimation play complementary roles. While point estimates provide single-value summaries, confidence intervals (CIs) quantify sampling uncertainty and are therefore essential for reliable statistical conclusions. In many practical situations, the performance of classical CI procedures based on normal approximations or large-sample theory can be unsatisfactory, particularly when the underlying data are highly skewed, heavy-tailed, or exhibit a substantial proportion of exact zeros. Developing CI methods with strong finite-sample properties for such data structures remains an important challenge in modern statistical inference.

Zero-inflated and semi-continuous data arise naturally in a wide range of applied fields, including economics, meteorology, biology, medicine, insurance claim modeling, environmental and hydrological studies, reliability and survival analysis, epidemiology, and transportation-related research. In these settings, observations often consist of a mixture of a point mass at zero and a continuous distribution for positive values. For instance, in traffic accident statistics, counts may be zero if no accident occurs, or they may reflect the number of accidents recorded. Research on zero-inflated models for several distributions has established confidence intervals addressing the needs of zero-inflated data, including the delta-lognormal (Zhou & Tu, 2000; Zhang et al., 2022) and delta-gamma (Guo et al., 2024) distributions.

Among skewed continuous distributions, the inverse Gaussian (IG) distribution has received considerable attention due to its analytical tractability and flexibility in capturing varying degrees of skewness. Chhikara and Folks (1977) noted that it offers

certain advantages over more complex distributions such as the lognormal distribution, being both more adaptable and computationally more straightforward. Chatzidiamantis et al. (2011) highlighted its effectiveness for modeling wind speed data and the benefits of parameter estimation, while Bardsley (1980) found it less complex than the three-parameter Weibull distribution.

The zero-inflated extension of the IG distribution, known as the delta-IG distribution, combines a point mass at zero with an inverse Gaussian component for positive values. This framework is suitable for modeling zero-inflated, right-skewed outcomes such as accident counts or waiting times, and has been applied to insurance claim estimation, precipitation levels, COVID-19 mortality rates, and accident frequencies (Chhikara & Folks, 1989). Research on confidence intervals for parameters of the delta-IG distribution has grown progressively. Jana and Gautam (2024) investigated CIs for means, their differences, and ratios within this distribution. Khumpasee et al. (2024) explored CIs for the coefficient of variation (CV) of the delta-IG distribution.

In statistics, variance quantifies the deviation from the mean and serves as an essential measure for assessing diversity across populations. The standard deviation, defined as the square root of the variance, further indicates the degree to which data values diverge from their mean. Prior research has investigated CIs for variance and standard deviation across various distributions. Chiou (2007) formulated CIs for variance in normal distributions using chi-squared methods, Wu (2011) examined CIs for the variance of exponential distributions, Paksaranuwat and Niwitpong (2010) established CIs for the variance and variance ratios in non-normal distributions with missing data, and Wongyai and Suwan (2019) established the confidence interval for the variance ratio of bivariate non-normal distributions with a kurtosis estimator. However, CIs for the variance of the delta-IG distribution have not yet been systematically addressed in the literature.

For comparative variance analysis between two populations, both difference-based and ratio-based approaches are commonly employed. In the difference-based framework, a confidence interval containing zero indicates no statistically significant difference in variability, whereas excluding zero suggests a significant difference. In the ratio-based approach, a confidence interval containing one implies no significant difference between the variances. Several researchers have addressed CIs for the difference and ratio of variances across various distributions. Bonett (2006) provided estimated CIs for the ratio of variances in bivariate non-normal distributions. Bebu and Mathew (2008) applied the GCI method to the variance ratio of bivariate lognormal distributions. Paksaranuwat and Niwitpong (2010) evaluated adaptive confidence intervals for the variance and variance ratios under non-normal distributions with missing data. Kooriphan et al. (2022) constructed CIs for the ratio of variances of two delta-gamma distributions using fiducial quantity methods, Bayesian credible intervals, and highest posterior density intervals. Ratasukharom et al. (2024) studied CIs for the difference and ratio of variances of zero-inflated Birnbaum-Saunders distributions. Nevertheless, confidence intervals for the difference and ratio of variances under delta-IG distributions have not yet been systematically investigated.

This study addresses these gaps by constructing and evaluating confidence intervals for the variance of delta-IG populations in two settings. For a single population variance, the methods considered are AGCI, PBPCI, FCI, and BCI. The AGCI, PBPCI, and FCI are combined with three estimation methods for the zero proportion: the variance-stabilizing transformation (VST) (Wu & Hsieh, 2014), Wilson's score (Wilson, 1927), and Hannig's method (Hannig, 2009). For the difference and ratio of two variances, the methods employed are the AGCI, FCI, BCI, MOVER, and NA. Performance of all methods is evaluated through Monte Carlo simulations focusing on coverage probability (CP) and average width (AW). The single variance methods are demonstrated using traffic fatality data from Bangkok, while the difference and ratio methods are illustrated using traffic fatality data from the northern and southern regions of Thailand.

3.2 The confidence interval for the variance of delta-IG distribution

Let $X = (X_1, X_2, \dots, X_n)$ be a non-negative random sample drawn from a delta-IG distribution with mean parameter (μ), shape parameter (λ), and the probability of zero values (δ) denoted as $X \sim \text{delta-IG}(\mu, \lambda, \delta)$ is

$$f(x; \mu, \lambda, \delta) = \delta I_{\{x=0\}} + (1 - \delta) I_{\{x>0\}} \sqrt{\frac{\lambda}{2\pi x^3}} \exp \left[\frac{-\lambda(x-\mu)^2}{2\mu^2 x} \right], \quad (3-1)$$

where $I(\cdot)$ is an indicator function, $\delta = P(X = 0)$, and $\mu, \lambda > 0$. The random sample consists of n observations, which can be partitioned into zero observations $n_{(0)}$ and nonzero observations $n_{(1)}$, such that $n = n_{(0)} + n_{(1)}$. The number of zeros follows a binomial distribution, $n_{(0)} \sim \text{Bin}(n, \delta)$.

The MLEs of the parameters are $\hat{\delta} = \frac{n_{(0)}}{n}$, $\hat{\mu} = \bar{X} = \frac{1}{n_{(1)}} \sum_{j=1}^{n_{(1)}} X_j$, and $n_{(1)} \hat{\lambda}^{-1} = V = \sum_{j=1}^{n_{(1)}} (X_j^{-1} - \bar{X}^{-1})$. In this section, the interest parameter is the variance of X which can be calculated as

$$\eta = \mu^2(1 - \delta) \left(\frac{\mu}{\lambda} + \delta \right). \quad (3-2)$$

The following section provides the methods for constructing confidence intervals for η .

3.2.1 Adjuster generalized confidence interval (AGCI)

In section 2.2.1, the GPQ for the parameter λ proposed by Ye, Ma, and Wang (2010) using in GCI method is presented and can be described as follows:

$$R_\lambda = \frac{n_{(1)}\lambda V}{n_{(1)}v} \sim \frac{\chi_{n_{(1)}-1}^2}{n_{(1)}v}, \quad (3-3)$$

where $\chi_{n_{(1)}-1}^2$ was a chi-square distribution with $n_{(1)} - 1$ degrees of freedom.

And in section 2.2.2, the GPQ for the parameter μ proposed by Krishnamoorthy and Tian (2008) using in the AGCI method instead of the conventional form of the GPQ of μ used in the standard GCI approach can be expressed as follows:

$$R_{\tilde{\mu}} = \frac{\bar{X}}{\max \left\{ 0, 1 + t_{n_{(1)}-1} \sqrt{\frac{\bar{X}v}{n_{(1)}-1}} \right\}}, \quad (3-4)$$

where $t_{n_{(1)}-1}$ is Student's t distribution with $n_{(1)} - 1$ degrees of freedom.

The methods utilized for estimating the zero proportion (δ) will be introduced in the following section.

3.2.1.1 Adjusted generalized confidence interval based on variance-stabilizing transformation method (AGCI-VST)

DasGupta (2008) demonstrated that the coverage probabilities of the VST approach are substantially higher than those of the Wald interval. Building on this result, Wu and Hsieh (2014) employed the GPQ of δ based on VST to construct confidence intervals, which can be expressed as follows:

$$R_{\delta}^{VST} = \sin^2 \left[\arcsin \sqrt{\hat{\delta}} - \frac{T}{2\sqrt{n}} \right], \quad (3-5)$$

where $T = 2\sqrt{n}(\arcsin \sqrt{\hat{\delta}} - \arcsin \sqrt{\delta}) \sim N(0,1)$, $n \rightarrow \infty$.

Hence, from the equations (3-3), (3-4), and (3-5), the GPQ of the single variance (η) can be expressed as

$$R_{\eta}^{AGCI.VST} = R_{\mu}^2 (1 - R_{\delta}^{VST}) \left(\frac{R_{\mu}}{R_{\lambda}} + R_{\delta}^{VST} \right). \quad (3-6)$$

Thus, the $100(1 - \alpha)\%$ two-sided confidence interval for η is determined from the empirical percentiles of the GPQs from the AGCI based on VST method is

$$CI_{\eta}^{AGCI.VST} = [L_{\eta}^{AGCI.VST}, U_{\eta}^{AGCI.VST}] = [R_{\eta}^{AGCI.VST}(\alpha/2), R_{\eta}^{AGCI.VST}(1 - \alpha/2)], \quad (3-7)$$

where $R_{\eta}^{AGCI.VST}(\alpha/2)$ and $R_{\eta}^{AGCI.VST}(1 - \alpha/2)$ are the $100\frac{\alpha}{2}$ -th and $100(1 - \frac{\alpha}{2})$ -th percentiles of the distribution of $R_{\eta}^{AGCI.VST}$, respectively.

3.2.1.2 Adjusted generalized confidence interval based on Wilson's score method (AGCI-WS)

The binomial confidence interval for estimating the CI of proportions introduced by Wilson (1927), commonly known as the Wilson's score interval, is widely used in practice. Then, Li, Zhou, and Tian (2013) achieved the GPQ for δ expressed as

$$R_{\delta}^{WS} = \frac{n_{(0)} + Z_W^2/2}{n + Z_W^2} - \frac{Z_W}{n - Z_W^2} \left[n_{(0)} \left(1 - \frac{n_{(0)}}{n} \right) + \frac{Z_W^2}{4} \right]^{1/2}, \quad (3-8)$$

where Z_W was the standard normal distribution.

From the GPQs of the parameters were R_{λ} , R_{μ} , and R_{δ}^{WS} , expressed as (3-3), (3-4), and (3-8), respectively. The GPQ for η is given by

$$R_{\eta}^{AGCI.WS} = R_{\mu}^2 (1 - R_{\delta}^{WS}) \left(\frac{R_{\mu}}{R_{\lambda}} + R_{\delta}^{WS} \right). \quad (3-9)$$

The $100(1 - \alpha)\%$ two-sided CI for η using the AGCI based on Wilson's score method is

$$CI_{\eta}^{AGCI.WS} = [L_{\eta}^{AGCI.WS}, U_{\eta}^{AGCI.WS}] = [R_{\eta}^{AGCI.WS}(\alpha/2), R_{\eta}^{AGCI.WS}(1 - \alpha/2)], \quad (3-10)$$

where $R_{\eta}^{AGCI.WS}(\alpha/2)$ and $R_{\eta}^{AGCI.WS}(1 - \alpha/2)$ are the $100\frac{\alpha}{2}$ -th and $100(1 - \frac{\alpha}{2})$ -th percentiles of the distribution of $R_{\eta}^{AGCI.WS}$, respectively.

3.2.1.3 Adjusted generalized confidence interval based on Hannig method (AGCI-HN)

Hannig (2009) introduced the concept of the generalized fiducial quantity (GFQ) for δ . Previous studies suggest that an appropriate form of the GFQ can be constructed

as a weighted combination of two beta distributions. Li, Zhou, and Tian (2013) subsequently adopted this formulation. This approach is also consistent with the GPQ framework. Therefore, the GPQ for δ is given as follows:

$$R_{\delta}^{HN} = 0.5\text{beta}(n_{(0)}, n_{(1)} + 1) + 0.5\text{beta}(n_{(0)} + 1, n_{(1)}). \quad (3-11)$$

Thus, from the GPQs of the parameters were R_{λ} , R_{μ} , and R_{δ}^{HN} , expressed as (3-3), (3-4), and (3-11), respectively. The GPQ for η is given by

$$R_{\eta}^{AGCI.HN} = R_{\mu}^2 (1 - R_{\delta}^{HN}) \left(\frac{R_{\mu}}{R_{\lambda}} + R_{\delta}^{HN} \right). \quad (3-12)$$

The $100(1-\alpha)\%$ two-sided CI for η using the AGCI based on the Hannig method is

$$CI_{\eta}^{AGCI.HN} = [L_{\eta}^{AGCI.HN}, U_{\eta}^{AGCI.HN}] = [R_{\eta}^{AGCI.HN}(\alpha/2), R_{\eta}^{AGCI.HN}(1 - \alpha/2)], \quad (3-13)$$

where $R_{\eta}^{AGCI.HN}(\alpha/2)$ and $R_{\eta}^{AGCI.HN}(1 - \alpha/2)$ are the $100\frac{\alpha}{2}$ -th and $100(1-\frac{\alpha}{2})$ -th percentiles of the distribution of $R_{\eta}^{AGCI.HN}$, respectively.

3.2.2 Parametric bootstrap percentile confidence interval (PBPCI)

From $x = (x_1, x_2, \dots, x_n)$ be a non-negative random sample of size n drawn from a delta-IG(μ, λ, δ) distribution. The MLE of μ and λ denoted as $\hat{\mu}$ and $\hat{\lambda}$, respectively. Let $x^* = (x_1^*, x_2^*, \dots, x_b^*)$ denote the bootstrap samples of size b . Define $b_{(0)}^*$ and $b_{(1)}^*$ as the numbers of zero and nonzero observations from bootstrap samples, respectively, such that $b = b_{(0)}^* + b_{(1)}^*$. The unbiased for $\hat{\mu}^*$ and $\hat{\lambda}^*$, adjusted by utilizing the bias estimates, are computed as follows

$$R_{\mu}^{PBPCI} = \hat{\mu}^* - 2\hat{B}(\hat{\mu}, \mu) \quad \text{and} \quad R_{\lambda}^{PBPCI} = \hat{\lambda}^* - 2\hat{B}(\hat{\lambda}, \lambda) \quad (3-14)$$

where $B(\hat{\mu}, \mu) = \bar{\mu}^* - \hat{\mu}$ and $B(\hat{\lambda}, \lambda) = \bar{\lambda}^* - \hat{\lambda}$, respectively, and $\bar{\mu}^* = \frac{1}{b} \sum_{j=1}^b \hat{\mu}_j^*$, $\bar{\lambda}^* = \frac{1}{b} \sum_{j=1}^b \hat{\lambda}_j^*$, $j = 1, 2, \dots, b$.

3.2.2.1 Parametric bootstrap percentile confidence interval based on variance-stabilizing transformation method (PBPCI-VST)

Applying the PBPCI method to estimate parameters μ and λ as shown in Equations (3-14), together with the estimation of the parameter δ using VST method in Equation (3-5), the estimator η of the delta-IG distribution can be expressed as follows:

$$R_{\eta}^{PBPCI.VST} = R_{\mu}^{PBPCI^2} (1 - R_{\delta}^{VST}) \left(\frac{R_{\mu}^{PBPCI}}{R_{\lambda}^{PBPCI}} + R_{\delta}^{VST} \right). \quad (3-15)$$

Hence, the $100(1 - \alpha)\%$ two-sided confidence interval for η is determined from the PBPCI based on VST method is

$$CI_{\eta}^{PBPCI.VST} = [R_{\eta}^{PBPCI.VST}(\alpha/2), R_{\eta}^{PBPCI.VST}(1 - \alpha/2)], \quad (3-16)$$

where $R_{\eta}^{PBPCI.VST}(\alpha)$ denotes the $100(\alpha)^{\text{th}}$ percentile of the distribution of $R_{\eta}^{PBPCI.VST}$.

3.2.2.2 Parametric bootstrap percentile confidence interval based on Wilson's score method (PBPCI-WS)

Using the PBPCI method to estimate parameters μ and λ as shown in Equations (3-14) and estimating the parameter δ using Wilson's score method in Equation (3-8), the estimator η of the delta-IG distribution can be expressed as follows

$$R_{\eta}^{PBPCI.WS} = R_{\mu}^{PBPCI^2} (1 - R_{\delta}^{WS}) \left(\frac{R_{\mu}^{PBPCI}}{R_{\lambda}^{PBPCI}} + R_{\delta}^{WS} \right). \quad (3-17)$$

The 100(1- α)% two-sided CI for η using the PBPCI based on Wilson's score method is

$$CI_{\eta}^{PBPCI.WS} = [R_{\eta}^{PBPCI.WS}(\alpha/2), R_{\eta}^{PBPCI.WS}(1 - \alpha/2)], \quad (3-18)$$

where $R_{\eta}^{PBPCI.WS}(\alpha)$ denotes the 100(α)th percentile of the distribution of $R_{\eta}^{PBPCI.WS}$.

3.2.2.3 Parametric bootstrap percentile confidence interval based on Hannig method (PBPCI-HN)

By estimating the PBPCI method to estimate parameters μ and λ as shown in Equations (3-14) and estimating the parameter δ using Hannig method in Equation (3-11), the estimator η of the delta-IG distribution can be expressed as follows:

$$R_{\eta}^{PBPCI.HN} = R_{\mu}^{PBPCI^2} (1 - R_{\delta}^{HN}) \left(\frac{R_{\mu}^{PBPCI}}{R_{\lambda}^{PBPCI}} + R_{\delta}^{HN} \right). \quad (3-19)$$

The 100(1- α)% two-sided CI for η using the PBPCI based on the Hannig method is

$$CI_{\eta}^{PBPCI.HN} = [R_{\eta}^{PBPCI.HN}(\alpha/2), R_{\eta}^{PBPCI.HN}(1 - \alpha/2)], \quad (3-20)$$

where $R_{\eta}^{PBPCI.HN}(\alpha)$ denotes the 100(α)th percentile of the distribution of $R_{\eta}^{PBPCI.HN}$.

3.2.3 Fiducial confidence interval (FCI)

Let $X = (X_1, X_2, \dots, X_n)$ be a random sample from the delta-IG distribution with parameters μ , λ , and δ . The variance (η) is defined as

$$\eta = h(\mu, \lambda, \delta) = \mu^2 (1 - \delta) \left(\frac{\mu}{\lambda} + \delta \right)$$

where $h(\cdot)$ is a known function depending on the parameterization of the delta-IG distribution.

To construct the fiducial distribution, the sampling distributions of the MLEs are used. Given $\hat{\mu}$ and $\hat{\lambda}$ were the MLEs of μ and λ , respectively. From the properties of the IG distribution, their sampling distributions can be approximated as: $\mu \sim IG(\hat{\mu}, n_{(1)}\hat{\lambda})$ and $\lambda \sim \frac{\hat{\lambda}}{n_{(1)}} \chi_{n_{(1)}-1}^2$. These distributions are treated as fiducial distributions by replacing the unknown parameters with their corresponding MLEs.

Since the $\eta = h(\mu, \lambda, \delta)$ is a function of μ , λ , and δ , the fiducial distribution of μ and λ for computing the variance can be obtained via simulation. A practical approach is to employ a Monte Carlo or Gibbs sampling scheme, then obtained R_{μ}^{FCI} and R_{λ}^{FCI} .

3.2.2.1 Fiducial confidence interval based on variance-stabilizing transformation method (FCI-VST)

Applying the FCI method to estimate parameters μ and λ denoted as R_{μ}^{FCI} and R_{λ}^{FCI} , along with the estimation of the parameter δ using VST method in Equation (3-5), the estimator η of the delta-IG distribution can be described as follows:

$$R_{\eta}^{FCI.VST} = R_{\mu}^{FCI^2} (1 - R_{\delta}^{VST}) \left(\frac{R_{\mu}^{FCI}}{R_{\lambda}^{FCI}} + R_{\delta}^{VST} \right). \quad (3-21)$$

Hence, the 100(1 - α)% two-sided confidence interval for η is determined from the FCI based on VST method is

$$CI_{\eta}^{FCI.VST} = [R_{\eta}^{FCI.VST}(\alpha/2), R_{\eta}^{FCI.VST}(1 - \alpha/2)], \quad (3-22)$$

where $R_\eta^{FCI.VST}(\alpha)$ denotes the $100(\alpha)^{\text{th}}$ percentile of the distribution of $R_\eta^{FCI.VST}$.

3.2.2.2 Fiducial confidence interval based on Wilson's score method (FCI-WS)

Utilizing the FCI method to estimate parameters μ and λ denoted as R_μ^{FCI} and R_λ^{FCI} , then estimating the parameter δ using Wilson's score method in Equation (3-8), the estimator η of the delta-IG distribution can be expressed as follows

$$R_\eta^{FCI.WS} = R_\mu^{FCI^2} (1 - R_\delta^{WS}) \left(\frac{R_\mu^{FCI}}{R_\lambda^{FCI}} + R_\delta^{WS} \right). \quad (3-23)$$

The $100(1-\alpha)\%$ two-sided CI for η using the FCI based on Wilson's score method is

$$CI_\eta^{FCI.WS} = [R_\eta^{FCI.WS}(\alpha/2), R_\eta^{FCI.WS}(1 - \alpha/2)], \quad (3-24)$$

where $R_\eta^{FCI.WS}(\alpha)$ denotes the $100(\alpha)^{\text{th}}$ percentile of the distribution of $R_\eta^{FCI.WS}$.

3.2.2.3 Fiducial confidence interval based on Hannig method (FCI-HN)

Using the FCI method to estimate parameters μ and λ denoted as R_μ^{FCI} and R_λ^{FCI} , together with estimating the parameter δ using Hannig method in Equation (3-11), the estimator η of the delta-IG distribution can be expressed as follows:

$$R_\eta^{FCI.HN} = R_\mu^{FCI^2} (1 - R_\delta^{HN}) \left(\frac{R_\mu^{FCI}}{R_\lambda^{FCI}} + R_\delta^{HN} \right). \quad (3-25)$$

The $100(1-\alpha)\%$ two-sided CI for η using the FCI based on the Hannig method is

$$CI_\eta^{FCI.HN} = [R_\eta^{FCI.HN}(\alpha/2), R_\eta^{FCI.HN}(1 - \alpha/2)], \quad (3-26)$$

where $R_\eta^{FCI.HN}(\alpha)$ denotes the $100(\alpha)^{\text{th}}$ percentile of the distribution of $R_\eta^{FCI.HN}$.

3.2.4 Bayesian credible interval (BCI)

A Bayesian credible interval provides a range of plausible values for an unknown parameter based on its posterior distribution, which combines prior information with the likelihood derived from observed data via Bayes' theorem. For the delta-IG distribution, the posterior distribution is obtained by combining the likelihood function with specified prior distributions. The choice of prior plays an important role, particularly in small samples, as it can influence the resulting interval. The likelihood function of the delta-IG distribution is given by

$$L(\eta|X) \propto \delta^{n_{(0)}} (1 - \delta)^{n_{(1)}} \lambda^{n_{(1)}/2} \exp \left(-\frac{\lambda}{2\mu^2} (t_1 - 2n_{(1)}\mu + t_2\mu^2) \right), \quad (3-27)$$

where $t_1 = \sum_{j=1}^{n_{(1)}} X_j$, and $t_2 = \sum_{j=1}^{n_{(1)}} X_j^{-1}$.

The posterior density is given by

$$f(\eta|X) \propto f(\eta)L(\eta|X). \quad (3-28)$$

Since the resulting posterior distributions are analytically intractable, Markov Chain Monte Carlo (MCMC) methods, specifically Gibbs sampling, are used to generate the posterior samples. In this study, a uniform prior is assigned to μ , a gamma prior to λ , and a beta prior to δ . The computation is implemented using the *R2OpenBUGS* package in the R programming language. Based on the posterior samples, the estimator of the variance for the delta-IG distribution is defined as

$$\hat{\eta}^{BS} = \hat{\mu}^{BS^2} (1 - \hat{\delta}^{BS}) \left(\frac{\hat{\mu}^{BS}}{\hat{\lambda}^{BS}} + \hat{\delta}^{BS} \right). \quad (3-29)$$

Finally, the $100(1-\alpha)\%$ two-sided CI of η for the delta-IG distribution based on the BCI method is given by

$$CI_{\eta}^{BCI} = [\hat{\eta}^{BS}(\alpha/2), \hat{\eta}^{BS}(1 - \alpha/2)], \quad (3-30)$$

where $\hat{\eta}^{BS}(\alpha/2)$ and $\hat{\eta}^{BS}(1 - \alpha/2)$ are the $100\frac{\alpha}{2}$ -th and $100(1-\frac{\alpha}{2})$ -th percentiles of the distribution of $\hat{\eta}^{BS}$.

3.3 The confidence intervals for the difference and the ratio of the variances of delta-IG distributions

This section extends the analysis to confidence intervals for the difference and ratio of two coefficients of variation, developed using the frameworks of AGCI, FCI, BCI, MOVER, and NA. Let $X_{ij} = (X_{i1}, X_{i2}, \dots, X_{in_i})$, for $i = 1, 2, j = 1, 2, \dots, n_i$ be a non-negative random sample from a delta-IG distribution stated as $X_{ij} \sim \text{delta-IG}(\delta_i, \mu_i, \lambda_i)$. The variance of delta-IG distributions was

$$\text{Var}(X_{ij}) = \eta_i = \mu_i^2(1 - \delta_i) \left(\frac{\mu_i}{\lambda_i} + \delta_i \right), i = 1, 2. \quad (3-31)$$

Therefore, the difference and ratio of the variances are parameters that we will calculate for its interval, denoted as

$$\eta_{diff} = \eta_1 - \eta_2 = \mu_1^2(1 - \delta_1) \left(\frac{\mu_1}{\lambda_1} + \delta_1 \right) - \mu_2^2(1 - \delta_2) \left(\frac{\mu_2}{\lambda_2} + \delta_2 \right), \quad (3-32)$$

and

$$\eta_{ratio} = \frac{\eta_1}{\eta_2} = \frac{\mu_1^2(1 - \delta_1) \left(\frac{\mu_1}{\lambda_1} + \delta_1 \right)}{\mu_2^2(1 - \delta_2) \left(\frac{\mu_2}{\lambda_2} + \delta_2 \right)}, \quad (3-33)$$

respectively.

The following section provides the methods for constructing confidence intervals for η_{diff} and η_{ratio} .

3.3.1 Adjusted generalized confidence interval (AGCI)

To construct confidence intervals for η_{diff} and η_{ratio} using the AGCI method, it is first necessary to define a GPQ for each parameter. The GPQ for the parameter λ_i introduced by Ye, Ma, and Wang (2010) for the delta-IG distribution can be expressed as

$$R_{\lambda_i} = \frac{n_{i(1)}\lambda_i V_i}{n_{i(1)}v_i} \sim \frac{\chi_{n_{i(1)}-1}^2}{n_{i(1)}v_i} \quad (3-34)$$

The GPQ for μ_i , as proposed by Krishnamoorthy and Tian (2008), is defined as

$$R_{\mu_i} = \frac{\bar{X}_i}{\max \left\{ 0, 1 + t_{n_{i(1)}-1} \sqrt{\frac{\bar{X}_i v_i}{n_{i(1)}-1}} \right\}}, \quad (3-35)$$

where $t_{n_{i(1)}-1}$ is Student's t distribution with $n_{i(1)} - 1$ degrees of freedom. The GPQ for δ_i from the VST method is defined as

$$R_{\delta_i} = \sin^2 \left[\arcsin \sqrt{\hat{\delta}_i} - \frac{T_i}{2\sqrt{n_i}} \right], \quad (3-36)$$

where $T_i = 2\sqrt{n_i}(\arcsin \sqrt{\hat{\delta}_i} - \arcsin \sqrt{\delta_i}) \sim N(0,1)$, $n_i \rightarrow \infty$, $i = 1, 2$.

Therefore, from the Equations (3-34), (3-35), and (3-36), the GPQ of the difference and the ratio between two variances (η_{diff} and η_{ratio}) are given as

$$R_{\tilde{\eta}_{diff}} = R_{\tilde{\mu}_1}^2 (1 - R_{\delta_1}) \left(\frac{R_{\tilde{\mu}_1}}{R_{\lambda_1}} + R_{\delta_1} \right) - R_{\tilde{\mu}_2}^2 (1 - R_{\delta_2}) \left(\frac{R_{\tilde{\mu}_2}}{R_{\lambda_2}} + R_{\delta_2} \right), \quad (3-37)$$

and

$$R_{\tilde{\eta}_{ratio}} = \frac{R_{\tilde{\mu}_1}^2 (1 - R_{\delta_1}) \left(\frac{R_{\tilde{\mu}_1}}{R_{\lambda_1}} + R_{\delta_1} \right)}{R_{\tilde{\mu}_2}^2 (1 - R_{\delta_2}) \left(\frac{R_{\tilde{\mu}_2}}{R_{\lambda_2}} + R_{\delta_2} \right)}. \quad (3-38)$$

Therefore, the $100(1 - \alpha)\%$ two-sided confidence intervals for η_{diff} and η_{ratio} from the AGCI method are expressed by

$$CI_{\eta_{diff}}^{AGCI} = [L_{\eta_{diff}}^{AGCI}, U_{\eta_{diff}}^{AGCI}] = [R_{\tilde{\eta}_{diff}}(\alpha/2), R_{\tilde{\eta}_{diff}}(1 - \alpha/2)], \quad (3-39)$$

and

$$CI_{\eta_{ratio}}^{AGCI} = [L_{\eta_{ratio}}^{AGCI}, U_{\eta_{ratio}}^{AGCI}] = [R_{\tilde{\eta}_{ratio}}(\alpha/2), R_{\tilde{\eta}_{ratio}}(1 - \alpha/2)], \quad (3-40)$$

where $R_{\tilde{\eta}_{diff}}(\alpha)$ and $R_{\tilde{\eta}_{ratio}}(\alpha)$ denote the $100(\alpha)^{\text{th}}$ percentiles of $R_{\tilde{\eta}_{diff}}$ and $R_{\tilde{\eta}_{ratio}}$, respectively.

3.3.2 Fiducial confidence interval (FCI)

Let $X_i = (X_{i1}, X_{i2}, \dots, X_{in_i}), i = 1, 2$ be a random sample from the delta-IG distribution with parameters μ_i, λ_i , and δ_i . The difference and ratio between two variances are defined as

$$\eta_{diff} = p(\mu_i, \lambda_i, \delta_i) = \mu_1^2 (1 - \delta_1) \left(\frac{\mu_1}{\lambda_1} + \delta_1 \right) - \mu_2^2 (1 - \delta_2) \left(\frac{\mu_2}{\lambda_2} + \delta_2 \right), \quad (3-41)$$

and

$$\eta_{ratio} = q(\mu_i, \lambda_i, \delta_i) = \frac{\mu_1^2 (1 - \delta_1) \left(\frac{\mu_1}{\lambda_1} + \delta_1 \right)}{\mu_2^2 (1 - \delta_2) \left(\frac{\mu_2}{\lambda_2} + \delta_2 \right)}, \quad (3-42)$$

where $p(\cdot)$ and $q(\cdot)$ are known function depending on the parameterization of the delta-IG distribution.

To construct the fiducial distribution, the sampling distributions of the MLEs are utilized. Let $\hat{\mu}_i$ and $\hat{\lambda}_i$ denote the MLEs of μ_i and λ_i , respectively. Based on the properties of the inverse Gaussian distribution, their sampling distributions can be approximated as: $\mu_i \sim IG(\hat{\mu}_i, n_{i(1)} \hat{\lambda}_i)$ and $\lambda_i \sim \frac{\hat{\lambda}_i}{n_{i(1)}} \chi_{n_{i(1)} - 1}^2$. These distributions are treated as fiducial distributions by replacing the unknown parameters with their corresponding MLEs.

Since the $\eta_{diff} = p(\mu_i, \lambda_i, \delta_i)$ and $\eta_{ratio} = q(\mu_i, \lambda_i, \delta_i)$ are function of μ_i, λ_i , and δ_i , the fiducial distribution of the difference and ratio between two variances can be obtained via simulation. A practical approach is to employ a Monte Carlo or Gibbs sampling scheme.

The $100(1 - \alpha)\%$ FCI for the difference and ratio of two variances (η_{diff} and η_{ratio}) defined as

$$\eta_{diff_L} = \text{quantile}_{\frac{\alpha}{2}}(\eta_{diff}^{(r)}), \eta_{diff_U} = \text{quantile}_{1 - \frac{\alpha}{2}}(\eta_{diff}^{(r)}), \quad (3-43)$$

and

$$\eta_{ratio_L} = \text{quantile}_{\alpha/2}(\eta_{ratio}^{(r)}), \eta_{ratio_U} = \text{quantile}_{1-\alpha/2}(\eta_{ratio}^{(r)}). \quad (3-44)$$

Thus, the fiducial confidence interval of η_{diff} and η_{ratio} are given by

$$CI_{\eta_{diff}}^{FCI} = [L_{FCI}, U_{FCI}] = (\eta_{diff_L}, \eta_{diff_U}), \quad (3-45)$$

and

$$CI_{\eta_{ratio}}^{FCI} = [L_{FCI}, U_{FCI}] = (\eta_{ratio_L}, \eta_{ratio_U}). \quad (3-46)$$

3.3.3 Bayesian credible interval (BCI)

Let $X_{ij} = (X_{i1}, X_{i2}, \dots, X_{in_i})$, for $i = 1, 2, j = 1, 2, \dots, n_i$ be a non-negative random sample from a delta-IG distribution stated as $X_{ij} \sim \text{delta-IG}(\delta_i, \mu_i, \lambda_i)$. The likelihood function of the delta-IG distribution is given by

$$L(\eta|X_i) \propto \delta_i^{n_{i(0)}} (1 - \delta_i)^{n_{i(1)}} \lambda_i^{n_{i(1)}/2} \exp\left(-\frac{\lambda_i}{2\mu_i^2} (t_1 - 2n_{i(1)}\mu_i + t_2\mu_i^2)\right), \quad (3-47)$$

where $t_1 = \sum_{j=1}^{n_{i(1)}} X_{ij}$, and $t_2 = \sum_{j=1}^{n_{i(1)}} X_{ij}^{-1}$, $i = 1, 2$.

The posterior density is given by

$$f(\eta|X_i) \propto f(\eta)L(\eta|X_i). \quad (3-48)$$

Since the resulting posterior distributions are analytically intractable, Markov Chain Monte Carlo (MCMC) methods, specifically Gibbs sampling, are used to generate the posterior samples. In this study, a uniform prior is assigned to μ , a gamma prior to λ , and a beta prior to δ . The computation is implemented using the *R2OpenBUGS* package in the R programming language.

Based on the posterior samples, the estimator of the difference and ratio between two variances (η_{diff} and η_{ratio}) for the delta-IG distribution are defined as

$$\hat{\eta}_{diff}^{BS} = \hat{\mu}_1^{BS^2} (1 - \hat{\delta}_1^{BS}) \left(\frac{\hat{\mu}_1^{BS}}{\hat{\lambda}_1^{BS}} + \hat{\delta}_1^{BS} \right) - \hat{\mu}_2^{BS^2} (1 - \hat{\delta}_2^{BS}) \left(\frac{\hat{\mu}_2^{BS}}{\hat{\lambda}_2^{BS}} + \hat{\delta}_2^{BS} \right), \quad (3-49)$$

and

$$\hat{\eta}_{ratio}^{BS} = \frac{\hat{\mu}_1^{BS^2} (1 - \hat{\delta}_1^{BS}) \left(\frac{\hat{\mu}_1^{BS}}{\hat{\lambda}_1^{BS}} + \hat{\delta}_1^{BS} \right)}{\hat{\mu}_2^{BS^2} (1 - \hat{\delta}_2^{BS}) \left(\frac{\hat{\mu}_2^{BS}}{\hat{\lambda}_2^{BS}} + \hat{\delta}_2^{BS} \right)}. \quad (3-50)$$

Therefore, the $100(1-\alpha)\%$ two-sided CI of η_{diff} and η_{ratio} for the delta-IG distribution based on the BCI method are given by

$$CI_{\eta_{diff}}^{BCI} = [\hat{\eta}_{diff}^{BS}(\alpha/2), \hat{\eta}_{diff}^{BS}(1 - \alpha/2)], \quad (3-51)$$

and

$$CI_{\eta_{ratio}}^{BCI} = [\hat{\eta}_{ratio}^{BS}(\alpha/2), \hat{\eta}_{ratio}^{BS}(1 - \alpha/2)], \quad (3-52)$$

where $\hat{\eta}_{diff}^{BS}(\alpha)$ and $\hat{\eta}_{ratio}^{BS}(\alpha)$ are the $100(\alpha)^{\text{th}}$ percentiles of the distribution of $\hat{\eta}_{diff}^{BS}$ and $\hat{\eta}_{ratio}^{BS}$, respectively.

3.3.4 Method of variance estimates recovery (MOVER)

The method of variance estimates recovery (MOVER), proposed by Zou and Donner (2009), is a flexible and efficient technique for constructing confidence intervals for a wide range of parameters, particularly the difference or ratio of two independent quantities. The MOVER method provides the lower and upper confidence

limits for the difference of two parameters or $\eta_{diff} = \eta_1 - \eta_2$, which are expressed as follows

$$L_{diff} = \hat{\eta}_1 - \hat{\eta}_2 - \sqrt{(\hat{\eta}_1 - l_1)^2 + (u_2 - \hat{\eta}_2)^2}, \quad (3-53)$$

and

$$U_{diff} = \hat{\eta}_1 - \hat{\eta}_2 + \sqrt{(u_1 - \hat{\eta}_1)^2 + (\hat{\eta}_2 - l_2)^2}, \quad (3-54)$$

where (l_1, u_1) and (l_2, u_2) are 100(1- α)% confidence limits for the parameters η_1 and η_2 .

And the MOVER method also provides the lower and upper confidence limits for the ratio of two parameters or $\eta_{ratio} = \eta_1/\eta_2$, which are expressed as follows

$$L_{ratio} = \frac{\hat{\eta}_1 \hat{\eta}_2 - \sqrt{(\hat{\eta}_1 \hat{\eta}_2)^2 - l_1 u_2 (2\hat{\eta}_1 - l_2)(2\hat{\eta}_2 - u_2)}}{u_2 (2\hat{\eta}_2 - u_2)}, \quad (3-55)$$

and

$$U_{ratio} = \frac{\hat{\eta}_1 \hat{\eta}_2 + \sqrt{(\hat{\eta}_1 \hat{\eta}_2)^2 - u_1 l_2 (2\hat{\eta}_1 - u_1)(2\hat{\eta}_2 - l_2)}}{l_2 (2\hat{\eta}_2 - l_2)}, \quad (3-56)$$

where (l_1, u_1) and (l_2, u_2) are 100(1- α)% confidence limits for the parameters η_1 and η_2 .

By utilizing CI computed from the AGCI method as shown in equations (3-39) and (3-40), we obtain the 100(1- α)% two-sided confidence intervals for η_{diff} and η_{ratio} as follows.

From the AGCI method, the GPQ for η_{diff} can be defined as

$$R_{\tilde{\eta}_{diff}} = R_{\tilde{\mu}_1}^2 (1 - R_{\delta_1}) \left(\frac{R_{\tilde{\mu}_1}}{R_{\lambda_1}} + R_{\delta_1} \right) - R_{\tilde{\mu}_2}^2 (1 - R_{\delta_2}) \left(\frac{R_{\tilde{\mu}_2}}{R_{\lambda_2}} + R_{\delta_2} \right), \quad (3-57)$$

and the GPQ for η_{ratio} can be defined as

$$R_{\tilde{\eta}_{ratio}} = \frac{R_{\tilde{\mu}_1}^2 (1 - R_{\delta_1}) \left(\frac{R_{\tilde{\mu}_1}}{R_{\lambda_1}} + R_{\delta_1} \right)}{R_{\tilde{\mu}_2}^2 (1 - R_{\delta_2}) \left(\frac{R_{\tilde{\mu}_2}}{R_{\lambda_2}} + R_{\delta_2} \right)}. \quad (3-58)$$

The 100(1- α)% two-sided CI for η_{diff} using the AGCI method is

$$CI_{\eta_{diff}}^{AGCI} = [L_{\eta_{diff}}^{AGCI}, U_{\eta_{diff}}^{AGCI}] = [R_{\tilde{\eta}_{diff}}(\alpha/2), R_{\tilde{\eta}_{diff}}(1 - \alpha/2)], \quad (3-59)$$

And the 100(1- α)% two-sided CI for η_{ratio} using the AGCI method is

$$CI_{\eta_{ratio}}^{AGCI} = [L_{\eta_{ratio}}^{AGCI}, U_{\eta_{ratio}}^{AGCI}] = [R_{\tilde{\eta}_{ratio}}(\alpha/2), R_{\tilde{\eta}_{ratio}}(1 - \alpha/2)], \quad (3-60)$$

Therefore, the 100(1- α)% two-sided confidence intervals for η_{diff} and η_{ratio} based on the MOVER are

$$CI_{\eta_{diff}}^{MOVER} = [L_{\eta}^{MOVER}, U_{\eta}^{MOVER}] = [\hat{\eta}_1 - \hat{\eta}_2 - \sqrt{(\hat{\eta}_1 - l_1)^2 + (u_2 - \hat{\eta}_2)^2}, \hat{\eta}_1 - \hat{\eta}_2 + \sqrt{(u_1 - \hat{\eta}_1)^2 + (\hat{\eta}_2 - l_2)^2}], \quad (3-61)$$

and

$$CI_{\eta_{ratio}}^{MOVER} = [L_{\eta}^{MOVER}, U_{\eta}^{MOVER}] = \left[\frac{\hat{\eta}_1 \hat{\eta}_2 - \sqrt{(\hat{\eta}_1 \hat{\eta}_2)^2 - l_1 u_2 (2\hat{\eta}_1 - l_2)(2\hat{\eta}_2 - u_2)}}{u_2 (2\hat{\eta}_2 - u_2)}, \frac{\hat{\eta}_1 \hat{\eta}_2 + \sqrt{(\hat{\eta}_1 \hat{\eta}_2)^2 - u_1 l_2 (2\hat{\eta}_1 - u_1)(2\hat{\eta}_2 - l_2)}}{l_2 (2\hat{\eta}_2 - l_2)} \right]. \quad (3-62)$$

where (l_1, u_1) and (l_2, u_2) are 100(1- α)% confidence limits computed from the AGCI method for the parameters η_1 and η_2 .

3.3.5 Normal approximation (NA)

In the normal approximation method, we utilize the delta method to show that the estimator η has a normal distribution as the limiting distribution. This approach relies on large-sample theory and serves as a conventional benchmark due to its analytical simplicity and closed-form implementation. This method may be briefly outlined as follows:

Let $u(v_1, v_2, v_3)$ be a differentiable scalar function of three variables. An estimator, $U = u(v_1, v_2, v_3)$, depends on three parameters: V_1 , V_2 , and V_3 . Using the delta method, we may determine the asymptotic distribution of U , which requires providing a stochastic representation of U . We employ the delta method to illustrate the asymptotic normality of U as sample sizes approach infinity and to compute the asymptotic mean and variance of the estimators η . In the delta method, the function u is expressed as a Taylor series at the points $\varphi_1, \varphi_2, \varphi_3$ of V_1, V_2, V_3 as follows:

$$u(v_1, v_2, v_3) \approx u(\varphi_1, \varphi_2, \varphi_3) + \frac{\partial u(\varphi_1, \varphi_2, \varphi_3)}{\partial v_1} (V_1 - \varphi_1) + \frac{\partial u(\varphi_1, \varphi_2, \varphi_3)}{\partial v_2} (V_2 - \varphi_2) + \frac{\partial u(\varphi_1, \varphi_2, \varphi_3)}{\partial v_3} (V_3 - \varphi_3). \quad (3-63)$$

The essential statistics are denoted as $V_1 = \hat{\mu}$, $V_2 = \hat{\lambda}$, and $V_3 = \hat{\delta}$, where $\hat{\mu}$, $\hat{\lambda}$, and $\hat{\delta}$ signify the MLEs of μ, λ , and δ , respectively. Consequently, $\hat{\eta} = u(\hat{\mu}, \hat{\lambda}, \hat{\delta})$, where the function $u(\mu, \lambda, \delta)$ is articulated as $\mu^2(1 - \delta)\left(\frac{\mu}{\lambda} + \delta\right)$. The partial derivatives are listed as follows.

$$\begin{aligned} \frac{\partial u(\mu, \lambda, \delta)}{\partial \mu} &= (1 - \delta) \left[\mu^2 \left(\frac{1}{\lambda} \right) + \left(\frac{\mu}{\lambda} + \delta \right) 2\mu \right], \\ \frac{\partial u(\mu, \lambda, \delta)}{\partial \lambda} &= \mu^2 (1 - \delta) \left(-\frac{\mu}{\lambda^2} \right), \\ \frac{\partial u(\mu, \lambda, \delta)}{\partial \delta} &= \mu^2 \left[(1 - \delta) - \left(\frac{\mu}{\lambda} + \delta \right) \right]. \end{aligned} \quad (3-64)$$

Thus,

$$\begin{aligned} \hat{\eta} &\approx u(\mu, \lambda, \delta) + \frac{\partial u(\mu, \lambda, \delta)}{\partial \mu} (\hat{\mu} - \mu) + \frac{\partial u(\mu, \lambda, \delta)}{\partial \lambda} (\hat{\lambda} - \lambda) + \frac{\partial u(\mu, \lambda, \delta)}{\partial \delta} (\hat{\delta} - \delta) \\ &= f(\mu, \lambda, \delta) + (1 - \delta) \left[\mu^2 \left(\frac{1}{\lambda} \right) + \left(\frac{\mu}{\lambda} + \delta \right) 2\mu \right] (\hat{\mu} - \mu) \\ &\quad + \mu^2 (1 - \delta) \left(-\frac{\mu}{\lambda^2} \right) (\hat{\lambda} - \lambda) + \mu^2 \left[(1 - \delta) - \left(\frac{\mu}{\lambda} + \delta \right) \right] (\hat{\delta} - \delta). \end{aligned} \quad (3-65)$$

Taking the expectation and variance in Equation (3-65), the estimated mean and variance for $\hat{\eta}$ are expressed as

$$E(\hat{\eta}) = \eta, \quad (3-66)$$

$$\begin{aligned} Var(\hat{\eta}) &\approx Var\{f(\mu, \lambda, \delta) + (1 - \delta) \left[\mu^2 \left(\frac{1}{\lambda} \right) + \left(\frac{\mu}{\lambda} + \delta \right) 2\mu \right] (\hat{\mu} - \mu) \\ &\quad + \mu^2 (1 - \delta) \left(-\frac{\mu}{\lambda^2} \right) (\hat{\lambda} - \lambda) + \mu^2 \left[(1 - \delta) - \left(\frac{\mu}{\lambda} + \delta \right) \right] (\hat{\delta} - \delta)\}. \end{aligned} \quad (3-67)$$

Consequently, when $n_i \rightarrow \infty$, $\hat{\eta}$ approximates a normal distribution, denoted as $N(E(\hat{\eta}), Var(\hat{\eta}))$. The values of μ, λ , and δ are uncertain when estimating the parameter function η solely from the available samples. Thereafter, we utilize the plug-in estimators for $E(\hat{\eta}_i)$ and $Var(\hat{\eta}_i)$, as shown below.

$$\hat{E}(\hat{\eta}_i) = \hat{\mu}_i^2 (1 - \hat{\delta}_i) \left(\frac{\hat{\mu}_i}{\hat{\lambda}_i} + \hat{\delta}_i \right), \quad (3-68)$$

$$\begin{aligned}
V\hat{ar}(\hat{\eta}_i) &= \left\{ (1 - \hat{\delta}_i) \left[\hat{\mu}_i^2 \left(\frac{1}{\hat{\lambda}_i} \right) + \left(\frac{\hat{\mu}_i}{\hat{\lambda}_i} + \hat{\delta}_i \right) 2\hat{\mu}_i \right] \right\}^2 \left(\frac{\hat{\mu}_i^3}{\hat{\lambda}_i n_{i(1)}} \right) \\
&+ \left\{ \hat{\mu}_i^2 (1 - \hat{\delta}_i) \left(-\frac{\hat{\mu}_i}{\hat{\lambda}_i^2} \right) \right\}^2 \left(\frac{2\hat{\lambda}_i^2}{n_{i(1)}} \right) \\
&+ \left\{ \hat{\mu}_i^2 \left[(1 - \hat{\delta}_i) - \left(\frac{\hat{\mu}_i}{\hat{\lambda}_i} + \hat{\delta}_i \right) \right] \right\}^2 \left(\frac{\hat{\delta}_i(1 - \hat{\delta}_i)}{n_i} \right).
\end{aligned} \quad (3-69)$$

The estimated variance of the estimated difference between two variances of delta-IG may be articulated as follows:

$$V\hat{ar}(\hat{\eta}_{diff}) = V\hat{ar}(\hat{\eta}_1) + V\hat{ar}(\hat{\eta}_2), \quad (3-70)$$

where $\text{cov}(\hat{\eta}_1, \hat{\eta}_2) = 0$.

The estimated variance of the estimated ratio between two variances of delta-IG can be determined as follows:

Let

$$\eta_{ratio} = \frac{\eta_1}{\eta_2} = g(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2) = \frac{\mu_1^2(1-\delta_1)\left(\frac{\mu_1}{\lambda_1} + \delta_1\right)}{\mu_2^2(1-\delta_2)\left(\frac{\mu_2}{\lambda_2} + \delta_2\right)}. \quad (3-71)$$

Consider the estimator of the parameter η_{ratio} , which is equal to

$$\hat{\eta}_{ratio} = \frac{\hat{\eta}_1}{\hat{\eta}_2} = g(\hat{\mu}_1, \hat{\lambda}_1, \hat{\delta}_1, \hat{\mu}_2, \hat{\lambda}_2, \hat{\delta}_2) = \frac{\hat{\mu}_1^2(1-\hat{\delta}_1)\left(\frac{\hat{\mu}_1}{\hat{\lambda}_1} + \hat{\delta}_1\right)}{\hat{\mu}_2^2(1-\hat{\delta}_2)\left(\frac{\hat{\mu}_2}{\hat{\lambda}_2} + \hat{\delta}_2\right)}. \quad (3-72)$$

Using the Delta method, the first-order Taylor series expansion can be written as follows:

$$\begin{aligned}
g(\hat{\mu}_1, \hat{\lambda}_1, \hat{\delta}_1, \hat{\mu}_2, \hat{\lambda}_2, \hat{\delta}_2) &= g(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2) + \frac{\partial g(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \mu_1} (\hat{\mu}_1 - \mu_1) \\
&+ \frac{\partial g(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \lambda_1} (\hat{\lambda}_1 - \lambda_1) + \frac{\partial g(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \delta_1} (\hat{\delta}_1 - \delta_1) \\
&+ \frac{\partial g(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \mu_2} (\hat{\mu}_2 - \mu_2) + \frac{\partial g(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \lambda_2} (\hat{\lambda}_2 - \lambda_2) \\
&+ \frac{\partial g(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \delta_2} (\hat{\delta}_2 - \delta_2) + \text{Reminder}.
\end{aligned} \quad (3-73)$$

The partial derivatives are as follows:

$$\begin{aligned}
\frac{\partial g(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \mu_1} &= \frac{\left[\mu_2^2(1-\delta_2)\left(\frac{\mu_2}{\lambda_2} + \delta_2\right) \right] \left\{ (1-\delta_1) \left[\mu_1^2 \left(\frac{1}{\lambda_1} \right) + \left(\frac{\mu_1}{\lambda_1} + \delta_1 \right) 2\mu_1 \right] \right\}}{\left[\mu_2^2(1-\delta_2)\left(\frac{\mu_2}{\lambda_2} + \delta_2\right) \right]^2} = G_{\mu_1}. \\
\frac{\partial g(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \lambda_1} &= \frac{\left[\mu_2^2(1-\delta_2)\left(\frac{\mu_2}{\lambda_2} + \delta_2\right) \right] \left[\mu_1^2(1-\delta_1) \left(-\frac{\mu_1}{\lambda_1^2} \right) \right]}{\left[\mu_2^2(1-\delta_2)\left(\frac{\mu_2}{\lambda_2} + \delta_2\right) \right]^2} = G_{\lambda_1}. \\
\frac{\partial g(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \delta_1} &= \frac{\left[\mu_2^2(1-\delta_2)\left(\frac{\mu_2}{\lambda_2} + \delta_2\right) \right] \left\{ \mu_1^2 \left[(1-\delta_1) - \left(\frac{\mu_1}{\lambda_1} + \delta_1 \right) \right] \right\}}{\left[\mu_2^2(1-\delta_2)\left(\frac{\mu_2}{\lambda_2} + \delta_2\right) \right]^2} = G_{\delta_1}. \\
\frac{\partial g(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \mu_2} &= \frac{-\left[\mu_1^2(1-\delta_1)\left(\frac{\mu_1}{\lambda_1} + \delta_1\right) \right] \left\{ (1-\delta_2) \left[\mu_2^2 \left(\frac{1}{\lambda_2} \right) + \left(\frac{\mu_2}{\lambda_2} + \delta_2 \right) 2\mu_2 \right] \right\}}{\left[\mu_2^2(1-\delta_2)\left(\frac{\mu_2}{\lambda_2} + \delta_2\right) \right]^2} = G_{\mu_2}. \\
\frac{\partial g(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \lambda_2} &= \frac{-\left[\mu_1^2(1-\delta_1)\left(\frac{\mu_1}{\lambda_1} + \delta_1\right) \right] \left[\mu_2^2(1-\delta_2) \left(-\frac{\mu_2}{\lambda_2^2} \right) \right]}{\left[\mu_2^2(1-\delta_2)\left(\frac{\mu_2}{\lambda_2} + \delta_2\right) \right]^2} = G_{\lambda_2}.
\end{aligned}$$

$$\frac{\partial g(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \delta_2} = \frac{-[\mu_1^2(1-\delta_1)(\frac{\mu_1}{\lambda_1} + \delta_1)]\{\mu_2^2[(1-\delta_2) - (\frac{\mu_2}{\lambda_2} + \delta_2)]\}}{[\mu_2^2(1-\delta_2)(\frac{\mu_2}{\lambda_2} + \delta_2)]^2} = G_{\delta_2}.$$

Therefore,

$$\begin{aligned} \hat{\eta}_{ratio} &\approx g(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2) + \frac{\partial g(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \mu_1}(\hat{\mu}_1 - \mu_1) \\ &+ \frac{\partial g(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \lambda_1}(\hat{\lambda}_1 - \lambda_1) + \frac{\partial g(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \delta_1}(\hat{\delta}_1 - \delta_1) \\ &+ \frac{\partial g(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \mu_2}(\hat{\mu}_2 - \mu_2) + \frac{\partial g(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \lambda_2}(\hat{\lambda}_2 - \lambda_2) \\ &+ \frac{\partial g(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2)}{\partial \delta_2}(\hat{\delta}_2 - \delta_2) \\ &= g(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2) + G_{\mu_1}(\hat{\mu}_1 - \mu_1) + G_{\lambda_1}(\hat{\lambda}_1 - \lambda_1) \\ &+ G_{\delta_1}(\hat{\delta}_1 - \delta_1) + G_{\mu_2}(\hat{\mu}_2 - \mu_2) + G_{\lambda_2}(\hat{\lambda}_2 - \lambda_2) + G_{\delta_2}(\hat{\delta}_2 - \delta_2). \end{aligned} \quad (3-74)$$

Then, taking the expectation and variance in the equation, the approximate mean and variance for the $\hat{\eta}_{ratio}$ is given by

$$E(\hat{\eta}_{ratio}) = \hat{\eta}_{ratio} \quad (3-75)$$

$$\begin{aligned} Var(\hat{\eta}_{ratio}) &= Var(g(\mu_1, \lambda_1, \delta_1, \mu_2, \lambda_2, \delta_2) + G_{\mu_1}(\hat{\mu}_1 - \mu_1) + G_{\lambda_1}(\hat{\lambda}_1 - \lambda_1) \\ &+ G_{\delta_1}(\hat{\delta}_1 - \delta_1) + G_{\mu_2}(\hat{\mu}_2 - \mu_2) + G_{\lambda_2}(\hat{\lambda}_2 - \lambda_2) + G_{\delta_2}(\hat{\delta}_2 - \delta_2)) \\ &= G_{\mu_1}^2 Var(\hat{\mu}_1 - \mu_1) + G_{\lambda_1}^2 Var(\hat{\lambda}_1 - \lambda_1) + G_{\delta_1}^2 Var(\hat{\delta}_1 - \delta_1) \\ &+ G_{\mu_2}^2 Var(\hat{\mu}_2 - \mu_2) + G_{\lambda_2}^2 Var(\hat{\lambda}_2 - \lambda_2) + G_{\delta_2}^2 Var(\hat{\delta}_2 - \delta_2) \\ &= G_{\mu_1}^2 \left(\frac{\mu_1^3}{\lambda_1 n_{1(1)}} \right) + G_{\lambda_1}^2 \left(\frac{2\lambda_1^2}{n_{1(1)}} \right) + G_{\delta_1}^2 \left(\frac{\delta_1(1-\delta_1)}{n_1} \right) \\ &+ G_{\mu_2}^2 \left(\frac{\mu_2^3}{\lambda_2 n_{2(1)}} \right) + G_{\lambda_2}^2 \left(\frac{2\lambda_2^2}{n_{2(1)}} \right) + G_{\delta_2}^2 \left(\frac{\delta_2(1-\delta_2)}{n_2} \right). \end{aligned} \quad (3-76)$$

The estimate of $Var(\hat{\eta}_{ratio})$ can be written as follows:

$$\begin{aligned} \hat{Var}(\hat{\eta}_{ratio}) &= G_{\hat{\mu}_1}^2 \left(\frac{\hat{\mu}_1^3}{\hat{\lambda}_1 n_{1(1)}} \right) + G_{\hat{\lambda}_1}^2 \left(\frac{2\hat{\lambda}_1^2}{n_{1(1)}} \right) + G_{\hat{\delta}_1}^2 \left(\frac{\hat{\delta}_1(1-\hat{\delta}_1)}{n_1} \right) \\ &+ G_{\hat{\mu}_2}^2 \left(\frac{\hat{\mu}_2^3}{\hat{\lambda}_2 n_{2(1)}} \right) + G_{\hat{\lambda}_2}^2 \left(\frac{2\hat{\lambda}_2^2}{n_{2(1)}} \right) + G_{\hat{\delta}_2}^2 \left(\frac{\hat{\delta}_2(1-\hat{\delta}_2)}{n_2} \right). \end{aligned} \quad (3-77)$$

The $100(1 - \alpha)\%$ confidence interval for η_{diff} using normal approximation is

$$CI_{\eta_{diff}}^{NA} = (\hat{\eta}_1 - \hat{\eta}_2) \pm z_{1-\frac{\alpha}{2}} \sqrt{\hat{Var}(\hat{\eta}_{diff})}, \quad (3-78)$$

where $z_{1-\frac{\alpha}{2}}$ is the $(1 - \frac{\alpha}{2})$ th percentile of the standard normal distribution.

The $100(1 - \alpha)\%$ confidence interval for η_{ratio} using normal approximation is

$$CI_{\eta_{ratio}}^{NA} = (\hat{\eta}_1 / \hat{\eta}_2) \pm z_{1-\frac{\alpha}{2}} \sqrt{\hat{Var}(\hat{\eta}_{ratio})}, \quad (3-79)$$

where $z_{1-\frac{\alpha}{2}}$ is the $(1 - \frac{\alpha}{2})$ th percentile of the standard normal distribution.

3.4 Simulation studies and results

This study assesses the performance of several methods for constructing confidence intervals for the variance, as well as for the difference and ratio of variances under the delta-IG (delta-IG) distribution. For a single variance, the methods considered include AGCI-VST, AGCI-WS, AGCI-HN PBPCI-VST, PBPCI-WS, PBPCI-HN, FCI-VST, FCI-WS, FCI-HN, and BCI. For the difference and ratio between two variances,

the methods AGCI, FCI, BCI, MOVER, and NA are employed. A Monte Carlo simulation study, implemented in R, is conducted to evaluate these methods in terms of coverage probabilities (CPs) and average widths (AWs). An effective confidence interval method is expected to achieve coverage probabilities close to the nominal confidence level. Among methods with comparable coverage performance, the preferred method is the one that produces the shortest interval width. The simulation is based on 5,000 replications. For the single variance, samples are generated from a delta-IG distribution, $X \sim \text{delta-IG}(\mu, \lambda, \delta)$, with sample sizes $n = 30, 50, 100$, or 200 . The mean μ is set to 0.5 or 1.0 , the shape parameter λ to $1, 2, 5$, or 10 , and the zero-inflation probability δ to 0.3 or 0.5 .

For the difference and ratio of two variances, data are generated from two independent delta-IG populations, $X_{ij} \sim \text{delta-IG}(\mu_i, \lambda_i, \delta_i)$, for $i = 1, 2$ and $j = 1, \dots, n_i$. The sample size combinations (n_1, n_2) include $(30, 30)$, $(50, 50)$, $(100, 100)$, $(200, 200)$, $(30, 50)$, $(50, 100)$, and $(100, 200)$. The mean parameters (μ_1, μ_2) are set to $(0.5, 0.5)$, $(0.5, 1.0)$, and $(1.0, 1.0)$, while the shape parameters (λ_1, λ_2) are $(1, 1)$, $(1, 5)$, $(5, 5)$, $(5, 10)$, and $(10, 10)$. The zero-inflation probabilities (δ_1, δ_2) are specified as $(0.3, 0.3)$, $(0.3, 0.5)$, and $(0.5, 0.5)$.

The performance of the competing methods for constructing confidence intervals for a single variance was evaluated using CPs and AWs, as reported in Tables 3-1 and 3-2 and illustrated in Figure 3-1. Overall, the AGCI approaches consistently achieved CPs close to or exceeding the nominal level of 0.95 across most scenarios. For larger sample sizes, the FCI based on Wilson's score method also attained comparable coverage. Among the AGCI variants, the VST and Wilson's score methods outperformed the Hannig approach in terms of coverage. The PBPCI method showed improved performance as sample sizes increased, reflecting standard large-sample behavior. In terms of interval width, AGCI based on the VST method produced the narrowest intervals, followed by the Hannig- and Wilson-based AGCI methods, while PBPCI, FCI, and BCI yielded wider intervals. Considering both criteria, AGCI with VST provided the best overall performance.

For the difference and ratio of two variances, the results are summarized in Tables 3-3 to 3-6 and Figures 3-2 and 3-3. For the difference of variances, CPs approached the nominal level as sample sizes increased, with AGCI and MOVER showing the most stable performance across both balanced and unbalanced designs. Interval widths decreased when increasing sample size for all methods, with AGCI and MOVER achieving the most substantial reductions, while FCI, BCI, and NA produced wider intervals, particularly in small samples. For the ratio of variances, CPs exhibited greater variability, especially in small or unbalanced samples. Among the methods, MOVER maintained coverage closest to the nominal level. Interval widths for ratio-based inference were generally larger and decreased more slowly compared to the difference case. Across most scenarios, AGCI and MOVER provided the most reliable results, combining adequate coverage with relatively narrow intervals. MOVER tended to perform better when zero-inflation probabilities were equal or sample sizes were large and unbalanced, whereas AGCI showed advantages when the probabilities differed.

Overall, AGCI and MOVER emerged as the most effective methods, with MOVER demonstrating strong robustness in more complex sampling conditions.



TABLE 3-1 The CPs of the 95% confidence intervals for the variance of a delta-IG distribution.

n	μ	λ	δ	Coverage Probability									
				AGCI.VST	AGCI.WS	AGCI.HN	PBPCI.VST	PBPCI.WS	PBPCI.HN	FCL.VST	FCL.WS	FCL.HN	BCI
30	0.5	1	0.3	0.9494	0.9510	0.9502	0.8456	0.8370	0.8380	0.9216	0.9254	0.9254	0.9382
			0.5	0.9500	0.9502	0.9382	0.8242	0.8176	0.8138	0.9220	0.9030	0.9078	0.9316
		2	0.3	0.9466	0.9508	0.9482	0.8560	0.8538	0.8526	0.9292	0.9270	0.9308	0.9308
			0.5	0.9490	0.9500	0.9426	0.8514	0.8334	0.8320	0.9322	0.9068	0.9106	0.9399
		5	0.3	0.9492	0.9588	0.9200	0.8862	0.8856	0.8702	0.9368	0.8854	0.8968	0.9366
			0.5	0.9500	0.9450	0.9368	0.8786	0.8528	0.8504	0.9308	0.8886	0.9036	0.9313
		10	0.3	0.9476	0.9718	0.8722	0.9058	0.9258	0.8804	0.9404	0.7764	0.7984	0.9425
			0.5	0.9506	0.9274	0.9226	0.8948	0.8632	0.8614	0.9278	0.8560	0.8830	0.9250
	1	1	0.3	0.9492	0.9520	0.9482	0.8318	0.8310	0.8322	0.9194	0.9192	0.9178	0.9282
			0.5	0.9516	0.9476	0.9384	0.7934	0.7902	0.7888	0.9044	0.8986	0.9024	0.9190
		2	0.3	0.9494	0.9510	0.9502	0.8456	0.8370	0.8380	0.9216	0.9254	0.9254	0.9247
			0.5	0.9500	0.9502	0.9382	0.8242	0.8176	0.8138	0.9220	0.9030	0.9078	0.9336
		5	0.3	0.9474	0.9506	0.9442	0.8652	0.8584	0.8582	0.9336	0.9230	0.9264	0.9358
			0.5	0.9480	0.9516	0.9434	0.8558	0.8346	0.8340	0.9342	0.9042	0.9090	0.9321
		10	0.3	0.9492	0.9588	0.9200	0.8862	0.8856	0.8702	0.9368	0.8854	0.8968	0.9361
			0.5	0.9500	0.9450	0.9368	0.8786	0.8528	0.8504	0.9308	0.8886	0.9036	0.9300
50	0.5	1	0.3	0.9454	0.9526	0.9514	0.8870	0.8844	0.8852	0.9452	0.9324	0.9320	0.9356
			0.5	0.9466	0.9502	0.9364	0.8620	0.8614	0.8602	0.9268	0.9168	0.9236	0.9262
		2	0.3	0.9472	0.9548	0.9512	0.8978	0.8900	0.8906	0.9436	0.9340	0.9346	0.9384
			0.5	0.9482	0.9502	0.9390	0.8828	0.8814	0.8804	0.9306	0.9222	0.9270	0.9410
		5	0.3	0.9472	0.9538	0.9158	0.9116	0.9114	0.8934	0.9460	0.8840	0.8988	0.9407
			0.5	0.9470	0.9466	0.9368	0.8950	0.8964	0.8962	0.9324	0.9064	0.9200	0.9322
		10	0.3	0.9484	0.9512	0.8634	0.9230	0.9356	0.8946	0.9446	0.7542	0.7910	0.9368
			0.5	0.9478	0.9342	0.9272	0.9090	0.8990	0.8976	0.9334	0.8852	0.9068	0.9360
	1	1	0.3	0.9462	0.9496	0.9480	0.8694	0.8732	0.8732	0.9290	0.9252	0.9270	0.9215
			0.5	0.9462	0.9504	0.9384	0.8496	0.8462	0.8422	0.9220	0.9154	0.9154	0.9258
		2	0.3	0.9454	0.9526	0.9514	0.8870	0.8844	0.8852	0.9452	0.9324	0.9320	0.9340
			0.5	0.9466	0.9502	0.9364	0.8620	0.8614	0.8602	0.9268	0.9168	0.9236	0.9297

TABLE 3-1 (CONTINUED)

n	μ	λ	δ	Coverage Probability									
				AGCI.VST	AGCI.WS	AGCI.HN	PBPCI.VST	PBPCI.WS	PBPCI.HN	FCL.VST	FCL.WS	FCL.HN	BCI
50	1	5	0.3	0.9482	0.9554	0.9472	0.9006	0.8948	0.8932	0.9454	0.9276	0.9312	0.9372
			0.5	0.9474	0.9506	0.9408	0.8846	0.8878	0.8864	0.9302	0.9200	0.9290	0.9414
		10	0.3	0.9472	0.9538	0.9158	0.9116	0.9114	0.8934	0.9460	0.8840	0.8988	0.9401
			0.5	0.9470	0.9466	0.9368	0.8950	0.8964	0.8962	0.9324	0.9064	0.9200	0.9371
100	0.5	1	0.3	0.9514	0.9524	0.9512	0.9128	0.9098	0.9106	0.9420	0.9418	0.9432	0.9349
			0.5	0.9554	0.9542	0.9458	0.9052	0.8998	0.8982	0.9428	0.9286	0.9296	0.9292
		2	0.3	0.9530	0.9524	0.9512	0.9194	0.9120	0.9120	0.9408	0.9408	0.9376	0.9378
			0.5	0.9554	0.9550	0.9422	0.9178	0.9072	0.9068	0.9436	0.9316	0.9330	0.9327
		5	0.3	0.9522	0.9542	0.9110	0.9232	0.9298	0.9142	0.9426	0.8854	0.8970	0.9404
			0.5	0.9540	0.9518	0.9414	0.9238	0.9224	0.9222	0.9430	0.9276	0.9364	0.9361
		10	0.3	0.9536	0.9540	0.8638	0.9306	0.9454	0.9036	0.9460	0.7504	0.7792	0.9401
			0.5	0.9538	0.9416	0.9314	0.9296	0.9234	0.9204	0.9410	0.9178	0.9248	0.9441
	1	1	0.3	0.9534	0.9510	0.9476	0.9090	0.9024	0.9018	0.9408	0.9404	0.9400	0.9375
			0.5	0.9550	0.9522	0.9426	0.8972	0.8964	0.8946	0.9360	0.9354	0.9332	0.9356
		2	0.3	0.9514	0.9524	0.9512	0.9128	0.9098	0.9106	0.9420	0.9418	0.9432	0.9408
			0.5	0.9554	0.9542	0.9458	0.9052	0.8998	0.8982	0.9428	0.9286	0.9296	0.9341
		5	0.3	0.9530	0.9522	0.9478	0.9238	0.9174	0.9148	0.9440	0.9364	0.9356	0.9469
			0.5	0.9540	0.9550	0.9430	0.9190	0.9110	0.9092	0.9414	0.9282	0.9346	0.9431
		10	0.3	0.9522	0.9542	0.9110	0.9232	0.9298	0.9142	0.9426	0.8854	0.8970	0.9461
			0.5	0.9540	0.9518	0.9414	0.9238	0.9224	0.9222	0.9430	0.9276	0.9364	0.9441
200	0.5	1	0.3	0.9476	0.9532	0.9514	0.9332	0.9324	0.9326	0.9526	0.9448	0.9448	0.9404
			0.5	0.9528	0.9444	0.9310	0.9298	0.9236	0.9228	0.9482	0.9374	0.9414	0.9424
		2	0.3	0.9464	0.9492	0.9460	0.9398	0.9328	0.9340	0.9536	0.9446	0.9436	0.9470
			0.5	0.9518	0.9464	0.9332	0.9390	0.9310	0.9288	0.9504	0.9362	0.9420	0.9415
		5	0.3	0.9464	0.9462	0.9002	0.9394	0.9412	0.9278	0.9494	0.8876	0.8996	0.9433
			0.5	0.9516	0.9470	0.9356	0.9410	0.9346	0.9318	0.9530	0.9328	0.9356	0.9466

TABLE 3-1 (CONTINUED)

n	μ	λ	δ	Coverage Probability									
				AGCI.VST	AGCI.WS	AGCI.HN	PBPCI.VST	PBPCI.WS	PBPCI.HN	FCL.VST	FCL.WS	FCL.HN	BCI
200	0.5	10	0.3	0.9476	0.9532	0.9514	0.9332	0.9324	0.9326	0.9526	0.9448	0.9448	0.9404
			0.5	0.9528	0.9444	0.9310	0.9298	0.9236	0.9228	0.9482	0.9374	0.9414	0.9424
	1	1	0.3	0.9466	0.9458	0.8558	0.9400	0.9530	0.9076	0.9496	0.7414	0.7770	0.9460
			0.5	0.9516	0.9414	0.9292	0.9408	0.9318	0.9316	0.9504	0.9282	0.9330	0.9454
		2	0.3	0.9470	0.9542	0.9514	0.9316	0.9270	0.9278	0.9474	0.9444	0.9464	0.9410
			0.5	0.9538	0.9454	0.9314	0.9232	0.9156	0.9156	0.9466	0.9366	0.9356	0.9454
		5	0.3	0.9476	0.9532	0.9514	0.9332	0.9324	0.9326	0.9526	0.9448	0.9448	0.9433
			0.5	0.9528	0.9444	0.9310	0.9298	0.9236	0.9228	0.9482	0.9374	0.9414	0.9431
		10	0.3	0.9464	0.9486	0.9418	0.9394	0.9354	0.9340	0.9530	0.9412	0.9388	0.9421
			0.5	0.9530	0.9466	0.9332	0.9398	0.9282	0.9264	0.9538	0.9370	0.9400	0.9457

Note: Bold values indicate coverage probabilities (CP) that satisfy the nominal confidence level of 0.95 ($CP \geq 0.945$, rounded to 0.950).

TABLE 3-2 The AWs of the 95% confidence intervals for the variance of a delta-IG distribution.

n	μ	λ	δ	Average Widths									
				AGCI.VST	AGCI.WS	AGCI.HN	PBPCI.VST	PBPCI.WS	PBPCI.HN	FCL.VST	FCL.WS	FCL.HN	BCI
30	0.5	1	0.3	0.1335	0.1326	0.1308	0.2421	0.2357	0.2354	0.3353	0.3236	0.3239	0.5990
			0.5	0.1275	0.1359	0.1293	0.2331	0.2285	0.2260	0.3677	0.3464	0.3478	0.6958
		2	0.3	0.0668	0.0685	0.0667	0.1064	0.1050	0.1045	0.1383	0.1340	0.1343	0.2611
			0.5	0.0637	0.0705	0.0660	0.1095	0.1087	0.1073	0.1529	0.1453	0.1462	0.2715
		5	0.3	0.0267	0.0358	0.0320	0.0441	0.0494	0.0466	0.0531	0.0513	0.0516	0.1122
			0.5	0.0254	0.0333	0.0292	0.0503	0.0516	0.0502	0.0619	0.0588	0.0593	0.1240
		10	0.3	0.0134	0.0285	0.0232	0.0255	0.0365	0.0319	0.0291	0.0281	0.0283	0.0597
			0.5	0.0127	0.0222	0.0180	0.0313	0.0343	0.0323	0.0356	0.0338	0.0341	0.0628
	1	1	0.3	1.0691	1.0710	1.0520	2.6079	2.4860	2.4750	3.9740	3.7510	3.7500	5.5163
			0.5	1.0197	1.0750	1.0300	2.3709	2.3450	2.3180	4.5222	4.2720	4.2790	5.3978
		2	0.3	0.5341	0.5302	0.5230	0.9685	0.9427	0.9415	1.3412	1.2940	1.2960	2.4700
			0.5	0.5099	0.5435	0.5174	0.9324	0.9139	0.9040	1.4707	1.3860	1.3910	3.2201
		5	0.3	0.2137	0.2266	0.2178	0.3373	0.3364	0.3325	0.4305	0.4157	0.4170	0.9172
			0.5	0.2038	0.2308	0.2138	0.3547	0.3541	0.3491	0.4786	0.4564	0.4596	0.9198
	10	10	0.3	0.1069	0.1430	0.1279	0.1763	0.1975	0.1865	0.2125	0.2052	0.2062	0.4572
			0.5	0.1018	0.1330	0.1167	0.2014	0.2063	0.2006	0.2476	0.2350	0.2371	0.5261
50	0.5	1	0.3	0.0934	0.0943	0.0934	0.1885	0.1895	0.1894	0.2260	0.2269	0.2273	0.4264
			0.5	0.0841	0.0920	0.0873	0.1842	0.1820	0.1806	0.2345	0.2257	0.2268	0.4497
		2	0.3	0.0467	0.0486	0.0479	0.0849	0.0856	0.0853	0.0984	0.0981	0.0983	0.2121
			0.5	0.0421	0.0473	0.0443	0.0876	0.0877	0.0869	0.1047	0.1017	0.1024	0.1990
		5	0.3	0.0187	0.0263	0.0232	0.0352	0.0400	0.0378	0.0391	0.0387	0.0389	0.0835
			0.5	0.0168	0.0217	0.0191	0.0403	0.0410	0.0401	0.0450	0.0435	0.0438	0.0862
		10	0.3	0.0093	0.0222	0.0176	0.0203	0.0294	0.0256	0.0218	0.0215	0.0217	0.0498
			0.5	0.0084	0.0142	0.0115	0.0249	0.0265	0.0253	0.0267	0.0258	0.0260	0.0539
	1	1	0.3	0.7472	0.7644	0.7521	1.8923	1.9230	1.9190	2.3966	2.4260	2.4290	4.0013

TABLE 3-2 (CONTINUED)

n	μ	λ	δ	Average Widths									
				AGCI.VST	AGCI.WS	AGCI.HN	PBPCI.VST	PBPCI.WS	PBPCI.HN	FCI.VST	FCI.WS	FCI.HN	BCI
50	1	1	0.5	0.6736	0.7299	0.6962	1.8226	1.7830	1.7700	2.5316	2.4140	2.4260	5.1454
			0.3	0.3738	0.3770	0.3734	0.7541	0.7580	0.7578	0.9041	0.9077	0.9092	1.8844
		5	0.5	0.3365	0.3678	0.3492	0.7368	0.7279	0.7222	0.9378	0.9028	0.9070	1.8296
			0.3	0.1495	0.1612	0.1553	0.2699	0.2751	0.2726	0.3092	0.3074	0.3083	0.5742
		10	0.5	0.1346	0.1541	0.1431	0.2847	0.2851	0.2822	0.3342	0.3243	0.3265	0.6135
			0.3	0.0748	0.1053	0.0929	0.1409	0.1599	0.1514	0.1565	0.1549	0.1555	0.3043
			0.5	0.0673	0.0868	0.0766	0.1611	0.1638	0.1606	0.1798	0.1739	0.1752	0.3769
			0.3	0.0673	0.0868	0.0766	0.1611	0.1638	0.1606	0.1798	0.1739	0.1752	0.3769
100	0.5	1	0.3	0.0617	0.0620	0.0616	0.1340	0.1331	0.1331	0.1464	0.1452	0.1454	0.3066
			0.5	0.0537	0.0595	0.0564	0.1307	0.1316	0.1304	0.1462	0.1444	0.1447	0.3001
		2	0.3	0.0309	0.0319	0.0313	0.0611	0.0612	0.0610	0.0658	0.0653	0.0653	0.1424
			0.5	0.0269	0.0303	0.0284	0.0637	0.0642	0.0635	0.0693	0.0683	0.0685	0.1386
		5	0.3	0.0123	0.0180	0.0156	0.0253	0.0287	0.0271	0.0268	0.0266	0.0266	0.0588
			0.5	0.0107	0.0133	0.0120	0.0293	0.0298	0.0294	0.0308	0.0304	0.0305	0.0568
		10	0.3	0.0062	0.0160	0.0124	0.0145	0.0211	0.0183	0.0151	0.0150	0.0151	0.0377
			0.5	0.0054	0.0083	0.0068	0.0181	0.0188	0.0183	0.0186	0.0183	0.0184	0.0399
	1	1	0.3	0.4940	0.5047	0.4974	1.3148	1.3150	1.3120	1.4647	1.4590	1.4600	3.7294
			0.5	0.4298	0.4731	0.4500	1.2273	1.2200	1.2110	1.4258	1.3980	1.3990	2.5945
		2	0.3	0.2469	0.2478	0.2463	0.5362	0.5325	0.5324	0.5858	0.5809	0.5815	1.2469
			0.5	0.2149	0.2379	0.2256	0.5228	0.5262	0.5217	0.5847	0.5778	0.5787	1.2411
		5	0.3	0.0988	0.1061	0.1023	0.1941	0.1967	0.1949	0.2082	0.2064	0.2067	0.4864
			0.5	0.0859	0.0981	0.1023	0.2072	0.2092	0.1949	0.2233	0.2203	0.2067	0.4818
		10	0.3	0.0494	0.0720	0.0624	0.1014	0.1147	0.1084	0.1071	0.1064	0.1066	0.2252
			0.5	0.0430	0.0533	0.0479	0.1173	0.1191	0.1174	0.1234	0.1215	0.1219	0.2630
200	0.5	1	0.3	0.0424	0.0426	0.0424	0.0947	0.0953	0.0953	0.0991	0.0992	0.0994	0.2013
			0.5	0.0363	0.0404	0.0383	0.0931	0.0941	0.0933	0.0980	0.0973	0.0974	0.2065
		2	0.3	0.0212	0.0218	0.0215	0.0436	0.0442	0.0441	0.0452	0.0453	0.0454	0.0962
			0.5	0.0181	0.0204	0.0192	0.0406	0.0463	0.0459	0.0476	0.0472	0.0473	0.0990
		5	0.3	0.0085	0.0126	0.0108	0.0182	0.0206	0.0195	0.0187	0.0187	0.0187	0.0406
			0.5	0.0085	0.0126	0.0108	0.0182	0.0206	0.0195	0.0187	0.0187	0.0187	0.0406
		10	0.3	0.0085	0.0126	0.0108	0.0182	0.0206	0.0195	0.0187	0.0187	0.0187	0.0406
			0.5	0.0085	0.0126	0.0108	0.0182	0.0206	0.0195	0.0187	0.0187	0.0187	0.0406

TABLE 3-2 (CONTINUED)

n	μ	λ	δ	Average Widths									
				AGCI.VST	AGCI.WS	AGCI.HN	PBPCI.VST	PBPCI.WS	PBPCI.HN	FCL.VST	FCL.WS	FCL.HN	BCI
200	0.5	5	0.5	<i>0.0073</i>	0.0086	0.0079	0.2102	0.0213	0.0211	0.0215	0.0214	0.0214	0.0405
			10	<i>0.0042</i>	0.0115	0.0088	0.0104	0.0151	0.0131	0.0106	0.0106	0.0106	0.0269
			0.5	<i>0.0036</i>	0.0050	0.0043	0.0129	0.0133	0.0130	0.0131	0.0130	0.0130	0.0308
	1	1	0.3	<i>0.3393</i>	0.3478	0.3430	0.9076	0.9211	0.9194	0.9597	0.9655	0.9668	2.6801
			0.5	<i>0.2903</i>	0.3219	0.3058	0.8483	0.8577	0.8518	0.9089	0.9062	0.9073	2.3529
			2	<i>0.1696</i>	0.1704	0.1697	0.3789	0.3813	0.3812	0.3964	0.3969	0.3974	0.8109
		5	0.3	<i>0.1451</i>	0.1614	0.1531	0.3724	0.3763	0.3731	0.3921	0.3892	0.3896	0.7289
			0.5	<i>0.0679</i>	0.0728	0.0703	0.1389	0.1420	0.1409	0.1437	0.1438	0.1439	0.2817
			0.5	<i>0.0581</i>	0.0656	0.0617	0.1491	0.1509	0.1496	0.1544	0.1532	0.1535	0.2965
		10	0.3	<i>0.0339</i>	0.0505	0.0433	0.0727	0.0824	0.0780	0.0747	0.0747	0.0747	0.1420
			0.5	<i>0.0290</i>	0.0344	0.0316	0.0841	0.0854	0.0845	0.0862	0.0855	0.0856	0.1895

Note: The shortest AWs are in italic.

TABLE 3-3 The CPs of the 95% confidence intervals for the difference between two variances of delta-IG distributions.

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities				
				AGCI	FCI	BCI	MOVER	NA
(30,30)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9436	0.9522	0.9560	0.9434	1.0000
			(0.3,0.5)	0.9508	0.9462	0.9416	0.9508	1.0000
			(0.5,0.5)	0.9540	0.9496	0.9418	0.9538	1.0000
		(1,5)	(0.3,0.3)	0.9444	0.9306	0.9354	0.9438	0.9212
			(0.3,0.5)	0.9482	0.9334	0.9435	0.9480	0.9458
			(0.5,0.5)	0.9526	0.9464	0.9453	0.9526	0.9644
		(5,5)	(0.3,0.3)	0.9466	0.9500	0.9432	0.9460	0.9966
			(0.3,0.5)	0.9514	0.9432	0.9441	0.9512	0.9990
			(0.5,0.5)	0.9514	0.9470	0.9431	0.9512	0.9996
		(5,10)	(0.3,0.3)	0.9452	0.9446	0.9466	0.9450	0.9868
			(0.3,0.5)	0.9522	0.9424	0.9353	0.9522	0.9968
			(0.5,0.5)	0.9564	0.9430	0.9408	0.9562	0.9988
		(10,10)	(0.3,0.3)	0.9448	0.9448	0.9386	0.9446	0.9870
			(0.3,0.5)	0.9502	0.9432	0.9401	0.9498	0.9970
			(0.5,0.5)	0.9518	0.9442	0.9364	0.9514	0.9992
	(0.5,1)	(1,1)	(0.3,0.3)	0.9534	0.9184	0.9484	0.9534	0.8690
			(0.3,0.5)	0.9562	0.9314	0.9424	0.9554	0.8962
			(0.5,0.5)	0.9548	0.9384	0.9355	0.9544	0.9044
		(1,5)	(0.3,0.3)	0.9484	0.9508	0.9488	0.9482	0.9930
			(0.3,0.5)	0.9522	0.9496	0.9399	0.9518	0.9962
			(0.5,0.5)	0.9550	0.9536	0.9442	0.9548	0.9988
		(5,5)	(0.3,0.3)	0.9532	0.9380	0.9375	0.9530	0.9310
			(0.3,0.5)	0.9556	0.9424	0.9401	0.9552	0.9672
			(0.5,0.5)	0.9554	0.9524	0.9381	0.9552	0.9690
		(5,10)	(0.3,0.3)	0.9536	0.9450	0.9463	0.9532	0.9604
			(0.3,0.5)	0.9556	0.9438	0.9422	0.9552	0.9826
			(0.5,0.5)	0.9526	0.9520	0.9357	0.9524	0.9850
		(10,10)	(0.3,0.3)	0.9504	0.9436	0.9396	0.9502	0.9514
			(0.3,0.5)	0.9546	0.9430	0.9359	0.9544	0.9812
			(0.5,0.5)	0.9542	0.9526	0.9386	0.9540	0.9822
	(1,1)	(1,1)	(0.3,0.3)	0.9478	0.9486	0.9424	0.9476	1.0000
			(0.3,0.5)	0.9518	0.9480	0.9450	0.9518	0.9998
			(0.5,0.5)	0.9532	0.9472	0.9473	0.9530	1.0000
		(1,5)	(0.3,0.3)	0.9458	0.9194	0.9493	0.9454	0.8848
			(0.3,0.5)	0.9468	0.9238	0.9352	0.9468	0.9096
			(0.5,0.5)	0.9536	0.9468	0.9486	0.9532	0.9374
		(5,5)	(0.3,0.3)	0.9476	0.9544	0.9456	0.9474	0.9990
			(0.3,0.5)	0.9502	0.9466	0.9402	0.9500	1.0000
			(0.5,0.5)	0.9522	0.9498	0.9486	0.9522	1.0000
		(5,10)	(0.3,0.3)	0.9460	0.9466	0.9398	0.9458	0.9862
			(0.3,0.5)	0.9520	0.9416	0.9446	0.9518	0.9960
			(0.5,0.5)	0.9566	0.9458	0.9470	0.9564	0.9972
		(10,10)	(0.3,0.3)	0.9466	0.9500	0.9382	0.9460	0.9942
			(0.3,0.5)	0.9514	0.9432	0.9493	0.9512	0.9982
			(0.5,0.5)	0.9514	0.9470	0.9498	0.9512	0.9996

Note: Bold values indicate coverage probabilities (CP) that satisfy the nominal confidence level of 0.95 ($CP \geq 0.945$, rounded to 0.950).

TABLE 3-3 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities						
				AGCI	FCI	BCI	MOVER	NA		
(50,50)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9488	0.9520	0.9493	0.9486	1.0000		
			(0.3,0.5)	0.9524	0.9498	0.9382	0.9520	1.0000		
			(0.5,0.5)	0.9468	0.9490	0.9471	0.9464	1.0000		
			(1,5)	(0.3,0.3)	0.9500	0.9496	0.9449	0.9498	0.9360	
				(0.3,0.5)	0.9482	0.9330	0.9422	0.9480	0.9582	
				(0.5,0.5)	0.9514	0.9370	0.9387	0.9514	0.9764	
			(5,5)	(0.3,0.3)	0.9500	0.9472	0.9420	0.9496	0.9946	
				(0.3,0.5)	0.9508	0.9440	0.9385	0.9506	0.9992	
				(0.5,0.5)	0.9478	0.9468	0.9380	0.9476	1.0000	
		(5,10)	(0.3,0.3)	0.9472	0.9498	0.9381	0.9470	0.9846		
			(0.3,0.5)	0.9530	0.9430	0.9495	0.9530	0.9976		
			(0.5,0.5)	0.9528	0.9468	0.9363	0.9524	0.9986		
		(10,10)	(0.3,0.3)	0.9502	0.9446	0.9407	0.9502	0.9852		
			(0.3,0.5)	0.9510	0.9436	0.9385	0.9508	0.9968		
			(0.5,0.5)	0.9468	0.9498	0.9368	0.9468	0.9998		
		(0.5,1)	(1,1)	(0.3,0.3)	0.9516	0.9486	0.9484	0.9512	0.8982	
				(0.3,0.5)	0.9480	0.9246	0.9374	0.9478	0.9328	
				(0.5,0.5)	0.9510	0.9378	0.9425	0.9508	0.9356	
			(1,5)	(0.3,0.3)	0.9490	0.9520	0.9439	0.9488	0.9934	
				(0.3,0.5)	0.9504	0.9460	0.9479	0.9500	0.9970	
				(0.5,0.5)	0.9462	0.9522	0.9438	0.9460	0.9990	
			(5,5)	(0.3,0.3)	0.9530	0.9476	0.9490	0.9530	0.9414	
				(0.3,0.5)	0.9476	0.9390	0.9391	0.9474	0.9814	
				(0.5,0.5)	0.9500	0.9434	0.9375	0.9498	0.9798	
			(5,10)	(0.3,0.3)	0.9522	0.9492	0.9381	0.9516	0.9686	
				(0.3,0.5)	0.9490	0.9380	0.9389	0.9488	0.9902	
				(0.5,0.5)	0.9500	0.9478	0.9407	0.9498	0.9924	
			(10,10)	(0.3,0.3)	0.9542	0.9488	0.9484	0.9542	0.9620	
				(0.3,0.5)	0.9474	0.9372	0.9462	0.9472	0.9874	
				(0.5,0.5)	0.9504	0.9452	0.9476	0.9502	0.9906	
			(1,1)	(1,1)	(0.3,0.3)	0.9480	0.9484	0.9380	0.9476	1.0000
					(0.3,0.5)	0.9498	0.9480	0.9480	0.9494	1.0000
					(0.5,0.5)	0.9476	0.9492	0.9432	0.9468	1.0000
		(1,5)		(0.3,0.3)	0.9490	0.9430	0.9493	0.9488	0.9110	
				(0.3,0.5)	0.9472	0.9170	0.9489	0.9472	0.9260	
				(0.5,0.5)	0.9478	0.9290	0.9485	0.9472	0.9522	
		(5,5)		(0.3,0.3)	0.9488	0.9496	0.9396	0.9486	0.9986	
				(0.3,0.5)	0.9508	0.9450	0.9463	0.9506	0.9992	
				(0.5,0.5)	0.9476	0.9506	0.9397	0.9472	1.0000	
		(5,10)		(0.3,0.3)	0.9472	0.9556	0.9471	0.9468	0.9854	
				(0.3,0.5)	0.9522	0.9444	0.9362	0.9522	0.9950	
				(0.5,0.5)	0.9516	0.9484	0.9494	0.9516	0.9978	
		(10,10)		(0.3,0.3)	0.9500	0.9472	0.9433	0.9496	0.9940	
				(0.3,0.5)	0.9508	0.9440	0.9482	0.9506	0.9982	
				(0.5,0.5)	0.9478	0.9468	0.9385	0.9476	1.0000	

TABLE 3-3 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities				
				AGCI	FCI	BCI	MOVER	NA
(100,100)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9492	0.9456	0.9375	0.9490	0.9996
			(0.3,0.5)	0.9490	0.9484	0.9474	0.9488	0.9998
			(0.5,0.5)	0.9474	0.9424	0.9427	0.9470	1.0000
		(1,5)	(0.3,0.3)	0.9518	0.9370	0.9451	0.9516	0.9608
			(0.3,0.5)	0.9462	0.9342	0.9480	0.9458	0.9650
			(0.5,0.5)	0.9522	0.9366	0.9496	0.9522	0.9874
		(5,5)	(0.3,0.3)	0.9504	0.9484	0.9391	0.9500	0.9910
			(0.3,0.5)	0.9472	0.9414	0.9399	0.9472	0.9984
			(0.5,0.5)	0.9450	0.9396	0.9443	0.9448	0.9998
		(5,10)	(0.3,0.3)	0.9502	0.9468	0.9446	0.9498	0.9848
			(0.3,0.5)	0.9488	0.9452	0.9416	0.9486	0.9980
			(0.5,0.5)	0.9488	0.9434	0.9394	0.9486	0.9994
		(10,10)	(0.3,0.3)	0.9504	0.9520	0.9467	0.9502	0.9854
			(0.3,0.5)	0.9470	0.9386	0.9427	0.9468	0.9984
			(0.5,0.5)	0.9468	0.9420	0.9430	0.9466	0.9998
	(0.5,1)	(1,1)	(0.3,0.3)	0.9482	0.9380	0.9445	0.9480	0.9302
			(0.3,0.5)	0.9500	0.9084	0.9428	0.9496	0.9622
			(0.5,0.5)	0.9500	0.9170	0.9363	0.9494	0.9678
		(1,5)	(0.3,0.3)	0.9514	0.9496	0.9459	0.9512	0.9930
			(0.3,0.5)	0.9468	0.9384	0.9479	0.9466	0.9962
			(0.5,0.5)	0.9460	0.9374	0.9449	0.9458	0.9996
		(5,5)	(0.3,0.3)	0.9482	0.9480	0.9420	0.9480	0.9662
			(0.3,0.5)	0.9496	0.9298	0.9450	0.9496	0.9900
			(0.5,0.5)	0.9512	0.9294	0.9352	0.9508	0.9920
		(5,10)	(0.3,0.3)	0.9504	0.9508	0.9352	0.9502	0.9750
			(0.3,0.5)	0.9464	0.9290	0.9481	0.9462	0.9940
			(0.5,0.5)	0.9484	0.9308	0.9382	0.9482	0.9958
		(10,10)	(0.3,0.3)	0.9468	0.9482	0.9463	0.9468	0.9768
			(0.3,0.5)	0.9486	0.9274	0.9396	0.9478	0.9962
			(0.5,0.5)	0.9508	0.9302	0.9459	0.9506	0.9938
	(1,1)	(1,1)	(0.3,0.3)	0.9486	0.9482	0.9469	0.9482	1.0000
			(0.3,0.5)	0.9480	0.9500	0.9407	0.9480	1.0000
			(0.5,0.5)	0.9452	0.9486	0.9448	0.9450	1.0000
		(1,5)	(0.3,0.3)	0.9512	0.9350	0.9438	0.9508	0.9454
			(0.3,0.5)	0.9472	0.9216	0.9480	0.9470	0.9460
			(0.5,0.5)	0.9506	0.9268	0.9410	0.9506	0.9706
		(5,5)	(0.3,0.3)	0.9496	0.9502	0.9468	0.9494	0.9976
			(0.3,0.5)	0.9468	0.9444	0.9414	0.9466	0.9988
			(0.5,0.5)	0.9450	0.9420	0.9357	0.9446	1.0000
		(5,10)	(0.3,0.3)	0.9498	0.9462	0.9396	0.9492	0.9862
			(0.3,0.5)	0.9484	0.9472	0.9418	0.9480	0.9954
			(0.5,0.5)	0.9494	0.9438	0.9496	0.9492	0.9982
		(10,10)	(0.3,0.3)	0.9504	0.9484	0.9384	0.9500	0.9902
			(0.3,0.5)	0.9472	0.9414	0.9379	0.9472	0.9984
			(0.5,0.5)	0.9450	0.9396	0.9414	0.9448	0.9996

TABLE 3-3 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities				
				AGCI	FCI	BCI	MOVER	NA
(200,200)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9466	0.9504	0.9461	0.9466	0.9956
			(0.3,0.5)	0.9466	0.9508	0.9374	0.9466	0.9988
			(0.5,0.5)	0.9534	0.9448	0.9371	0.9534	1.0000
		(1,5)	(0.3,0.3)	0.9484	0.9462	0.9418	0.9484	0.9694
			(0.3,0.5)	0.9440	0.9502	0.9416	0.9436	0.9774
			(0.5,0.5)	0.9492	0.9334	0.9456	0.9486	0.9910
		(5,5)	(0.3,0.3)	0.9456	0.9496	0.9495	0.9456	0.9888
			(0.3,0.5)	0.9492	0.9556	0.9427	0.9486	0.9980
			(0.5,0.5)	0.9534	0.9490	0.9408	0.9532	0.9998
		(5,10)	(0.3,0.3)	0.9470	0.9510	0.9391	0.9466	0.9868
			(0.3,0.5)	0.9492	0.9530	0.9372	0.9488	0.9962
			(0.5,0.5)	0.9542	0.9462	0.9449	0.9542	0.9994
		(10,10)	(0.3,0.3)	0.9456	0.9478	0.9380	0.9452	0.9812
			(0.3,0.5)	0.9494	0.9504	0.9479	0.9492	0.9980
			(0.5,0.5)	0.9528	0.9460	0.9479	0.9528	0.9998
		(0.5,1)	(1,1)	(0.3,0.3)	0.9478	0.9332	0.9499	0.9470
			(1,1)	(0.3,0.5)	0.9474	0.9446	0.9406	0.9472
			(1,1)	(0.5,0.5)	0.9504	0.9254	0.9477	0.9502
		(1,5)	(1,5)	(0.3,0.3)	0.9484	0.9444	0.9463	0.9478
			(1,5)	(0.3,0.5)	0.9488	0.9482	0.9359	0.9484
			(1,5)	(0.5,0.5)	0.9502	0.9480	0.9462	0.9500
		(5,5)	(5,5)	(0.3,0.3)	0.9480	0.9366	0.9365	0.9478
			(5,5)	(0.3,0.5)	0.9498	0.9442	0.9427	0.9496
			(5,5)	(0.5,0.5)	0.9522	0.9402	0.9402	0.9518
		(5,10)	(5,10)	(0.3,0.3)	0.9470	0.9432	0.9391	0.9470
			(5,10)	(0.3,0.5)	0.9468	0.9462	0.9447	0.9466
			(5,10)	(0.5,0.5)	0.9534	0.9412	0.9371	0.9530
		(10,10)	(10,10)	(0.3,0.3)	0.9472	0.9434	0.9365	0.9466
			(10,10)	(0.3,0.5)	0.9498	0.9450	0.9496	0.9494
			(10,10)	(0.5,0.5)	0.9522	0.9384	0.9463	0.9522
		(1,1)	(1,1)	(0.3,0.3)	0.9470	0.9506	0.9376	0.9468
			(1,1)	(0.3,0.5)	0.9488	0.9514	0.9395	0.9486
			(1,1)	(0.5,0.5)	0.9532	0.9408	0.9486	0.9526
		(1,5)	(1,5)	(0.3,0.3)	0.9472	0.9382	0.9351	0.9470
			(1,5)	(0.3,0.5)	0.9446	0.9464	0.9493	0.9440
			(1,5)	(0.5,0.5)	0.9482	0.9234	0.9396	0.9482
		(5,5)	(5,5)	(0.3,0.3)	0.9466	0.9472	0.9416	0.9462
			(5,5)	(0.3,0.5)	0.9468	0.9556	0.9359	0.9466
			(5,5)	(0.5,0.5)	0.9538	0.9518	0.9363	0.9536
		(5,10)	(5,10)	(0.3,0.3)	0.9472	0.9496	0.9420	0.9466
			(5,10)	(0.3,0.5)	0.9490	0.9524	0.9394	0.9488
			(5,10)	(0.5,0.5)	0.9536	0.9472	0.9356	0.9536
		(10,10)	(10,10)	(0.3,0.3)	0.9456	0.9496	0.9450	0.9456
			(10,10)	(0.3,0.5)	0.9492	0.9556	0.9482	0.9486
			(10,10)	(0.5,0.5)	0.9534	0.9490	0.9470	0.9532

TABLE 3-3 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities				
				AGCI	FCI	BCI	MOVER	NA
(30,50)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9530	0.9476	0.9434	0.9526	0.9998
			(0.3,0.5)	0.9524	0.9520	0.9471	0.9520	1.0000
			(0.5,0.5)	0.9526	0.9388	0.9462	0.9526	1.0000
		(1,5)	(0.3,0.3)	0.9526	0.9388	0.9491	0.9526	0.9092
			(0.3,0.5)	0.9528	0.9450	0.9360	0.9526	0.9334
			(0.5,0.5)	0.9504	0.9156	0.9423	0.9500	0.9480
		(5,5)	(0.3,0.3)	0.9522	0.9472	0.9388	0.9522	0.9906
			(0.3,0.5)	0.9526	0.9478	0.9430	0.9526	0.9988
			(0.5,0.5)	0.9520	0.9332	0.9397	0.9518	0.9990
		(5,10)	(0.3,0.3)	0.9528	0.9436	0.9426	0.9526	0.9784
			(0.3,0.5)	0.9526	0.9520	0.9462	0.9524	0.9898
			(0.5,0.5)	0.9538	0.9324	0.9470	0.9538	0.9958
		(10,10)	(0.3,0.3)	0.9518	0.9458	0.9423	0.9514	0.9830
			(0.3,0.5)	0.9522	0.9534	0.9378	0.9522	0.9920
			(0.5,0.5)	0.9510	0.9334	0.9418	0.9506	0.9996
		(0.5,1)	(1,1)	0.9540	0.9380	0.9377	0.9540	0.9014
			(0.3,0.5)	0.9522	0.9452	0.9490	0.9518	0.9274
			(0.5,0.5)	0.9568	0.9066	0.9437	0.9560	0.9450
		(1,5)	(0.3,0.3)	0.9522	0.9498	0.9468	0.9520	0.9996
			(0.3,0.5)	0.9532	0.9462	0.9387	0.9530	0.9990
			(0.5,0.5)	0.9512	0.9378	0.9415	0.9504	0.9994
		(5,5)	(0.3,0.3)	0.9534	0.9468	0.9388	0.9532	0.9538
			(0.3,0.5)	0.9504	0.9464	0.9440	0.9498	0.9836
			(0.5,0.5)	0.9532	0.9312	0.9482	0.9528	0.9860
		(5,10)	(0.3,0.3)	0.9546	0.9454	0.9417	0.9542	0.9728
			(0.3,0.5)	0.9528	0.9514	0.9477	0.9526	0.9918
			(0.5,0.5)	0.9552	0.9270	0.9463	0.9550	0.9944
		(10,10)	(0.3,0.3)	0.9548	0.9462	0.9471	0.9546	0.9670
			(0.3,0.5)	0.9504	0.9518	0.9485	0.9500	0.9884
			(0.5,0.5)	0.9536	0.9262	0.9389	0.9536	0.9906
		(1,1)	(0.3,0.3)	0.9522	0.9452	0.9416	0.9516	1.0000
			(0.3,0.5)	0.9512	0.9434	0.9373	0.9508	1.0000
			(0.5,0.5)	0.9550	0.9422	0.9355	0.9548	1.0000
		(1,5)	(0.3,0.3)	0.9516	0.9312	0.9445	0.9514	0.8782
			(0.3,0.5)	0.9530	0.9400	0.9373	0.9530	0.8942
			(0.5,0.5)	0.9534	0.9016	0.9425	0.9532	0.9086
		(5,5)	(0.3,0.3)	0.9526	0.9482	0.9442	0.9524	0.9970
			(0.3,0.5)	0.9542	0.9498	0.9360	0.9538	0.9994
			(0.5,0.5)	0.9508	0.9364	0.9452	0.9506	0.9998
		(5,10)	(0.3,0.3)	0.9520	0.9436	0.9437	0.9514	0.9716
			(0.3,0.5)	0.9512	0.9504	0.9480	0.9512	0.9908
			(0.5,0.5)	0.9530	0.9312	0.9477	0.9528	0.9936
		(10,10)	(0.3,0.3)	0.9522	0.9472	0.9373	0.9522	0.9896
			(0.3,0.5)	0.9526	0.9478	0.9379	0.9526	0.9976
			(0.5,0.5)	0.9520	0.9332	0.9386	0.9518	0.9994

TABLE 3-3 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities				
				AGCI	FCI	BCI	MOVER	NA
(50,100)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9506	0.9490	0.9366	0.9504	0.9966
			(0.3,0.5)	0.9438	0.9482	0.9388	0.9432	0.9998
			(0.5,0.5)	0.9448	0.9482	0.9399	0.9446	1.0000
		(1,5)	(0.3,0.3)	0.9538	0.9356	0.9410	0.9536	0.9244
			(0.3,0.5)	0.9486	0.9412	0.9457	0.9476	0.9382
			(0.5,0.5)	0.9490	0.9498	0.9463	0.9486	0.9652
		(5,5)	(0.3,0.3)	0.9504	0.9440	0.9406	0.9502	0.9896
			(0.3,0.5)	0.9462	0.9442	0.9351	0.9458	0.9984
			(0.5,0.5)	0.9426	0.9478	0.9482	0.9426	0.9990
		(5,10)	(0.3,0.3)	0.9508	0.9426	0.9378	0.9506	0.9732
			(0.3,0.5)	0.9452	0.9454	0.9465	0.9448	0.9892
			(0.5,0.5)	0.9476	0.9480	0.9364	0.9474	0.9970
		(10,10)	(0.3,0.3)	0.9492	0.9448	0.9479	0.9486	0.9790
			(0.3,0.5)	0.9436	0.9472	0.9355	0.9434	0.9932
			(0.5,0.5)	0.9444	0.9454	0.9476	0.9442	0.9990
	(0.5,1)	(1,1)	(0.3,0.3)	0.9490	0.9192	0.9366	0.9488	0.9410
			(0.3,0.5)	0.9526	0.9372	0.9430	0.9524	0.9676
			(0.5,0.5)	0.9514	0.9326	0.9366	0.9512	0.9700
		(1,5)	(0.3,0.3)	0.9502	0.9418	0.9405	0.9502	0.9990
			(0.3,0.5)	0.9476	0.9428	0.9418	0.9468	0.9996
			(0.5,0.5)	0.9484	0.9430	0.9442	0.9482	0.9998
		(5,5)	(0.3,0.3)	0.9506	0.9318	0.9491	0.9506	0.9690
			(0.3,0.5)	0.9546	0.9414	0.9451	0.9540	0.9902
			(0.5,0.5)	0.9508	0.9444	0.9425	0.9506	0.9922
		(5,10)	(0.3,0.3)	0.9508	0.9314	0.9405	0.9508	0.9816
			(0.3,0.5)	0.9542	0.9388	0.9422	0.9542	0.9940
			(0.5,0.5)	0.9508	0.9436	0.9431	0.9508	0.9980
		(10,10)	(0.3,0.3)	0.9498	0.9308	0.9368	0.9488	0.9748
			(0.3,0.5)	0.9534	0.9398	0.9380	0.9534	0.9960
			(0.5,0.5)	0.9508	0.9450	0.9362	0.9506	0.9974
	(1,1)	(1,1)	(0.3,0.3)	0.9492	0.9474	0.9370	0.9488	1.0000
			(0.3,0.5)	0.9432	0.9478	0.9396	0.9426	1.0000
			(0.5,0.5)	0.9464	0.9492	0.9420	0.9460	1.0000
		(1,5)	(0.3,0.3)	0.9536	0.9260	0.9456	0.9532	0.9096
			(0.3,0.5)	0.9484	0.9436	0.9362	0.9484	0.9110
			(0.5,0.5)	0.9496	0.9424	0.9448	0.9494	0.9368
		(5,5)	(0.3,0.3)	0.9492	0.9460	0.9396	0.9490	0.9930
			(0.3,0.5)	0.9432	0.9512	0.9353	0.9428	0.9992
			(0.5,0.5)	0.9442	0.9498	0.9385	0.9440	0.9994
		(5,10)	(0.3,0.3)	0.9502	0.9450	0.9441	0.9502	0.9668
			(0.3,0.5)	0.9446	0.9486	0.9492	0.9446	0.9852
			(0.5,0.5)	0.9464	0.9488	0.9462	0.9460	0.9934
		(10,10)	(0.3,0.3)	0.9504	0.9440	0.9355	0.9502	0.9884
			(0.3,0.5)	0.9462	0.9442	0.9408	0.9458	0.9968
			(0.5,0.5)	0.9426	0.9478	0.9409	0.9426	0.9992

TABLE 3-3 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities				
				AGCI	FCI	BCI	MOVER	NA
(100,200)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9524	0.9464	0.9469	0.9522	0.9920
			(0.3,0.5)	0.9468	0.9472	0.9465	0.9462	0.9998
			(0.5,0.5)	0.9506	0.9512	0.9418	0.9502	1.0000
		(1,5)	(0.3,0.3)	0.9552	0.9232	0.9440	0.9548	0.9560
			(0.3,0.5)	0.9502	0.9388	0.9459	0.9498	0.9590
			(0.5,0.5)	0.9526	0.9476	0.9408	0.9526	0.9802
		(5,5)	(0.3,0.3)	0.9502	0.9370	0.9359	0.9498	0.9852
			(0.3,0.5)	0.9496	0.9462	0.9376	0.9494	0.9974
			(0.5,0.5)	0.9506	0.9432	0.9358	0.9506	0.9988
		(5,10)	(0.3,0.3)	0.9546	0.9300	0.9429	0.9544	0.9796
			(0.3,0.5)	0.9526	0.9424	0.9498	0.9524	0.9912
			(0.5,0.5)	0.9518	0.9426	0.9500	0.9518	0.9986
		(10,10)	(0.3,0.3)	0.9502	0.9336	0.9406	0.9502	0.9834
			(0.3,0.5)	0.9484	0.9466	0.9393	0.9482	0.9946
			(0.5,0.5)	0.9516	0.9418	0.9425	0.9510	0.9996
	(0.5,1)	(1,1)	(0.3,0.3)	0.9602	0.9216	0.9480	0.9600	0.9600
			(0.3,0.5)	0.9520	0.9324	0.9376	0.9520	0.9832
			(0.5,0.5)	0.9488	0.9402	0.9429	0.9488	0.9844
		(1,5)	(0.3,0.3)	0.9500	0.9464	0.9427	0.9494	0.9956
			(0.3,0.5)	0.9516	0.9508	0.9365	0.9516	0.9996
			(0.5,0.5)	0.9500	0.9462	0.9493	0.9500	1.0000
		(5,5)	(0.3,0.3)	0.9596	0.9368	0.9490	0.9594	0.9774
			(0.3,0.5)	0.9544	0.9452	0.9352	0.9544	0.9928
			(0.5,0.5)	0.9480	0.9472	0.9464	0.9478	0.9964
		(5,10)	(0.3,0.3)	0.9614	0.9348	0.9491	0.9614	0.9840
			(0.3,0.5)	0.9558	0.9448	0.9375	0.9556	0.9968
			(0.5,0.5)	0.9512	0.9438	0.9392	0.9506	0.9974
		(10,10)	(0.3,0.3)	0.9602	0.9342	0.9392	0.9602	0.9782
			(0.3,0.5)	0.9554	0.9450	0.9436	0.9548	0.9958
			(0.5,0.5)	0.9476	0.9434	0.9438	0.9474	0.9972
	(1,1)	(1,1)	(0.3,0.3)	0.9520	0.9424	0.9381	0.9516	0.9966
			(0.3,0.5)	0.9482	0.9500	0.9380	0.9480	1.0000
			(0.5,0.5)	0.9498	0.9504	0.9419	0.9494	1.0000
		(1,5)	(0.3,0.3)	0.9550	0.9036	0.9419	0.9546	0.9346
			(0.3,0.5)	0.9498	0.9274	0.9413	0.9492	0.9410
			(0.5,0.5)	0.9508	0.9374	0.9430	0.9508	0.9632
		(5,5)	(0.3,0.3)	0.9470	0.9410	0.9430	0.9470	0.9900
			(0.3,0.5)	0.9486	0.9466	0.9426	0.9486	0.9990
			(0.5,0.5)	0.9520	0.9458	0.9372	0.9514	0.9992
		(5,10)	(0.3,0.3)	0.9528	0.9306	0.9474	0.9526	0.9760
			(0.3,0.5)	0.9518	0.9442	0.9361	0.9516	0.9906
			(0.5,0.5)	0.9518	0.9474	0.9499	0.9516	0.9982
		(10,10)	(0.3,0.3)	0.9502	0.9370	0.9396	0.9498	0.9882
			(0.3,0.5)	0.9496	0.9462	0.9450	0.9494	0.9972
			(0.5,0.5)	0.9506	0.9432	0.9389	0.9506	0.9990

TABLE 3-4 The AWs of the 95% confidence intervals for the difference between two variances of delta-IG distributions.

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths				
				AGCI	FCI	BCI	MOVER	NA
(30,30)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.2032	0.5358	0.9134	0.2029	<i>0.5054</i>
			(0.3,0.5)	0.1385	0.3497	0.6884	<i>0.1383</i>	0.6020
			(0.5,0.5)	0.0896	0.2191	0.4222	<i>0.0895</i>	0.6900
		(1,5)	(0.3,0.3)	<i>0.1380</i>	0.3443	0.6670	0.1378	0.3468
			(0.3,0.5)	0.0960	0.2314	0.4776	<i>0.0959</i>	0.3553
			(0.5,0.5)	0.0633	0.1509	0.3066	<i>0.0632</i>	0.4688
		(5,5)	(0.3,0.3)	<i>0.0406</i>	0.0784	0.2052	<i>0.0406</i>	0.0990
			(0.3,0.5)	<i>0.0277</i>	0.0567	0.1335	<i>0.0277</i>	0.1257
			(0.5,0.5)	<i>0.0179</i>	0.0385	0.0874	<i>0.0179</i>	0.1501
		(5,10)	(0.3,0.3)	0.0314	0.0622	0.1482	<i>0.0313</i>	0.0840
			(0.3,0.5)	0.0216	0.0454	0.1067	<i>0.0215</i>	0.0970
			(0.5,0.5)	<i>0.0140</i>	0.0311	0.0713	<i>0.0140</i>	0.1244
		(10,10)	(0.3,0.3)	0.0203	0.0422	0.1001	0.0203	<i>0.0682</i>
			(0.3,0.5)	<i>0.0138</i>	0.0313	0.0695	<i>0.0138</i>	0.0821
			(0.5,0.5)	<i>0.0090</i>	0.0216	0.0433	<i>0.0090</i>	0.0943
	(0.5,1)	(1,1)	(0.3,0.3)	1.0779	3.9289	5.2874	<i>1.0764</i>	3.4710
			(0.3,0.5)	0.7552	2.4676	3.8485	<i>0.7543</i>	4.4720
			(0.5,0.5)	0.4990	1.4910	2.5111	<i>0.4984</i>	4.4980
		(1,5)	(0.3,0.3)	0.2678	0.5981	1.3545	<i>0.2674</i>	0.6174
			(0.3,0.5)	0.1833	0.4074	0.9097	<i>0.1830</i>	0.7957
			(0.5,0.5)	0.1190	0.2637	0.5955	<i>0.1188</i>	0.8851
		(5,5)	(0.3,0.3)	0.2156	0.4312	1.0310	<i>0.2153</i>	0.4861
			(0.3,0.5)	0.1511	0.3126	0.7114	<i>0.1509</i>	0.7164
			(0.5,0.5)	0.0998	0.2102	0.4753	<i>0.0997</i>	0.7060
		(5,10)	(0.3,0.3)	0.1119	0.2200	0.5316	<i>0.1118</i>	0.2817
			(0.3,0.5)	0.0779	0.1622	0.3702	<i>0.0778</i>	0.4118
			(0.5,0.5)	0.0513	0.1110	0.2430	<i>0.0512</i>	0.4244
		(10,10)	(0.3,0.3)	0.1078	0.2135	0.5087	<i>0.1077</i>	0.2794
			(0.3,0.5)	0.0755	0.1582	0.3824	<i>0.0754</i>	0.4119
			(0.5,0.5)	<i>0.0499</i>	0.1085	0.2387	<i>0.0499</i>	0.4138
	(1,1)	(1,1)	(0.3,0.3)	1.6257	6.6087	2.9601	<i>1.6231</i>	5.4600
			(0.3,0.5)	1.1075	3.9162	5.5739	<i>1.1060</i>	6.4530
			(0.5,0.5)	0.7170	2.2669	3.4048	<i>0.7161</i>	6.9950
		(1,5)	(0.3,0.3)	1.1039	3.9747	5.2099	<i>1.1023</i>	3.5320
			(0.3,0.5)	0.7676	2.4451	3.6789	<i>0.7666</i>	3.6680
			(0.5,0.5)	0.5061	1.5002	2.4653	<i>0.5055</i>	4.5900
		(5,5)	(0.3,0.3)	0.3250	0.6527	1.6359	<i>0.3245</i>	0.7047
			(0.3,0.5)	0.2216	0.4569	1.1113	<i>0.2213</i>	0.8939
			(0.5,0.5)	0.1434	0.3020	0.6823	<i>0.1432</i>	1.0450
		(5,10)	(0.3,0.3)	0.2513	0.5009	1.1851	<i>0.2509</i>	0.5676
			(0.3,0.5)	0.1724	0.3542	0.8246	<i>0.1722</i>	0.6495
			(0.5,0.5)	0.1123	0.2377	0.5668	<i>0.1122</i>	0.8377
		(10,10)	(0.3,0.3)	0.1625	0.3134	0.7993	<i>0.1622</i>	0.3933
			(0.3,0.5)	0.1108	0.2269	0.5566	<i>0.1106</i>	0.5021
			(0.5,0.5)	0.0717	0.1539	0.3375	<i>0.0716</i>	0.5974

Note: The shortest AW values are in italic.

TABLE 3-4 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths				
				AGCI	FCI	BCI	MOVER	NA
(50,50)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.0607	0.1445	0.3050	0.0606	0.3735
			(0.3,0.5)	0.1725	0.4443	0.8345	0.1722	0.4407
			(0.5,0.5)	0.1162	0.2873	0.5657	0.1160	0.5022
			(1,5)	(0.3,0.3)	0.0433	0.1017	0.2162	0.0432
			(0.3,0.5)	0.1355	0.3361	0.6603	0.1353	0.2653
			(0.5,0.5)	0.0951	0.2301	0.4813	0.0950	0.3474
			(5,5)	(0.3,0.3)	0.0121	0.0266	0.0575	0.0121
			(0.3,0.5)	0.0345	0.0682	0.1712	0.0345	0.0963
			(0.5,0.5)	0.0232	0.0484	0.1144	0.0232	0.1142
		(5,10)	(0.3,0.3)	0.0095	0.0216	0.0483	0.0095	0.0649
			(0.3,0.5)	0.0291	0.0583	0.1420	0.0291	0.0740
			(0.5,0.5)	0.0201	0.0424	0.0955	0.0200	0.0954
		(10,10)	(0.3,0.3)	0.0061	0.0151	0.0299	0.0061	0.0523
			(0.3,0.5)	0.0172	0.0372	0.0812	0.0172	0.0630
			(0.5,0.5)	0.0116	0.0269	0.0588	0.0116	0.0722
		(0.5,1)	(1,1)	(0.3,0.3)	0.3415	0.9692	1.6811	0.3411
			(0.3,0.5)	0.7675	2.5050	3.7803	0.7665	3.2520
			(0.5,0.5)	0.5051	1.4968	2.5595	0.5045	3.2320
			(1,5)	(0.3,0.3)	0.0807	0.1778	0.3853	0.0806
			(0.3,0.5)	0.2130	0.4962	1.0702	0.2127	0.6143
			(0.5,0.5)	0.1414	0.3260	0.6760	0.1412	0.6628
			(5,5)	(0.3,0.3)	0.0683	0.1450	0.3324	0.0682
			(0.3,0.5)	0.1534	0.3162	0.7423	0.1532	0.5485
			(0.5,0.5)	0.1010	0.2121	0.4958	0.1009	0.5472
		(5,10)	(0.3,0.3)	0.0350	0.0771	0.1717	0.0350	0.2175
			(0.3,0.5)	0.0816	0.1679	0.3994	0.0815	0.3195
			(0.5,0.5)	0.0536	0.1149	0.2560	0.0536	0.3264
		(10,10)	(0.3,0.3)	0.0341	0.0755	0.1616	0.0341	0.2151
			(0.3,0.5)	0.0767	0.1601	0.3846	0.0766	0.3187
			(0.5,0.5)	0.0505	0.1094	0.2425	0.0504	0.3214
		(1,1)	(1,1)	(0.3,0.3)	0.4852	1.4275	2.4407	0.4847
			(0.3,0.5)	1.3794	5.2961	6.5355	1.3773	4.3640
			(0.5,0.5)	0.9293	3.1140	4.5143	0.9281	5.0360
			(1,5)	(0.3,0.3)	0.3461	0.9804	1.6284	0.3457
			(0.3,0.5)	1.0829	3.9188	5.1871	1.0813	2.6260
			(0.5,0.5)	0.7607	2.4482	3.7343	0.7597	3.2860
			(5,5)	(0.3,0.3)	0.0971	0.2059	0.4649	0.0969
			(0.3,0.5)	0.2760	0.5592	1.3668	0.2756	0.6754
			(0.5,0.5)	0.1858	0.3851	0.8948	0.1856	0.7951
		(5,10)	(0.3,0.3)	0.0764	0.1633	0.3653	0.0763	0.4310
			(0.3,0.5)	0.2329	0.4680	1.1580	0.2325	0.4954
			(0.5,0.5)	0.1606	0.3323	0.7975	0.1604	0.6403
		(10,10)	(0.3,0.3)	0.0485	0.1065	0.2352	0.0485	0.3022
			(0.3,0.5)	0.1381	0.2728	0.6947	0.1379	0.3842
			(0.5,0.5)	0.0929	0.1935	0.4653	0.0928	0.4548

TABLE 3-4 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths				
				AGCI	FCI	BCI	MOVER	NA
(100,100)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.0762	0.1828	0.3867	0.0761	0.2532
			(0.3,0.5)	0.2002	0.5795	0.9779	0.1999	0.3019
			(0.5,0.5)	0.1323	0.3561	0.6245	0.1321	0.3403
		(1,5)	(0.3,0.3)	0.0625	0.1480	0.3116	0.0624	0.1798
			(0.3,0.5)	0.1375	0.3493	0.6702	0.1373	0.1853
			(0.5,0.5)	0.0954	0.2336	0.4793	0.0952	0.2399
		(5,5)	(0.3,0.3)	0.0152	0.0330	0.0732	0.0152	0.0529
			(0.3,0.5)	0.0400	0.0855	0.2032	0.0400	0.0677
			(0.5,0.5)	0.0265	0.0611	0.1293	0.0264	0.0799
		(5,10)	(0.3,0.3)	0.0132	0.0290	0.0656	0.0132	0.0452
			(0.3,0.5)	0.0312	0.0663	0.1529	0.0312	0.0517
			(0.5,0.5)	0.0211	0.0482	0.1040	0.0211	0.0667
		(10,10)	(0.3,0.3)	0.0076	0.0186	0.0367	0.0076	0.0365
			(0.3,0.5)	0.0200	0.0474	0.0965	0.0200	0.0439
			(0.5,0.5)	0.0132	0.0350	0.0625	0.0132	0.0506
	(0.5,1)	(1,1)	(0.3,0.3)	0.3447	0.9702	1.7077	0.3443	1.6950
			(0.3,0.5)	1.0250	5.3466	5.0533	1.0233	2.1550
			(0.5,0.5)	0.6786	2.4892	3.2223	0.6776	2.1560
		(1,5)	(0.3,0.3)	0.0934	0.2115	0.4418	0.0933	0.3161
			(0.3,0.5)	0.2615	0.6466	1.3169	0.2611	0.4293
			(0.5,0.5)	0.1716	0.4279	0.8249	0.1713	0.4564
		(5,5)	(0.3,0.3)	0.0689	0.1463	0.3293	0.0689	0.2618
			(0.3,0.5)	0.2049	0.4829	1.0340	0.2045	0.3855
			(0.5,0.5)	0.1356	0.3354	0.6857	0.1354	0.3866
		(5,10)	(0.3,0.3)	0.0363	0.0797	0.1740	0.0362	0.1535
			(0.3,0.5)	0.1072	0.2546	0.5319	0.1070	0.2255
			(0.5,0.5)	0.0707	0.1843	0.3452	0.0706	0.2317
		(10,10)	(0.3,0.3)	0.0345	0.0763	0.1698	0.0344	0.1507
			(0.3,0.5)	0.1024	0.2486	0.5052	0.1022	0.2262
			(0.5,0.5)	0.0678	0.1806	0.3371	0.0677	0.2250
	(1,1)	(1,1)	(0.3,0.3)	0.6097	1.8484	3.0446	0.6089	2.5200
			(0.3,0.5)	1.6018	8.1861	7.9901	1.5991	2.8340
			(0.5,0.5)	1.0586	4.0019	5.1331	1.0571	3.1690
		(1,5)	(0.3,0.3)	0.5001	1.4748	2.5065	0.4995	1.7180
			(0.3,0.5)	1.1003	4.1301	5.5312	1.0987	1.7430
			(0.5,0.5)	0.7627	2.4725	3.6037	0.7617	2.1680
		(5,5)	(0.3,0.3)	0.1219	0.2570	0.6123	0.1218	0.3706
			(0.3,0.5)	0.3205	0.6992	1.6277	0.3200	0.4706
			(0.5,0.5)	0.2118	0.4766	1.0108	0.2114	0.5525
		(5,10)	(0.3,0.3)	0.1054	0.2226	0.5103	0.1053	0.3017
			(0.3,0.5)	0.2499	0.5242	1.2100	0.2495	0.3472
			(0.5,0.5)	0.1689	0.3677	0.7986	0.1686	0.4488
		(10,10)	(0.3,0.3)	0.0610	0.1319	0.2867	0.0609	0.2114
			(0.3,0.5)	0.1602	0.3421	0.8077	0.1599	0.2692
			(0.5,0.5)	0.1059	0.2445	0.5195	0.1057	0.3187

TABLE 3-4 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths				
				AGCI	FCI	BCI	MOVER	NA
(200,200)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.0842	0.2183	0.4105	0.0841	0.1757
			(0.3,0.5)	0.0566	0.1443	0.2783	0.0566	0.2063
			(0.5,0.5)	0.1670	0.4538	0.7901	0.1667	0.2337
		(1,5)	(0.3,0.3)	0.0629	0.1503	0.2987	0.0628	0.1255
			(0.3,0.5)	0.0431	0.1023	0.2160	0.0430	0.1297
			(0.5,0.5)	0.1340	0.3346	0.6610	0.1338	0.1678
		(5,5)	(0.3,0.3)	0.0168	0.0414	0.0794	0.0168	0.0371
			(0.3,0.5)	0.0113	0.0288	0.0562	0.0113	0.0476
			(0.5,0.5)	0.0334	0.0723	0.1607	0.0333	0.0563
		(5,10)	(0.3,0.3)	0.0137	0.0329	0.0648	0.0137	0.0319
			(0.3,0.5)	0.0093	0.0230	0.0472	0.0093	0.0363
			(0.5,0.5)	0.0285	0.0605	0.1395	0.0285	0.0469
		(10,10)	(0.3,0.3)	0.0084	0.0242	0.0414	0.0084	0.0257
			(0.3,0.5)	0.0057	0.0170	0.0267	0.0057	0.0311
			(0.5,0.5)	0.0167	0.0405	0.0848	0.0167	0.0354
	(0.5,1)	(1,1)	(0.3,0.3)	0.4340	1.4403	2.1197	0.4335	1.1750
			(0.3,0.5)	0.2936	0.9285	1.3932	0.2933	1.4680
			(0.5,0.5)	0.7000	2.6513	3.3815	0.6990	1.4590
		(1,5)	(0.3,0.3)	0.1086	0.2758	0.5505	0.1085	0.2229
			(0.3,0.5)	0.0729	0.1869	0.3433	0.0728	0.2996
			(0.5,0.5)	0.2033	0.5184	0.9649	0.2030	0.3169
		(5,5)	(0.3,0.3)	0.0868	0.2253	0.4131	0.0867	0.1850
			(0.3,0.5)	0.0587	0.1558	0.2794	0.0587	0.2732
			(0.5,0.5)	0.1399	0.3458	0.6810	0.1397	0.2740
		(5,10)	(0.3,0.3)	0.0450	0.1264	0.2222	0.0449	0.1076
			(0.3,0.5)	0.0304	0.0883	0.1458	0.0303	0.1602
			(0.5,0.5)	0.0754	0.1917	0.3812	0.0753	0.1635
		(10,10)	(0.3,0.3)	0.0434	0.1242	0.2195	0.0434	0.1058
			(0.3,0.5)	0.0294	0.0869	0.1406	0.0293	0.1591
			(0.5,0.5)	0.0699	0.1848	0.3408	0.0698	0.1600
	(1,1)	(1,1)	(0.3,0.3)	0.6735	2.2374	3.1868	0.6727	1.7040
			(0.3,0.5)	0.4530	1.4049	2.1869	0.4524	1.9210
			(0.5,0.5)	1.3370	5.3507	6.4529	1.3349	2.1010
		(1,5)	(0.3,0.3)	0.5030	1.4895	2.4307	0.5024	1.1970
			(0.3,0.5)	0.3444	0.9824	1.6713	0.3440	1.2040
			(0.5,0.5)	1.0732	3.7995	5.2432	1.0716	1.4820
		(5,5)	(0.3,0.3)	0.1347	0.3132	0.6654	0.1345	0.2614
			(0.3,0.5)	0.0906	0.2143	0.4464	0.0905	0.3293
			(0.5,0.5)	0.2671	0.5785	1.3194	0.2667	0.3863
		(5,10)	(0.3,0.3)	0.1094	0.2453	0.5177	0.1093	0.2121
			(0.3,0.5)	0.0743	0.1694	0.3635	0.0742	0.2442
			(0.5,0.5)	0.2284	0.4768	1.1544	0.2281	0.3143
		(10,10)	(0.3,0.3)	0.0674	0.1655	0.3174	0.0673	0.1483
			(0.3,0.5)	0.0453	0.1150	0.2200	0.0452	0.1903
			(0.5,0.5)	0.1334	0.2891	0.6383	0.1332	0.2256

TABLE 3-4 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths				
				AGCI	FCI	BCI	MOVER	NA
(30,50)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.1116	0.2879	0.5261	0.1114	0.4418
			(0.3,0.5)	0.0728	0.1838	0.3646	0.0727	0.5057
			(0.5,0.5)	0.1986	0.5991	1.0105	0.1983	0.6038
		(1,5)	(0.3,0.3)	0.0945	0.2302	0.4684	0.0944	0.3322
			(0.3,0.5)	0.0622	0.1492	0.2975	0.0621	0.3519
			(0.5,0.5)	0.1316	0.3720	0.6327	0.1314	0.4604
		(5,5)	(0.3,0.3)	0.0223	0.0509	0.1054	0.0223	0.0874
			(0.3,0.5)	0.0146	0.0348	0.0696	0.0145	0.1071
			(0.5,0.5)	0.0397	0.0920	0.2004	0.0396	0.1328
		(5,10)	(0.3,0.3)	0.0197	0.0439	0.0961	0.0197	0.0785
			(0.3,0.5)	0.0129	0.0300	0.0629	0.0129	0.0865
			(0.5,0.5)	0.0305	0.0736	0.1476	0.0304	0.1164
		(10,10)	(0.3,0.3)	0.0112	0.0291	0.0554	0.0111	0.0607
			(0.3,0.5)	0.0073	0.0202	0.0362	0.0073	0.0705
			(0.5,0.5)	0.0199	0.0519	0.0998	0.0198	0.0843
		(0.5,1)	(1,1)	0.4440	1.4640	2.1364	0.4434	2.5740
			(0.3,0.5)	0.2977	0.9381	1.4985	0.2973	3.1660
			(0.5,0.5)	1.0323	4.6303	5.0934	1.0306	3.2760
		(1,5)	(0.3,0.3)	0.1326	0.3379	0.6651	0.1324	0.5244
			(0.3,0.5)	0.0866	0.2203	0.4366	0.0865	0.6674
			(0.5,0.5)	0.2613	0.6713	1.2680	0.2609	0.7475
		(5,5)	(0.3,0.3)	0.0888	0.2284	0.4328	0.0887	0.3761
			(0.3,0.5)	0.0595	0.1573	0.2802	0.0595	0.5497
			(0.5,0.5)	0.2065	0.4836	1.0165	0.2062	0.5543
		(5,10)	(0.3,0.3)	0.0480	0.1307	0.2319	0.0479	0.2224
			(0.3,0.5)	0.0319	0.0907	0.1569	0.0318	0.3274
			(0.5,0.5)	0.1078	0.2577	0.5142	0.1076	0.3354
		(10,10)	(0.3,0.3)	0.0444	0.1259	0.2168	0.0443	0.2160
			(0.3,0.5)	0.0298	0.0877	0.1437	0.0297	0.3189
			(0.5,0.5)	0.1031	0.2499	0.4891	0.1029	0.3237
		(1,1)	(0.3,0.3)	0.8925	3.1078	4.5070	0.8913	4.6100
			(0.3,0.5)	0.5821	1.8442	2.9230	0.5814	5.1840
			(0.5,0.5)	1.5870	7.8564	7.8876	1.5841	6.1900
		(1,5)	(0.3,0.3)	0.7558	2.4463	3.6390	0.7549	3.4810
			(0.3,0.5)	0.4975	1.4863	2.5295	0.4969	3.5180
			(0.5,0.5)	1.0521	4.4773	5.1289	1.0504	4.4100
		(5,5)	(0.3,0.3)	0.1785	0.3951	0.8609	0.1783	0.6262
			(0.3,0.5)	0.1165	0.2642	0.5724	0.1163	0.7442
			(0.5,0.5)	0.3177	0.7366	1.6044	0.3171	0.9280
		(5,10)	(0.3,0.3)	0.1579	0.3384	0.7779	0.1577	0.5345
			(0.3,0.5)	0.1036	0.2274	0.5217	0.1034	0.5906
			(0.5,0.5)	0.2437	0.5658	1.2228	0.2433	0.7990
		(10,10)	(0.3,0.3)	0.0892	0.2037	0.4333	0.0891	0.3513
			(0.3,0.5)	0.0583	0.1391	0.2879	0.0582	0.4275
			(0.5,0.5)	0.1587	0.3680	0.7949	0.1584	0.5312

TABLE 3-4 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths				
				AGCI	FCI	BCI	MOVER	NA
(50,100)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.1267	0.3719	0.5967	0.1265	0.3129
			(0.3,0.5)	0.0789	0.2220	0.3767	0.0788	0.3590
			(0.5,0.5)	0.0522	0.1436	0.2473	0.0522	0.4260
		(1,5)	(0.3,0.3)	0.0865	0.2433	0.4118	0.0864	0.2524
			(0.3,0.5)	0.0552	0.1528	0.2704	0.0551	0.2599
			(0.5,0.5)	0.0371	0.1007	0.1780	0.0370	0.3442
		(5,5)	(0.3,0.3)	0.0253	0.0660	0.1210	0.0253	0.0652
			(0.3,0.5)	0.0158	0.0445	0.0746	0.0157	0.0781
			(0.5,0.5)	0.0104	0.0307	0.0526	0.0104	0.0983
		(5,10)	(0.3,0.3)	0.0196	0.0537	0.0930	0.0196	0.0590
			(0.3,0.5)	0.0123	0.0366	0.0609	0.0123	0.0642
			(0.5,0.5)	0.0082	0.0254	0.0389	0.0082	0.0875
		(10,10)	(0.3,0.3)	0.0127	0.0387	0.0629	0.0127	0.0449
			(0.3,0.5)	0.0079	0.0267	0.0392	0.0079	0.0513
			(0.5,0.5)	0.0052	0.0187	0.0255	0.0052	0.0624
	(0.5,1)	(1,1)	(0.3,0.3)	0.6808	2.5759	3.4190	0.6798	1.7190
			(0.3,0.5)	0.4352	1.4522	2.0917	0.4347	2.1510
			(0.5,0.5)	0.2927	0.9224	1.3976	0.2924	2.1640
		(1,5)	(0.3,0.3)	0.1674	0.4419	0.8337	0.1671	0.3717
			(0.3,0.5)	0.1046	0.2795	0.5173	0.1044	0.4705
			(0.5,0.5)	0.0694	0.1864	0.3400	0.0694	0.5298
		(5,5)	(0.3,0.3)	0.1361	0.3406	0.6662	0.1360	0.2678
			(0.3,0.5)	0.0870	0.2272	0.4216	0.0869	0.3890
			(0.5,0.5)	0.0586	0.1562	0.2978	0.0585	0.3940
		(5,10)	(0.3,0.3)	0.0705	0.1878	0.3421	0.0704	0.1582
			(0.3,0.5)	0.0448	0.1280	0.2197	0.0447	0.2303
			(0.5,0.5)	0.0300	0.0891	0.1472	0.0300	0.2388
		(10,10)	(0.3,0.3)	0.0681	0.1832	0.3417	0.0680	0.1524
			(0.3,0.5)	0.0435	0.1253	0.2149	0.0435	0.2271
			(0.5,0.5)	0.0293	0.0874	0.1441	0.0293	0.2291
	(1,1)	(1,1)	(0.3,0.3)	1.0134	4.2070	4.8279	1.0119	3.2130
			(0.3,0.5)	0.6309	2.2606	3.0347	0.6300	3.4820
			(0.5,0.5)	0.4179	1.3642	2.0612	0.4174	4.1000
		(1,5)	(0.3,0.3)	0.6916	2.5744	3.3204	0.6906	2.5250
			(0.3,0.5)	0.4413	1.4837	2.1167	0.4407	2.5690
			(0.5,0.5)	0.2967	0.9235	1.4823	0.2963	3.2820
		(5,5)	(0.3,0.3)	0.2027	0.5032	0.9915	0.2024	0.4595
			(0.3,0.5)	0.1262	0.3278	0.6341	0.1260	0.5430
			(0.5,0.5)	0.0836	0.2215	0.4053	0.0835	0.6840
		(5,10)	(0.3,0.3)	0.1570	0.3955	0.7794	0.1567	0.4015
			(0.3,0.5)	0.0985	0.2619	0.4894	0.0983	0.4365
			(0.5,0.5)	0.0656	0.1781	0.3172	0.0656	0.5911
		(10,10)	(0.3,0.3)	0.1014	0.2641	0.5132	0.1012	0.2595
			(0.3,0.5)	0.0631	0.1781	0.3043	0.0630	0.3113
			(0.5,0.5)	0.0418	0.1230	0.1994	0.0417	0.3939

TABLE 3-4 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths				
				AGCI	FCI	BCI	MOVER	NA
(100,200)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.1639	0.4816	0.7854	<i>0.1637</i>	0.2174
			(0.3,0.5)	0.1037	0.2963	0.4974	<i>0.1035</i>	0.2432
			(0.5,0.5)	0.0663	0.1845	0.3253	<i>0.0662</i>	0.2908
		(1,5)	(0.3,0.3)	0.1285	0.3652	0.6115	<i>0.1283</i>	0.1759
			(0.3,0.5)	0.0846	0.2374	0.4238	<i>0.0845</i>	0.1799
			(0.5,0.5)	<i>0.0542</i>	0.1495	0.2578	<i>0.0542</i>	0.2353
		(5,5)	(0.3,0.3)	0.0328	0.0792	0.1639	<i>0.0327</i>	0.0456
			(0.3,0.5)	<i>0.0207</i>	0.0556	0.1033	<i>0.0207</i>	0.0546
			(0.5,0.5)	0.0133	0.0381	0.0640	<i>0.0132</i>	0.0690
		(5,10)	(0.3,0.3)	<i>0.0277</i>	0.0684	0.1368	<i>0.0277</i>	0.0412
			(0.3,0.5)	<i>0.0179</i>	0.0491	0.0883	<i>0.0179</i>	0.0449
			(0.5,0.5)	<i>0.0114</i>	0.0338	0.0565	<i>0.0114</i>	0.0616
		(10,10)	(0.3,0.3)	<i>0.0164</i>	0.0455	0.0780	<i>0.0164</i>	0.0316
			(0.3,0.5)	0.0104	0.0329	0.0496	<i>0.0103</i>	0.0360
			(0.5,0.5)	<i>0.0066</i>	0.0230	0.0324	<i>0.0066</i>	0.0436
	(0.5,1)	(1,1)	(0.3,0.3)	0.6942	2.6112	3.4646	<i>0.6932</i>	1.1920
			(0.3,0.5)	0.4387	1.4583	2.1868	<i>0.4381</i>	1.4650
			(0.5,0.5)	0.2963	0.9300	1.4607	<i>0.2960</i>	1.4740
		(1,5)	(0.3,0.3)	0.2008	0.5472	1.0100	<i>0.2004</i>	0.2583
			(0.3,0.5)	0.1257	0.3462	0.6224	<i>0.1255</i>	0.3271
			(0.5,0.5)	0.0811	0.2215	0.4134	<i>0.0810</i>	0.3618
		(5,5)	(0.3,0.3)	0.1388	0.3433	0.6834	<i>0.1386</i>	0.1873
			(0.3,0.5)	0.0877	0.2288	0.4295	<i>0.0876</i>	0.2735
			(0.5,0.5)	0.0593	0.1580	0.2891	<i>0.0592</i>	0.2764
		(5,10)	(0.3,0.3)	0.0748	0.1934	0.3807	<i>0.0747</i>	0.1110
			(0.3,0.5)	0.0470	0.1323	0.2368	<i>0.0469</i>	0.1628
			(0.5,0.5)	0.0313	0.0922	0.1499	<i>0.0312</i>	0.1682
		(10,10)	(0.3,0.3)	0.0693	0.1840	0.3500	<i>0.0692</i>	0.1076
			(0.3,0.5)	0.0439	0.1263	0.2126	<i>0.0438</i>	0.1603
			(0.5,0.5)	<i>0.0296</i>	0.0885	0.1442	<i>0.0296</i>	0.1626
	(1,1)	(1,1)	(0.3,0.3)	1.3099	5.9758	6.2398	<i>1.3077</i>	2.1190
			(0.3,0.5)	0.8301	3.2084	4.1907	<i>0.8289</i>	2.3060
			(0.5,0.5)	0.5307	1.8306	2.6098	<i>0.5300</i>	2.6580
		(1,5)	(0.3,0.3)	1.0268	4.4348	5.2364	<i>1.0251</i>	1.6930
			(0.3,0.5)	0.6778	2.5250	3.1904	<i>0.6769</i>	1.7170
			(0.5,0.5)	0.4341	1.4592	2.1860	<i>0.4336</i>	2.1340
		(5,5)	(0.3,0.3)	0.2620	0.6201	1.2658	<i>0.2616</i>	0.3196
			(0.3,0.5)	0.1659	0.4167	0.8058	<i>0.1656</i>	0.3791
			(0.5,0.5)	0.1061	0.2770	0.5171	<i>0.1060</i>	0.4738
		(5,10)	(0.3,0.3)	0.2214	0.5233	1.0845	<i>0.2210</i>	0.2817
			(0.3,0.5)	0.1433	0.3620	0.6898	<i>0.1431</i>	0.3082
			(0.5,0.5)	0.0915	0.2418	0.4534	<i>0.0914</i>	0.4170
		(10,10)	(0.3,0.3)	0.1310	0.3168	0.6461	<i>0.1308</i>	0.1829
			(0.3,0.5)	0.0829	0.2223	0.3949	<i>0.0828</i>	0.2180
			(0.5,0.5)	<i>0.0530</i>	0.1524	0.2655	<i>0.0530</i>	0.2751

TABLE 3-5 The CPs of the 95% confidence intervals for the ratio between two variances of delta-IG distributions.

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities				
				AGCI	FCI	BCI	MOVER	NA
(30,30)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9436	0.9522	0.9560	0.9434	0.2894
			(0.3,0.5)	0.9508	0.9462	0.9365	0.9508	0.3076
			(0.5,0.5)	0.9540	0.9496	0.9359	0.9538	0.3486
		(1,5)	(0.3,0.3)	0.9466	0.9394	0.9492	0.9464	0.1228
			(0.3,0.5)	0.9506	0.9364	0.9436	0.9502	0.1386
			(0.5,0.5)	0.9534	0.9464	0.9465	0.9528	0.1814
		(5,5)	(0.3,0.3)	0.9466	0.9500	0.9391	0.9460	0.1322
			(0.3,0.5)	0.9514	0.9432	0.9436	0.9512	0.1892
			(0.5,0.5)	0.9514	0.9470	0.9352	0.9512	0.2000
		(5,10)	(0.3,0.3)	0.9452	0.9458	0.9354	0.9452	0.1098
			(0.3,0.5)	0.9510	0.9442	0.9417	0.9508	0.1502
			(0.5,0.5)	0.9560	0.9438	0.9377	0.9558	0.1864
		(10,10)	(0.3,0.3)	0.9448	0.9448	0.9415	0.9446	0.1110
			(0.3,0.5)	0.9502	0.9432	0.9399	0.9498	0.1654
			(0.5,0.5)	0.9518	0.9442	0.9403	0.9514	0.1878
	(0.5,1)	(1,1)	(0.3,0.3)	0.9484	0.9454	0.9462	0.9480	0.9528
			(0.3,0.5)	0.9510	0.9456	0.9456	0.9508	0.9254
			(0.5,0.5)	0.9530	0.9490	0.9449	0.9526	0.9352
		(1,5)	(0.3,0.3)	0.9468	0.9474	0.9487	0.9464	0.8466
			(0.3,0.5)	0.9494	0.9398	0.9390	0.9490	0.8762
			(0.5,0.5)	0.9552	0.9474	0.9454	0.9550	0.9218
		(5,5)	(0.3,0.3)	0.9486	0.9500	0.9390	0.9484	0.9364
			(0.3,0.5)	0.9508	0.9450	0.9369	0.9508	0.9598
			(0.5,0.5)	0.9542	0.9514	0.9351	0.9538	0.9584
		(5,10)	(0.3,0.3)	0.9466	0.9500	0.9418	0.9460	0.9052
			(0.3,0.5)	0.9514	0.9432	0.9438	0.9512	0.9570
			(0.5,0.5)	0.9514	0.9470	0.9355	0.9512	0.9572
		(10,10)	(0.3,0.3)	0.9506	0.9492	0.9484	0.9506	0.9146
			(0.3,0.5)	0.9536	0.9452	0.9395	0.9536	0.9704
			(0.5,0.5)	0.9528	0.9466	0.9428	0.9526	0.9624
	(1,1)	(1,1)	(0.3,0.3)	0.9478	0.9486	0.9307	0.9476	0.9016
			(0.3,0.5)	0.9518	0.9480	0.9497	0.9518	0.8444
			(0.5,0.5)	0.9532	0.9472	0.9445	0.9530	0.8904
		(1,5)	(0.3,0.3)	0.9460	0.9302	0.9435	0.9456	0.5684
			(0.3,0.5)	0.9490	0.9318	0.9465	0.9486	0.5614
			(0.5,0.5)	0.9560	0.9496	0.9350	0.9560	0.6744
		(5,5)	(0.3,0.3)	0.9476	0.9544	0.9484	0.9474	0.6340
			(0.3,0.5)	0.9502	0.9466	0.9493	0.9500	0.7214
			(0.5,0.5)	0.9522	0.9498	0.9431	0.9522	0.7758
		(5,10)	(0.3,0.3)	0.9472	0.9472	0.9381	0.9468	0.4844
			(0.3,0.5)	0.9518	0.9446	0.9494	0.9518	0.5696
			(0.5,0.5)	0.9532	0.9466	0.9430	0.9532	0.6594
		(10,10)	(0.3,0.3)	0.9466	0.9500	0.9407	0.9460	0.4962
			(0.3,0.5)	0.9514	0.9432	0.9455	0.9512	0.6610
			(0.5,0.5)	0.9514	0.9470	0.9362	0.9512	0.7054

Note: Bold values indicate coverage probabilities (CP) that satisfy the nominal confidence level of 0.95 ($CP \geq 0.945$, rounded to 0.950).

TABLE 3-5 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities				
				AGCI	FCI	BCI	MOVER	NA
(50,50)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9488	0.9520	0.9439	0.9486	0.2764
			(0.3,0.5)	0.9524	0.9498	0.9369	0.9520	0.2792
			(0.5,0.5)	0.9468	0.9490	0.9352	0.9464	0.3228
			(1,5)	0.9472	0.9482	0.9439	0.9470	0.1276
			(0.3,0.5)	0.9514	0.9360	0.9441	0.9514	0.1470
			(0.5,0.5)	0.9510	0.9424	0.9454	0.9506	0.1930
		(5,5)	(0.3,0.3)	0.9500	0.9472	0.9362	0.9496	0.1390
			(0.3,0.5)	0.9508	0.9440	0.9351	0.9506	0.1826
			(0.5,0.5)	0.9478	0.9468	0.9434	0.9476	0.1954
		(5,10)	(0.3,0.3)	0.9488	0.9468	0.9362	0.9484	0.1100
			(0.3,0.5)	0.9536	0.9444	0.9383	0.9536	0.1548
			(0.5,0.5)	0.9520	0.9480	0.9441	0.9510	0.1914
		(10,10)	(0.3,0.3)	0.9502	0.9446	0.9454	0.9502	0.1094
			(0.3,0.5)	0.9510	0.9436	0.9489	0.9508	0.1610
			(0.5,0.5)	0.9468	0.9498	0.9434	0.9468	0.2002
		(0.5,1)	(1,1)	0.9492	0.9526	0.9413	0.9492	0.9730
			(0.3,0.5)	0.9514	0.9500	0.9387	0.9508	0.9598
			(0.5,0.5)	0.9458	0.9528	0.9421	0.9458	0.9702
			(1,5)	0.9488	0.9500	0.9394	0.9482	0.8312
			(0.3,0.5)	0.9522	0.9408	0.9367	0.9522	0.8980
			(0.5,0.5)	0.9482	0.9462	0.9420	0.9482	0.9328
		(5,5)	(0.3,0.3)	0.9494	0.9484	0.9455	0.9492	0.9556
			(0.3,0.5)	0.9504	0.9438	0.9451	0.9496	0.9820
			(0.5,0.5)	0.9466	0.9462	0.9496	0.9462	0.9742
		(5,10)	(0.3,0.3)	0.9500	0.9472	0.9498	0.9496	0.9124
			(0.3,0.5)	0.9508	0.9440	0.9381	0.9506	0.9678
			(0.5,0.5)	0.9478	0.9468	0.9432	0.9476	0.9720
		(10,10)	(0.3,0.3)	0.9502	0.9474	0.9366	0.9502	0.9390
			(0.3,0.5)	0.9506	0.9414	0.9487	0.9502	0.9802
			(0.5,0.5)	0.9454	0.9508	0.9457	0.9450	0.9832
		(1,1)	(0.3,0.3)	0.9480	0.9484	0.9452	0.9476	0.9258
			(0.3,0.5)	0.9498	0.9480	0.9489	0.9494	0.8616
			(0.5,0.5)	0.9474	0.9492	0.9461	0.9468	0.9264
		(1,5)	(0.3,0.3)	0.9482	0.9444	0.9421	0.9478	0.5744
			(0.3,0.5)	0.9488	0.9262	0.9398	0.9486	0.5838
			(0.5,0.5)	0.9490	0.9358	0.9391	0.9488	0.7024
		(5,5)	(0.3,0.3)	0.9488	0.9496	0.9432	0.9486	0.6342
			(0.3,0.5)	0.9508	0.9450	0.9473	0.9506	0.7328
			(0.5,0.5)	0.9476	0.9506	0.9426	0.9472	0.7810
		(5,10)	(0.3,0.3)	0.9484	0.9530	0.9387	0.9482	0.4790
			(0.3,0.5)	0.9524	0.9450	0.9489	0.9520	0.5856
			(0.5,0.5)	0.9474	0.9504	0.9397	0.9472	0.6758
		(10,10)	(0.3,0.3)	0.9500	0.9472	0.9391	0.9496	0.5090
			(0.3,0.5)	0.9508	0.9440	0.9482	0.9506	0.6590
			(0.5,0.5)	0.9478	0.9468	0.9450	0.9476	0.7090

TABLE 3-5 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities				
				AGCI	FCI	BCI	MOVER	NA
(100,100)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9492	0.9456	0.9438	0.9490	0.2690
			(0.3,0.5)	0.9482	0.9476	0.9466	0.9482	0.2804
			(0.5,0.5)	0.9468	0.9400	0.9393	0.9464	0.3316
		(1,5)	(0.3,0.3)	0.9508	0.9416	0.9407	0.9506	0.1282
			(0.3,0.5)	0.9486	0.9376	0.9393	0.9486	0.1396
			(0.5,0.5)	0.9536	0.9396	0.9436	0.9524	0.1874
		(5,5)	(0.3,0.3)	0.9504	0.9484	0.9434	0.9500	0.1322
			(0.3,0.5)	0.9462	0.9430	0.9377	0.9462	0.1916
			(0.5,0.5)	0.9446	0.9416	0.9411	0.9444	0.2114
		(5,10)	(0.3,0.3)	0.9492	0.9478	0.9464	0.9490	0.1098
			(0.3,0.5)	0.9492	0.9456	0.9421	0.9484	0.1460
			(0.5,0.5)	0.9490	0.9434	0.9402	0.9486	0.1886
		(10,10)	(0.3,0.3)	0.9504	0.9520	0.9436	0.9502	0.1112
			(0.3,0.5)	0.9462	0.9426	0.9361	0.9460	0.1640
			(0.5,0.5)	0.9468	0.9450	0.9494	0.9466	0.2108
	(0.5,1)	(1,1)	(0.3,0.3)	0.9514	0.9444	0.9459	0.9506	0.9954
			(0.3,0.5)	0.9488	0.9376	0.9466	0.9488	0.9876
			(0.5,0.5)	0.9460	0.9360	0.9402	0.9456	0.9924
		(1,5)	(0.3,0.3)	0.9480	0.9436	0.9380	0.9480	0.8356
			(0.3,0.5)	0.9478	0.9422	0.9472	0.9476	0.9170
			(0.5,0.5)	0.9474	0.9424	0.9469	0.9470	0.9414
		(5,5)	(0.3,0.3)	0.9518	0.9494	0.9473	0.9516	0.9760
			(0.3,0.5)	0.9468	0.9398	0.9444	0.9468	0.9938
			(0.5,0.5)	0.9442	0.9356	0.9390	0.9440	0.9924
		(5,10)	(0.3,0.3)	0.9504	0.9484	0.9478	0.9500	0.9392
			(0.3,0.5)	0.9462	0.9430	0.9364	0.9462	0.9798
			(0.5,0.5)	0.9446	0.9416	0.9381	0.9444	0.9802
		(10,10)	(0.3,0.3)	0.9516	0.9510	0.9497	0.9510	0.9656
			(0.3,0.5)	0.9476	0.9370	0.9450	0.9474	0.9928
			(0.5,0.5)	0.9444	0.9362	0.9381	0.9444	0.9908
	(1,1)	(1,1)	(0.3,0.3)	0.9486	0.9482	0.9480	0.9482	0.9422
			(0.3,0.5)	0.9498	0.9444	0.9485	0.9496	0.9050
			(0.5,0.5)	0.9458	0.9458	0.9487	0.9452	0.9470
		(1,5)	(0.3,0.3)	0.9494	0.9358	0.9489	0.9490	0.5936
			(0.3,0.5)	0.9480	0.9320	0.9413	0.9476	0.5868
			(0.5,0.5)	0.9488	0.9354	0.9369	0.9484	0.7220
		(5,5)	(0.3,0.3)	0.9496	0.9502	0.9484	0.9494	0.6316
			(0.3,0.5)	0.9468	0.9444	0.9372	0.9466	0.7186
			(0.5,0.5)	0.9450	0.9420	0.9382	0.9446	0.7778
		(5,10)	(0.3,0.3)	0.9500	0.9466	0.9431	0.9498	0.4768
			(0.3,0.5)	0.9478	0.9484	0.9361	0.9476	0.5936
			(0.5,0.5)	0.9482	0.9444	0.9479	0.9480	0.6828
		(10,10)	(0.3,0.3)	0.9504	0.9484	0.9364	0.9500	0.5120
			(0.3,0.5)	0.9462	0.9430	0.9419	0.9462	0.6564
			(0.5,0.5)	0.9446	0.9416	0.9435	0.9444	0.7220

TABLE 3-5 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities					
				AGCI	FCI	BCI	MOVER	NA	
(200,200)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9476	0.9494	0.9441	0.9476	0.2786	
			(0.3,0.5)	0.9464	0.9486	0.9409	0.9464	0.2698	
			(0.5,0.5)	0.9534	0.9472	0.9394	0.9534	0.3236	
		(1,5)	(0.3,0.3)	0.9480	0.9460	0.9498	0.9478	0.1262	
			(0.3,0.5)	0.9464	0.9502	0.9447	0.9460	0.1422	
			(0.5,0.5)	0.9506	0.9366	0.9368	0.9502	0.1828	
		(5,5)	(0.3,0.3)	0.9446	0.9490	0.9461	0.9446	0.1350	
			(0.3,0.5)	0.9506	0.9560	0.9418	0.9496	0.1938	
			(0.5,0.5)	0.9542	0.9480	0.9447	0.9538	0.2106	
		(5,10)	(0.3,0.3)	0.9470	0.9512	0.9387	0.9466	0.1074	
			(0.3,0.5)	0.9494	0.9532	0.9380	0.9486	0.1584	
			(0.5,0.5)	0.9544	0.9466	0.9412	0.9542	0.1846	
		(10,10)	(0.3,0.3)	0.9452	0.9492	0.9424	0.9450	0.1142	
			(0.3,0.5)	0.9488	0.9508	0.9401	0.9486	0.1722	
			(0.5,0.5)	0.9530	0.9468	0.9482	0.9528	0.2058	
		(0.5,1)	(1,1)	(0.3,0.3)	0.9464	0.9454	0.9447	0.9464	0.9986
				(0.3,0.5)	0.9488	0.9500	0.9355	0.9484	0.9970
				(0.5,0.5)	0.9528	0.9478	0.9474	0.9524	0.9966
		(1,5)	(1,1)	(0.3,0.3)	0.9466	0.9448	0.9465	0.9460	0.8324
				(0.3,0.5)	0.9494	0.9522	0.9355	0.9492	0.9276
				(0.5,0.5)	0.9514	0.9396	0.9361	0.9510	0.9428
		(5,5)	(1,1)	(0.3,0.3)	0.9472	0.9434	0.9492	0.9468	0.9868
				(0.3,0.5)	0.9492	0.9504	0.9424	0.9488	0.9970
				(0.5,0.5)	0.9528	0.9508	0.9369	0.9524	0.9968
		(5,10)	(1,1)	(0.3,0.3)	0.9446	0.9490	0.9369	0.9446	0.9454
				(0.3,0.5)	0.9506	0.9560	0.9491	0.9496	0.9910
				(0.5,0.5)	0.9542	0.9480	0.9408	0.9538	0.9874
		(10,10)	(1,1)	(0.3,0.3)	0.9470	0.9486	0.9407	0.9470	0.9714
				(0.3,0.5)	0.9506	0.9510	0.9398	0.9504	0.9956
				(0.5,0.5)	0.9518	0.9452	0.9394	0.9516	0.9986
		(1,1)	(1,1)	(0.3,0.3)	0.9478	0.9516	0.9369	0.9474	0.9606
				(0.3,0.5)	0.9482	0.9544	0.9472	0.9480	0.9234
				(0.5,0.5)	0.9536	0.9468	0.9433	0.9534	0.9692
		(1,5)	(1,5)	(0.3,0.3)	0.9486	0.9404	0.9483	0.9484	0.5904
				(0.3,0.5)	0.9494	0.9472	0.9485	0.9494	0.5842
				(0.5,0.5)	0.9496	0.9276	0.9490	0.9496	0.7146
		(5,5)	(5,5)	(0.3,0.3)	0.9466	0.9472	0.9423	0.9462	0.6306
				(0.3,0.5)	0.9468	0.9556	0.9453	0.9466	0.7130
				(0.5,0.5)	0.9538	0.9518	0.9444	0.9536	0.7814
		(5,10)	(5,10)	(0.3,0.3)	0.9468	0.9484	0.9417	0.9468	0.4988
				(0.3,0.5)	0.9490	0.9522	0.9440	0.9490	0.5878
				(0.5,0.5)	0.9534	0.9476	0.9395	0.9532	0.6806
		(10,10)	(10,10)	(0.3,0.3)	0.9446	0.9490	0.9355	0.9446	0.5104
				(0.3,0.5)	0.9506	0.9560	0.9383	0.9496	0.6702
				(0.5,0.5)	0.9542	0.9480	0.9420	0.9538	0.7238

TABLE 3-5 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities						
				AGCI	FCI	BCI	MOVER	NA		
(30,50)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9524	0.9480	0.9370	0.9522	0.2674		
			(0.3,0.5)	0.9520	0.9512	0.9484	0.9520	0.2646		
			(0.5,0.5)	0.9528	0.9388	0.9451	0.9526	0.3222		
			(1,5)	(0.3,0.3)	0.9524	0.9384	0.9475	0.9518	0.1210	
			(0.3,0.5)	0.9514	0.9446	0.9422	0.9506	0.1460		
			(0.5,0.5)	0.9508	0.9232	0.9423	0.9504	0.1854		
			(5,5)	(0.3,0.3)	0.9520	0.9478	0.9492	0.9516	0.1358	
			(0.3,0.5)	0.9520	0.9490	0.9424	0.9520	0.1776		
			(0.5,0.5)	0.9520	0.9332	0.9429	0.9518	0.2138		
		(5,10)	(0.3,0.3)	0.9530	0.9432	0.9456	0.9524	0.1138		
			(0.3,0.5)	0.9528	0.9524	0.9397	0.9528	0.1486		
			(0.5,0.5)	0.9528	0.9324	0.9384	0.9528	0.1838		
			(10,10)	(0.3,0.3)	0.9512	0.9450	0.9445	0.9508	0.1120	
			(0.3,0.5)	0.9512	0.9518	0.9351	0.9510	0.1608		
			(0.5,0.5)	0.9510	0.9334	0.9366	0.9506	0.2002		
			(0.5,1)	(1,1)	(0.3,0.3)	0.9524	0.9492	0.9416	0.9524	0.9780
				(0.3,0.5)	0.9534	0.9496	0.9401	0.9530	0.9540	
				(0.5,0.5)	0.9538	0.9318	0.9361	0.9534	0.9706	
		(1,5)		(0.3,0.3)	0.9526	0.9410	0.9393	0.9526	0.8248	
		(0.3,0.5)		0.9512	0.9450	0.9423	0.9512	0.8834		
		(0.5,0.5)		0.9536	0.9312	0.9401	0.9534	0.9324		
		(5,5)		(0.3,0.3)	0.9522	0.9470	0.9365	0.9514	0.9576	
		(0.3,0.5)		0.9556	0.9454	0.9376	0.9554	0.9798		
		(0.5,0.5)		0.9510	0.9344	0.9481	0.9510	0.9782		
		(5,10)	(0.3,0.3)	0.9520	0.9478	0.9455	0.9516	0.9066		
			(0.3,0.5)	0.9520	0.9490	0.9444	0.9520	0.9670		
			(0.5,0.5)	0.9520	0.9332	0.9374	0.9518	0.9676		
			(10,10)	(0.3,0.3)	0.9514	0.9460	0.9471	0.9512	0.9358	
			(0.3,0.5)	0.9548	0.9504	0.9424	0.9544	0.9798		
			(0.5,0.5)	0.9532	0.9332	0.9367	0.9532	0.9806		
			(1,1)	(1,1)	(0.3,0.3)	0.9512	0.9502	0.9398	0.9510	0.9570
				(0.3,0.5)	0.9520	0.9450	0.9426	0.9520	0.9004	
				(0.5,0.5)	0.9550	0.9422	0.9369	0.9548	0.9476	
		(1,5)		(0.3,0.3)	0.9538	0.9336	0.9488	0.9534	0.5938	
		(0.3,0.5)		0.9522	0.9396	0.9434	0.9520	0.5998		
		(0.5,0.5)		0.9540	0.9144	0.9438	0.9536	0.6810		
		(5,5)		(0.3,0.3)	0.9526	0.9482	0.9380	0.9524	0.6278	
		(0.3,0.5)		0.9542	0.9498	0.9449	0.9538	0.7084		
		(0.5,0.5)		0.9508	0.9364	0.9461	0.9506	0.7772		
		(5,10)	(0.3,0.3)	0.9528	0.9438	0.9460	0.9522	0.4934		
			(0.3,0.5)	0.9508	0.9510	0.9401	0.9508	0.5840		
			(0.5,0.5)	0.9526	0.9318	0.9437	0.9526	0.6792		
			(10,10)	(0.3,0.3)	0.9520	0.9478	0.9426	0.9516	0.4980	
			(0.3,0.5)	0.9520	0.9490	0.9352	0.9520	0.6326		
			(0.5,0.5)	0.9520	0.9332	0.9372	0.9518	0.7118		

TABLE 3-5 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities				
				AGCI	FCI	BCI	MOVER	NA
(50,100)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9506	0.9490	0.9461	0.9504	0.2638
			(0.3,0.5)	0.9438	0.9482	0.9442	0.9432	0.2644
			(0.5,0.5)	0.9448	0.9482	0.9499	0.9446	0.3230
		(1,5)	(0.3,0.3)	0.9520	0.9382	0.9478	0.9520	0.1270
			(0.3,0.5)	0.9478	0.9436	0.9436	0.9470	0.1406
			(0.5,0.5)	0.9478	0.9490	0.9489	0.9478	0.1928
		(5,5)	(0.3,0.3)	0.9504	0.9440	0.9490	0.9502	0.1362
			(0.3,0.5)	0.9462	0.9442	0.9427	0.9458	0.1804
			(0.5,0.5)	0.9426	0.9478	0.9494	0.9426	0.2080
		(5,10)	(0.3,0.3)	0.9516	0.9420	0.9364	0.9514	0.1014
			(0.3,0.5)	0.9444	0.9450	0.9471	0.9442	0.1462
			(0.5,0.5)	0.9460	0.9488	0.9390	0.9452	0.1904
		(10,10)	(0.3,0.3)	0.9492	0.9448	0.9472	0.9486	0.1146
			(0.3,0.5)	0.9436	0.9472	0.9408	0.9434	0.1582
			(0.5,0.5)	0.9444	0.9454	0.9416	0.9442	0.1932
		(0.5,1)	(1,1)	0.9500	0.9406	0.9487	0.9498	0.9940
			(0.3,0.5)	0.9456	0.9454	0.9395	0.9454	0.9880
			(0.5,0.5)	0.9458	0.9452	0.9461	0.9458	0.9918
		(1,5)	(0.3,0.3)	0.9510	0.9432	0.9368	0.9504	0.7814
			(0.3,0.5)	0.9466	0.9480	0.9416	0.9464	0.8852
			(0.5,0.5)	0.9434	0.9510	0.9495	0.9430	0.9314
		(5,5)	(0.3,0.3)	0.9494	0.9410	0.9367	0.9482	0.9736
			(0.3,0.5)	0.9500	0.9456	0.9460	0.9498	0.9896
			(0.5,0.5)	0.9488	0.9406	0.9442	0.9486	0.9870
		(5,10)	(0.3,0.3)	0.9504	0.9440	0.9395	0.9502	0.9160
			(0.3,0.5)	0.9462	0.9442	0.9380	0.9458	0.9724
			(0.5,0.5)	0.9426	0.9478	0.9451	0.9426	0.9808
		(10,10)	(0.3,0.3)	0.9490	0.9414	0.9482	0.9488	0.9412
			(0.3,0.5)	0.9510	0.9422	0.9440	0.9508	0.9890
			(0.5,0.5)	0.9484	0.9458	0.9475	0.9484	0.9910
		(1,1)	(0.3,0.3)	0.9492	0.9474	0.9419	0.9488	0.9752
			(0.3,0.5)	0.9432	0.9478	0.9351	0.9426	0.9366
			(0.5,0.5)	0.9464	0.9492	0.9465	0.9460	0.9754
		(1,5)	(0.3,0.3)	0.9532	0.9310	0.9484	0.9532	0.6020
			(0.3,0.5)	0.9468	0.9436	0.9443	0.9466	0.6042
			(0.5,0.5)	0.9448	0.9436	0.9437	0.9442	0.7136
		(5,5)	(0.3,0.3)	0.9492	0.9460	0.9413	0.9490	0.6144
			(0.3,0.5)	0.9434	0.9512	0.9402	0.9428	0.7062
			(0.5,0.5)	0.9442	0.9498	0.9389	0.9440	0.7660
		(5,10)	(0.3,0.3)	0.9508	0.9450	0.9447	0.9498	0.4866
			(0.3,0.5)	0.9452	0.9496	0.9360	0.9450	0.5596
			(0.5,0.5)	0.9446	0.9476	0.9414	0.9440	0.6746
		(10,10)	(0.3,0.3)	0.9504	0.9440	0.9353	0.9502	0.5046
			(0.3,0.5)	0.9462	0.9442	0.9433	0.9458	0.6346
			(0.5,0.5)	0.9426	0.9478	0.9442	0.9426	0.7202

TABLE 3-5 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Coverage Probabilities				
				AGCI	FCI	BCI	MOVER	NA
(100,200)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9524	0.9464	0.9366	0.9522	0.2720
			(0.3,0.5)	0.9468	0.9472	0.9444	0.9462	0.2602
			(0.5,0.5)	0.9506	0.9512	0.9470	0.9502	0.3110
		(1,5)	(0.3,0.3)	0.9556	0.9262	0.9386	0.9554	0.1314
			(0.3,0.5)	0.9512	0.9392	0.9491	0.9510	0.1508
			(0.5,0.5)	0.9526	0.9472	0.9469	0.9522	0.1956
		(5,5)	(0.3,0.3)	0.9502	0.9370	0.9350	0.9498	0.1386
			(0.3,0.5)	0.9496	0.9462	0.9467	0.9494	0.1812
			(0.5,0.5)	0.9506	0.9432	0.9360	0.9506	0.2168
		(5,10)	(0.3,0.3)	0.9544	0.9316	0.9432	0.9540	0.1144
			(0.3,0.5)	0.9516	0.9428	0.9463	0.9510	0.1474
			(0.5,0.5)	0.9520	0.9438	0.9442	0.9514	0.1920
		(10,10)	(0.3,0.3)	0.9502	0.9336	0.9481	0.9502	0.1132
			(0.3,0.5)	0.9484	0.9466	0.9396	0.9482	0.1468
			(0.5,0.5)	0.9516	0.9418	0.9367	0.9510	0.2050
	(0.5,1)	(1,1)	(0.3,0.3)	0.9512	0.9498	0.9450	0.9510	0.9984
			(0.3,0.5)	0.9484	0.9496	0.9388	0.9478	0.9974
			(0.5,0.5)	0.9492	0.9488	0.9376	0.9492	0.9980
		(1,5)	(0.3,0.3)	0.9554	0.9334	0.9381	0.9552	0.7746
			(0.3,0.5)	0.9524	0.9436	0.9417	0.9524	0.8764
			(0.5,0.5)	0.9514	0.9464	0.9481	0.9510	0.9210
		(5,5)	(0.3,0.3)	0.9520	0.9432	0.9435	0.9518	0.9812
			(0.3,0.5)	0.9514	0.9486	0.9360	0.9508	0.9942
			(0.5,0.5)	0.9508	0.9468	0.9365	0.9502	0.9976
		(5,10)	(0.3,0.3)	0.9502	0.9370	0.9400	0.9498	0.9108
			(0.3,0.5)	0.9496	0.9462	0.9366	0.9494	0.9844
			(0.5,0.5)	0.9506	0.9432	0.9433	0.9506	0.9796
		(10,10)	(0.3,0.3)	0.9544	0.9392	0.9485	0.9542	0.9512
			(0.3,0.5)	0.9534	0.9452	0.9417	0.9534	0.9920
			(0.5,0.5)	0.9500	0.9406	0.9367	0.9498	0.9946
	(1,1)	(1,1)	(0.3,0.3)	0.9520	0.9424	0.9398	0.9516	0.9832
			(0.3,0.5)	0.9482	0.9500	0.9369	0.9480	0.9476
			(0.5,0.5)	0.9498	0.9504	0.9365	0.9494	0.9826
		(1,5)	(0.3,0.3)	0.9582	0.9116	0.9361	0.9580	0.6132
			(0.3,0.5)	0.9514	0.9312	0.9363	0.9512	0.6050
			(0.5,0.5)	0.9506	0.9406	0.9409	0.9506	0.7346
		(5,5)	(0.3,0.3)	0.9470	0.9410	0.9401	0.9470	0.6142
			(0.3,0.5)	0.9486	0.9466	0.9413	0.9486	0.6902
			(0.5,0.5)	0.9520	0.9458	0.9452	0.9514	0.7672
		(5,10)	(0.3,0.3)	0.9528	0.9314	0.9362	0.9526	0.4986
			(0.3,0.5)	0.9514	0.9456	0.9470	0.9510	0.5654
			(0.5,0.5)	0.9522	0.9470	0.9437	0.9514	0.6908
		(10,10)	(0.3,0.3)	0.9502	0.9370	0.9472	0.9498	0.5102
			(0.3,0.5)	0.9496	0.9462	0.9390	0.9494	0.6256
			(0.5,0.5)	0.9506	0.9432	0.9368	0.9506	0.7190

Note: Bold values indicate coverage probabilities (CP) that satisfy the nominal confidence level of 0.95 ($CP \geq 0.945$, rounded to 0.950).

TABLE 3-6 The AWs of the 95% confidence intervals for the ratio between two variances of delta-IG distributions.

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths				
				AGCI	FCI	BCI	MOVER	NA
(30,30)	(0.5,0.5)	(1,1)	(0.3,0.3)	1.2973	3.9437	<i>3.0356</i>	1.2956	0.5054
			(0.3,0.5)	0.9211	2.5168	2.0473	<i>0.9199</i>	0.6020
			(0.5,0.5)	0.6167	1.5646	1.5244	<i>0.6160</i>	0.6900
		(1,5)	(0.3,0.3)	2.0369	5.2089	4.7651	<i>2.0340</i>	0.3468
			(0.3,0.5)	1.4311	3.4913	3.2759	<i>1.4292</i>	0.3553
			(0.5,0.5)	0.9492	2.2709	2.3015	<i>0.9481</i>	0.4688
		(5,5)	(0.3,0.3)	<i>0.5267</i>	1.0708	1.2783	<i>0.5259</i>	0.0990
			(0.3,0.5)	0.3736	0.7854	0.8904	<i>0.3731</i>	0.1257
			(0.5,0.5)	0.2488	0.5417	0.6038	<i>0.2485</i>	0.1501
		(5,10)	(0.3,0.3)	0.5071	1.0341	1.1738	<i>0.5064</i>	0.0840
			(0.3,0.5)	0.3545	0.7585	0.8529	<i>0.3540</i>	0.0970
			(0.5,0.5)	0.2337	0.5230	0.5205	<i>0.2334</i>	0.1244
		(10,10)	(0.3,0.3)	<i>0.3101</i>	0.6691	0.6911	0.3096	0.0682
			(0.3,0.5)	0.2175	0.5015	0.5084	<i>0.2173</i>	0.0821
			(0.5,0.5)	0.1436	0.3498	0.3214	<i>0.1434</i>	0.0943
	(0.5,1)	(1,1)	(0.3,0.3)	0.2245	0.9125	0.5430	<i>0.2242</i>	3.4710
			(0.3,0.5)	0.1591	0.5451	0.3711	<i>0.1589</i>	4.4720
			(0.5,0.5)	0.1065	0.3206	0.2559	<i>0.1063</i>	4.4980
		(1,5)	(0.3,0.3)	0.4439	1.1622	1.0211	<i>0.4433</i>	0.6174
			(0.3,0.5)	0.3132	0.7703	0.6933	<i>0.3128</i>	0.7957
			(0.5,0.5)	0.2086	0.4968	0.5055	<i>0.2083</i>	0.8851
		(5,5)	(0.3,0.3)	0.1314	0.2793	0.3056	<i>0.1312</i>	0.4861
			(0.3,0.5)	0.0956	0.2041	0.2333	<i>0.0954</i>	0.7164
			(0.5,0.5)	0.0649	0.1404	0.1432	<i>0.0648</i>	0.7060
		(5,10)	(0.3,0.3)	0.1317	0.2677	0.2955	<i>0.1315</i>	0.2817
			(0.3,0.5)	0.0934	0.1964	0.2311	<i>0.0933</i>	0.4118
			(0.5,0.5)	0.0622	0.1354	0.1374	<i>0.0621</i>	0.4244
		(10,10)	(0.3,0.3)	0.0896	0.1885	0.2060	<i>0.0895</i>	0.2794
			(0.3,0.5)	0.0648	0.1415	0.1462	<i>0.0647</i>	0.4119
			(0.5,0.5)	0.0437	0.0988	0.1046	<i>0.0436</i>	0.4138
	(1,1)	(1,1)	(0.3,0.3)	1.6384	8.7653	2.9341	<i>1.6361</i>	5.4600
			(0.3,0.5)	1.1496	4.7377	2.5755	<i>1.1481</i>	6.4530
			(0.5,0.5)	0.7648	2.6037	1.8186	<i>0.7639</i>	6.9950
		(1,5)	(0.3,0.3)	3.3885	12.6422	8.0805	<i>3.3836</i>	3.5320
			(0.3,0.5)	2.3840	7.6535	5.8014	<i>2.3809</i>	3.6680
			(0.5,0.5)	1.5860	4.6318	3.5059	<i>1.5841</i>	4.5900
		(5,5)	(0.3,0.3)	0.8254	1.7870	1.8230	<i>0.8242</i>	0.7047
			(0.3,0.5)	0.5889	1.2608	1.3478	<i>0.5882</i>	0.8939
			(0.5,0.5)	0.3948	0.8491	0.8703	<i>0.3944</i>	1.0450
		(5,10)	(0.3,0.3)	0.8919	1.8551	2.0199	<i>0.8906</i>	0.5676
			(0.3,0.5)	0.6275	1.3180	1.4738	<i>0.6267</i>	0.6495
			(0.5,0.5)	0.4162	0.8919	1.0172	<i>0.4157</i>	0.8377
		(10,10)	(0.3,0.3)	0.5267	1.0708	1.1762	<i>0.5259</i>	0.3933
			(0.3,0.5)	0.3736	0.7854	0.9216	<i>0.3731</i>	0.5021
			(0.5,0.5)	0.2488	0.5417	0.5737	<i>0.2485</i>	0.5974

Note: The shortest AW values are in italic.

TABLE 3-6 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths				
				AGCI	FCI	BCI	MOVER	NA
(50,50)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.4251	1.0340	1.0490	0.4246	0.3735
			(0.3,0.5)	1.1693	3.3202	2.6414	1.1677	0.4407
			(0.5,0.5)	0.8100	2.1059	1.8149	0.8090	0.5022
		(1,5)	(0.3,0.3)	0.6510	1.5320	1.4478	0.6502	0.2580
			(0.3,0.5)	1.9760	4.9840	4.6775	1.9732	0.2653
			(0.5,0.5)	1.3897	3.3931	3.4586	1.3879	0.3474
		(5,5)	(0.3,0.3)	0.1708	0.3780	0.4112	0.1706	0.0758
			(0.3,0.5)	0.4708	0.9545	1.1386	0.4701	0.0963
			(0.5,0.5)	0.3247	0.6869	0.7930	0.3243	0.1142
		(5,10)	(0.3,0.3)	0.1597	0.3649	0.3927	0.1595	0.0649
			(0.3,0.5)	0.4756	0.9666	1.1688	0.4749	0.0740
			(0.5,0.5)	0.3305	0.7061	0.7952	0.3300	0.0954
		(10,10)	(0.3,0.3)	0.0981	0.2456	0.2330	0.0980	0.0523
			(0.3,0.5)	0.2728	0.5980	0.6710	0.2724	0.0630
			(0.5,0.5)	0.1868	0.4378	0.4531	0.1866	0.0722
		(0.5,1)	(1,1)	0.0734	0.2049	0.1760	0.0733	2.5060
			(0.3,0.5)	0.1962	0.6900	0.4383	0.1959	3.2520
			(0.5,0.5)	0.1346	0.4030	0.2980	0.1344	3.2320
		(1,5)	(0.3,0.3)	0.1433	0.3337	0.3497	0.1431	0.4595
			(0.3,0.5)	0.4182	1.0719	0.9575	0.4176	0.6143
			(0.5,0.5)	0.2924	0.7161	0.6920	0.2920	0.6628
		(5,5)	(0.3,0.3)	0.0451	0.0978	0.1122	0.0450	0.3747
			(0.3,0.5)	0.1127	0.2350	0.2616	0.1126	0.5485
			(0.5,0.5)	0.0778	0.1653	0.1799	0.0778	0.5472
		(5,10)	(0.3,0.3)	0.0427	0.0945	0.0960	0.0427	0.2175
			(0.3,0.5)	0.1177	0.2386	0.2641	0.1175	0.3195
			(0.5,0.5)	0.0812	0.1717	0.1802	0.0811	0.3264
		(10,10)	(0.3,0.3)	0.0302	0.0694	0.0688	0.0302	0.2151
			(0.3,0.5)	0.0750	0.1599	0.1742	0.0749	0.3187
			(0.5,0.5)	0.0514	0.1150	0.1183	0.0513	0.3214
		(1,1)	(0.3,0.3)	0.5251	1.6053	1.2833	0.5245	3.8540
			(0.3,0.5)	1.4613	6.9263	3.3759	1.4593	4.3640
			(0.5,0.5)	1.0046	3.7180	2.3135	1.0033	5.0360
		(1,5)	(0.3,0.3)	1.0885	3.0135	2.5361	1.0872	2.5640
			(0.3,0.5)	3.2420	11.9997	7.2801	3.2375	2.6260
			(0.5,0.5)	2.2757	7.3488	5.3617	2.2727	3.2860
		(5,5)	(0.3,0.3)	0.2722	0.5849	0.6626	0.2719	0.5395
			(0.3,0.5)	0.7453	1.5726	1.8215	0.7443	0.6754
			(0.5,0.5)	0.5169	1.0946	1.2495	0.5162	0.7951
		(5,10)	(0.3,0.3)	0.2853	0.6157	0.6699	0.2850	0.4310
			(0.3,0.5)	0.8372	1.7219	1.8642	0.8361	0.4954
			(0.5,0.5)	0.5837	1.2250	1.3741	0.5829	0.6403
		(10,10)	(0.3,0.3)	0.1708	0.3780	0.3761	0.1706	0.3022
			(0.3,0.5)	0.4708	0.9545	1.1281	0.4701	0.3842
			(0.5,0.5)	0.3247	0.6869	0.7238	0.3243	0.4548

TABLE 3-6 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths				
				AGCI	FCI	BCI	MOVER	NA
(100,100)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.5369	1.3204	1.2699	0.5362	0.2532
			(0.3,0.5)	1.4227	4.7839	3.4285	1.4207	0.3019
			(0.5,0.5)	1.0106	3.0373	2.4893	1.0094	0.3403
		(1,5)	(0.3,0.3)	0.9160	2.1738	2.0714	0.9150	0.1798
			(0.3,0.5)	1.8624	4.9465	4.5410	1.8596	0.1853
			(0.5,0.5)	1.3045	3.3241	3.1800	1.3028	0.2399
		(5,5)	(0.3,0.3)	0.2153	0.4687	0.5282	0.2150	0.0529
			(0.3,0.5)	0.4660	1.0372	1.0629	0.4653	0.0677
			(0.5,0.5)	0.3239	0.7680	0.7197	0.3234	0.0799
		(5,10)	(0.3,0.3)	0.2178	0.4818	0.5237	0.2176	0.0452
			(0.3,0.5)	0.4362	0.9463	1.0545	0.4355	0.0517
			(0.5,0.5)	0.3014	0.7000	0.6969	0.3010	0.0667
		(10,10)	(0.3,0.3)	0.1235	0.3028	0.2730	0.1234	0.0365
			(0.3,0.5)	0.2588	0.6222	0.5970	0.2584	0.0439
			(0.5,0.5)	0.1771	0.4708	0.4089	0.1768	0.0506
	(0.5,1)	(1,1)	(0.3,0.3)	0.0893	0.2464	0.2193	0.0892	1.6950
			(0.3,0.5)	0.2747	1.3001	0.6499	0.2744	2.1550
			(0.5,0.5)	0.1958	0.7621	0.4684	0.1955	2.1560
		(1,5)	(0.3,0.3)	0.1932	0.4571	0.4354	0.1929	0.3161
			(0.3,0.5)	0.4309	1.2000	0.9934	0.4302	0.4293
			(0.5,0.5)	0.3028	0.7960	0.7002	0.3024	0.4564
		(5,5)	(0.3,0.3)	0.0523	0.1129	0.1178	0.0523	0.2618
			(0.3,0.5)	0.1240	0.2941	0.3019	0.1238	0.3855
			(0.5,0.5)	0.0884	0.2170	0.2178	0.0883	0.3866
		(5,10)	(0.3,0.3)	0.0538	0.1172	0.1280	0.0538	0.1535
			(0.3,0.5)	0.1165	0.2593	0.2590	0.1163	0.2255
			(0.5,0.5)	0.0810	0.1920	0.2020	0.0809	0.2317
		(10,10)	(0.3,0.3)	0.0345	0.0796	0.0808	0.0345	0.1507
			(0.3,0.5)	0.0770	0.1849	0.1908	0.0769	0.2262
			(0.5,0.5)	0.0539	0.1401	0.1271	0.0538	0.2250
	(1,1)	(1,1)	(0.3,0.3)	0.6634	2.1206	1.5632	0.6626	2.5200
			(0.3,0.5)	2.0000	12.6667	4.4066	1.9972	2.8340
			(0.5,0.5)	1.4115	6.5376	3.2232	1.4097	3.1690
		(1,5)	(0.3,0.3)	1.5016	4.3935	3.5057	1.4998	1.7180
			(0.3,0.5)	3.3040	13.3281	7.3678	3.2993	1.7430
			(0.5,0.5)	2.3219	7.8848	5.5421	2.3189	2.1680
		(5,5)	(0.3,0.3)	0.3434	0.7315	0.8230	0.3430	0.3706
			(0.3,0.5)	0.7926	1.8607	1.7604	0.7915	0.4706
			(0.5,0.5)	0.5584	1.3239	1.3405	0.5577	0.5525
		(5,10)	(0.3,0.3)	0.3854	0.8198	0.9199	0.3849	0.3017
			(0.3,0.5)	0.8056	1.7742	1.9970	0.8044	0.3472
			(0.5,0.5)	0.5611	1.2710	1.3563	0.5603	0.4488
		(10,10)	(0.3,0.3)	0.2153	0.4687	0.5219	0.2150	0.2114
			(0.3,0.5)	0.4660	1.0372	1.1181	0.4653	0.2692
			(0.5,0.5)	0.3239	0.7680	0.7357	0.3234	0.3187

TABLE 3-6 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths				
				AGCI	FCI	BCI	MOVER	NA
(200,200)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.6748	1.8439	1.5607	0.6740	0.1757
			(0.3,0.5)	0.4643	1.2144	1.1495	0.4638	0.2063
			(0.5,0.5)	1.2733	3.8381	2.9342	1.2716	0.2337
		(1,5)	(0.3,0.3)	0.8640	2.1294	1.9475	0.8630	0.1255
			(0.3,0.5)	0.5924	1.4443	1.3258	0.5917	0.1297
			(0.5,0.5)	1.8009	4.6451	4.2693	1.7983	0.1678
		(5,5)	(0.3,0.3)	0.2124	0.5249	0.4988	0.2121	0.0371
			(0.3,0.5)	0.1448	0.3672	0.3411	0.1447	0.0476
			(0.5,0.5)	0.4147	0.9154	0.9692	0.4141	0.0563
		(5,10)	(0.3,0.3)	0.1975	0.4795	0.4510	0.1973	0.0319
			(0.3,0.5)	0.1349	0.3358	0.3354	0.1347	0.0363
			(0.5,0.5)	0.4079	0.8757	1.0144	0.4073	0.0469
		(10,10)	(0.3,0.3)	0.1149	0.3274	0.2726	0.1148	0.0257
			(0.3,0.5)	0.0780	0.2304	0.1812	0.0779	0.0311
			(0.5,0.5)	0.2273	0.5514	0.5450	0.2269	0.0354
	(0.5,1)	(1,1)	(0.3,0.3)	0.1311	0.4303	0.3077	0.1309	1.1750
			(0.3,0.5)	0.0903	0.2718	0.2035	0.0902	1.4680
			(0.5,0.5)	0.2369	0.9231	0.5709	0.2366	1.4590
		(1,5)	(0.3,0.3)	0.2007	0.5036	0.4670	0.2004	0.2229
			(0.3,0.5)	0.1376	0.3396	0.3084	0.1374	0.2996
			(0.5,0.5)	0.4046	1.0779	0.9367	0.4040	0.3169
		(5,5)	(0.3,0.3)	0.0590	0.1473	0.1341	0.0589	0.1850
			(0.3,0.5)	0.0406	0.1028	0.0918	0.0405	0.2732
			(0.5,0.5)	0.1057	0.2450	0.2446	0.1055	0.2740
		(5,10)	(0.3,0.3)	0.0531	0.1312	0.1295	0.0530	0.1076
			(0.3,0.5)	0.0362	0.0918	0.0830	0.0362	0.1602
			(0.5,0.5)	0.1037	0.2288	0.2469	0.1035	0.1635
		(10,10)	(0.3,0.3)	0.0354	0.0973	0.0864	0.0354	0.1058
			(0.3,0.5)	0.0242	0.0684	0.0589	0.0242	0.1591
			(0.5,0.5)	0.0640	0.1561	0.1526	0.0639	0.1600
	(1,1)	(1,1)	(0.3,0.3)	0.9391	3.4359	2.2059	0.9380	1.7040
			(0.3,0.5)	0.6446	2.0902	1.5977	0.6439	1.9210
			(0.5,0.5)	1.7615	8.8732	4.1629	1.7591	2.1010
		(1,5)	(0.3,0.3)	1.5406	4.6646	3.6067	1.5387	1.1970
			(0.3,0.5)	1.0568	3.0434	2.4933	1.0556	1.2040
			(0.5,0.5)	3.1568	11.7079	7.5928	3.1523	1.4820
		(5,5)	(0.3,0.3)	0.3701	0.8792	0.8517	0.3696	0.2614
			(0.3,0.5)	0.2537	0.6068	0.6162	0.2534	0.3293
			(0.5,0.5)	0.7116	1.6077	1.6124	0.7106	0.3863
		(5,10)	(0.3,0.3)	0.3695	0.8514	0.8786	0.3690	0.2121
			(0.3,0.5)	0.2527	0.5898	0.6141	0.2524	0.2442
			(0.5,0.5)	0.7534	1.6227	1.7983	0.7523	0.3143
		(10,10)	(0.3,0.3)	0.2124	0.5249	0.5171	0.2121	0.1483
			(0.3,0.5)	0.1448	0.3672	0.3262	0.1447	0.1903
			(0.5,0.5)	0.4147	0.9154	0.9526	0.4141	0.2256

TABLE 3-6 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths				
				AGCI	FCI	BCI	MOVER	NA
(30,50)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.8879	2.4463	2.1937	0.8868	0.4418
			(0.3,0.5)	0.5899	1.5313	1.3729	0.5892	0.5057
			(0.5,0.5)	1.3335	4.8205	3.1358	1.3315	0.6038
		(1,5)	(0.3,0.3)	1.2746	3.1763	3.1755	1.2730	0.3322
			(0.3,0.5)	0.8416	2.0529	1.8595	0.8406	0.3519
			(0.5,0.5)	1.7654	5.2181	4.0880	1.7624	0.4604
		(5,5)	(0.3,0.3)	0.2861	0.6569	0.6820	0.2857	0.0874
			(0.3,0.5)	0.1887	0.4501	0.4419	0.1885	0.1071
			(0.5,0.5)	0.4728	1.1687	1.0738	0.4720	0.1328
		(5,10)	(0.3,0.3)	0.2853	0.6388	0.6480	0.2849	0.0785
			(0.3,0.5)	0.1879	0.4381	0.4193	0.1877	0.0865
			(0.5,0.5)	0.4318	1.0796	0.9520	0.4311	0.1164
		(10,10)	(0.3,0.3)	0.1556	0.4023	0.3472	0.1554	0.0607
			(0.3,0.5)	0.1023	0.2797	0.2298	0.1021	0.0705
			(0.5,0.5)	0.2686	0.7347	0.6523	0.2681	0.0843
		(0.5,1)	(1,1)	0.1638	0.5335	0.3700	0.1636	2.5740
			(0.3,0.5)	0.1092	0.3214	0.2720	0.1091	3.1660
			(0.5,0.5)	0.2556	1.2807	0.6025	0.2553	3.2760
		(1,5)	(0.3,0.3)	0.2846	0.7236	0.6869	0.2843	0.5244
			(0.3,0.5)	0.1883	0.4645	0.4442	0.1880	0.6674
			(0.5,0.5)	0.4074	1.2495	0.9337	0.4068	0.7475
		(5,5)	(0.3,0.3)	0.0726	0.1719	0.1680	0.0725	0.3761
			(0.3,0.5)	0.0484	0.1174	0.1201	0.0483	0.5497
			(0.5,0.5)	0.1282	0.3274	0.2823	0.1280	0.5543
		(5,10)	(0.3,0.3)	0.0715	0.1642	0.1608	0.0714	0.2224
			(0.3,0.5)	0.0472	0.1125	0.1155	0.0471	0.3274
			(0.5,0.5)	0.1182	0.2922	0.2780	0.1180	0.3354
		(10,10)	(0.3,0.3)	0.0433	0.1117	0.1059	0.0433	0.2160
			(0.3,0.5)	0.0287	0.0776	0.0681	0.0287	0.3189
			(0.5,0.5)	0.0825	0.2150	0.1951	0.0823	0.3237
		(1,1)	(0.3,0.3)	1.2188	4.8395	2.6926	1.2172	4.6100
			(0.3,0.5)	0.8084	2.7162	1.9197	0.8074	5.1840
			(0.5,0.5)	1.8113	12.5550	4.1975	1.8085	6.1900
		(1,5)	(0.3,0.3)	2.2273	7.4034	5.4126	2.2244	3.4810
			(0.3,0.5)	1.4723	4.4488	3.6798	1.4705	3.5180
			(0.5,0.5)	3.0834	14.2498	6.8237	3.0783	4.4100
		(5,5)	(0.3,0.3)	0.4946	1.1204	1.2364	0.4940	0.6262
			(0.3,0.5)	0.3277	0.7502	0.7766	0.3273	0.7442
			(0.5,0.5)	0.7791	1.9882	1.8567	0.7779	0.9280
		(5,10)	(0.3,0.3)	0.5280	1.1559	1.2327	0.5274	0.5345
			(0.3,0.5)	0.3484	0.7775	0.8704	0.3480	0.5906
			(0.5,0.5)	0.7839	1.9213	1.8228	0.7826	0.7990
		(10,10)	(0.3,0.3)	0.2861	0.6569	0.6470	0.2857	0.3513
			(0.3,0.5)	0.1887	0.4501	0.4689	0.1885	0.4275
			(0.5,0.5)	0.4728	1.1687	1.1314	0.4720	0.5312

TABLE 3-6 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths				
				AGCI	FCI	BCI	MOVER	NA
(50,100)	(0.5,0.5)	(1,1)	(0.3,0.3)	0.9126	2.9678	2.2801	0.9113	0.3129
			(0.3,0.5)	0.5975	1.7830	1.3444	0.5968	0.3590
			(0.5,0.5)	0.4067	1.1437	0.9866	0.4063	0.4260
		(1,5)	(0.3,0.3)	1.1762	3.4037	2.8476	1.1745	0.2524
			(0.3,0.5)	0.7559	2.1346	1.7698	0.7550	0.2599
			(0.5,0.5)	0.5097	1.4034	1.1804	0.5091	0.3442
		(5,5)	(0.3,0.3)	0.3174	0.8586	0.7887	0.3169	0.0652
			(0.3,0.5)	0.2039	0.5885	0.4844	0.2036	0.0781
			(0.5,0.5)	0.1372	0.4079	0.3322	0.1371	0.0983
		(5,10)	(0.3,0.3)	0.2842	0.7926	0.6463	0.2838	0.0590
			(0.3,0.5)	0.1805	0.5447	0.4450	0.1803	0.0642
			(0.5,0.5)	0.1210	0.3779	0.2900	0.1209	0.0875
		(10,10)	(0.3,0.3)	0.1773	0.5534	0.4190	0.1770	0.0449
			(0.3,0.5)	0.1126	0.3865	0.2601	0.1124	0.0513
			(0.5,0.5)	0.0753	0.2706	0.1769	0.0752	0.0624
		(0.5,1)	(1,1)	0.1765	0.7348	0.4338	0.1763	1.7190
			(0.3,0.5)	0.1163	0.4080	0.2650	0.1162	2.1510
			(0.5,0.5)	0.0796	0.2533	0.1891	0.0795	2.1640
		(1,5)	(0.3,0.3)	0.2736	0.8041	0.6184	0.2732	0.3717
			(0.3,0.5)	0.1766	0.4985	0.3997	0.1763	0.4705
			(0.5,0.5)	0.1192	0.3264	0.2660	0.1191	0.5298
		(5,5)	(0.3,0.3)	0.0895	0.2393	0.2002	0.0894	0.2678
			(0.3,0.5)	0.0593	0.1628	0.1410	0.0592	0.3890
			(0.5,0.5)	0.0406	0.1127	0.0994	0.0405	0.3940
		(5,10)	(0.3,0.3)	0.0793	0.2147	0.1967	0.0792	0.1582
			(0.3,0.5)	0.0510	0.1471	0.1206	0.0509	0.2303
			(0.5,0.5)	0.0343	0.1020	0.0775	0.0343	0.2388
		(10,10)	(0.3,0.3)	0.0566	0.1621	0.1255	0.0565	0.1524
			(0.3,0.5)	0.0370	0.1129	0.0865	0.0369	0.2271
			(0.5,0.5)	0.0251	0.0790	0.0615	0.0251	0.2291
		(1,1)	(1,1)	1.2297	6.3208	2.7568	1.2281	3.2130
			(0.3,0.5)	0.8003	3.1940	1.8876	0.7993	3.4820
			(0.5,0.5)	0.5438	1.8654	1.2855	0.5432	4.1000
		(1,5)	(0.3,0.3)	2.0667	8.0032	4.6692	2.0638	2.5250
			(0.3,0.5)	1.3335	4.5408	2.9531	1.3318	2.5690
			(0.5,0.5)	0.9007	2.8107	2.0413	0.8996	3.2820
		(5,5)	(0.3,0.3)	0.5302	1.3957	1.1705	0.5295	0.4595
			(0.3,0.5)	0.3448	0.9241	0.7612	0.3444	0.5430
			(0.5,0.5)	0.2336	0.6279	0.5372	0.2334	0.6840
		(5,10)	(0.3,0.3)	0.5217	1.3589	1.2050	0.5210	0.4015
			(0.3,0.5)	0.3341	0.9072	0.7907	0.3336	0.4365
			(0.5,0.5)	0.2247	0.6178	0.5190	0.2244	0.5911
		(10,10)	(0.3,0.3)	0.3174	0.8586	0.7685	0.3169	0.2595
			(0.3,0.5)	0.2039	0.5885	0.4970	0.2036	0.3113
			(0.5,0.5)	0.1372	0.4079	0.3199	0.1371	0.3939

TABLE 3-6 (CONTINUED)

(n_1, n_2)	(μ_1, μ_2)	(λ_1, λ_2)	(δ_1, δ_2)	Average Widths				
				AGCI	FCI	BCI	MOVER	NA
(100,200)	(0.5,0.5)	(1,1)	(0.3,0.3)	1.2174	4.0898	2.6883	<i>1.2155</i>	0.2174
			(0.3,0.5)	0.7999	2.4267	1.8946	<i>0.7988</i>	0.2432
			(0.5,0.5)	0.5188	1.4930	1.2662	<i>0.5181</i>	0.2908
		(1,5)	(0.3,0.3)	1.7256	5.0527	4.1256	<i>1.7228</i>	0.1759
			(0.3,0.5)	1.1408	3.2465	2.8087	<i>1.1392</i>	0.1799
			(0.5,0.5)	0.7331	2.0417	1.8102	<i>0.7322</i>	0.2353
		(5,5)	(0.3,0.3)	0.4166	1.0446	1.0116	<i>0.4159</i>	0.0456
			(0.3,0.5)	0.2701	0.7368	0.5988	<i>0.2698</i>	0.0546
			(0.5,0.5)	0.1746	0.5064	0.4206	<i>0.1744</i>	0.0690
		(5,10)	(0.3,0.3)	0.4005	1.0096	0.9855	<i>0.3998</i>	0.0412
			(0.3,0.5)	0.2610	0.7240	0.6337	<i>0.2606</i>	0.0449
			(0.5,0.5)	0.1675	0.4982	0.3851	<i>0.1673</i>	0.0616
		(10,10)	(0.3,0.3)	0.2313	0.6561	0.5246	<i>0.2309</i>	0.0316
			(0.3,0.5)	0.1487	0.4769	0.3386	<i>0.1485</i>	0.0360
			(0.5,0.5)	0.0957	0.3336	0.2213	<i>0.0956</i>	0.0436
	(0.5,1)	(1,1)	(0.3,0.3)	0.2256	0.9519	0.5557	<i>0.2253</i>	1.1920
			(0.3,0.5)	0.1479	0.5235	0.3456	<i>0.1477</i>	1.4650
			(0.5,0.5)	0.0964	0.3087	0.2176	<i>0.0963</i>	1.4740
		(1,5)	(0.3,0.3)	0.3878	1.1639	0.9169	<i>0.3872</i>	0.2583
			(0.3,0.5)	0.2554	0.7335	0.6371	<i>0.2551</i>	0.3271
			(0.5,0.5)	0.1645	0.4594	0.4057	<i>0.1643</i>	0.3618
		(5,5)	(0.3,0.3)	0.1085	0.2758	0.2584	<i>0.1084</i>	0.1873
			(0.3,0.5)	0.0712	0.1901	0.1638	<i>0.0711</i>	0.2735
			(0.5,0.5)	0.0470	0.1305	0.1091	<i>0.0469</i>	0.2764
		(5,10)	(0.3,0.3)	0.1041	0.2612	0.2345	<i>0.1040</i>	0.1110
			(0.3,0.5)	0.0675	0.1842	0.1580	<i>0.0674</i>	0.1628
			(0.5,0.5)	0.0437	0.1266	0.1020	<i>0.0436</i>	0.1682
		(10,10)	(0.3,0.3)	0.0673	0.1828	0.1543	<i>0.0672</i>	0.1076
			(0.3,0.5)	<i>0.0436</i>	0.1302	0.1065	<i>0.0436</i>	0.1603
			(0.5,0.5)	0.0287	0.0910	0.0651	<i>0.0287</i>	0.1626
	(1,1)	(1,1)	(0.3,0.3)	1.6422	9.8489	4.1016	<i>1.6396</i>	2.1190
			(0.3,0.5)	1.0753	4.7944	2.6628	<i>1.0738</i>	2.3060
			(0.5,0.5)	0.6936	2.5718	1.7290	<i>0.6928</i>	2.6580
		(1,5)	(0.3,0.3)	2.9949	13.6144	7.3688	<i>2.9902</i>	1.6930
			(0.3,0.5)	1.9858	7.5414	4.9435	<i>1.9831</i>	1.7170
			(0.5,0.5)	1.2765	4.3296	2.9089	<i>1.2749</i>	2.1340
		(5,5)	(0.3,0.3)	0.7006	1.7571	1.5945	<i>0.6995</i>	0.3196
			(0.3,0.5)	0.4590	1.1832	1.0924	<i>0.4584</i>	0.3791
			(0.5,0.5)	0.2976	0.7891	0.7015	<i>0.2972</i>	0.4738
		(5,10)	(0.3,0.3)	0.7329	1.7929	1.6289	<i>0.7317</i>	0.2817
			(0.3,0.5)	0.4805	1.2343	1.0669	<i>0.4799</i>	0.3082
			(0.5,0.5)	0.3090	0.8249	0.7688	<i>0.3086</i>	0.4170
		(10,10)	(0.3,0.3)	0.4166	1.0446	0.9744	<i>0.4159</i>	0.1829
			(0.3,0.5)	0.2701	0.7368	0.6749	<i>0.2698</i>	0.2180
			(0.5,0.5)	0.1746	0.5064	0.3959	<i>0.1744</i>	0.2751

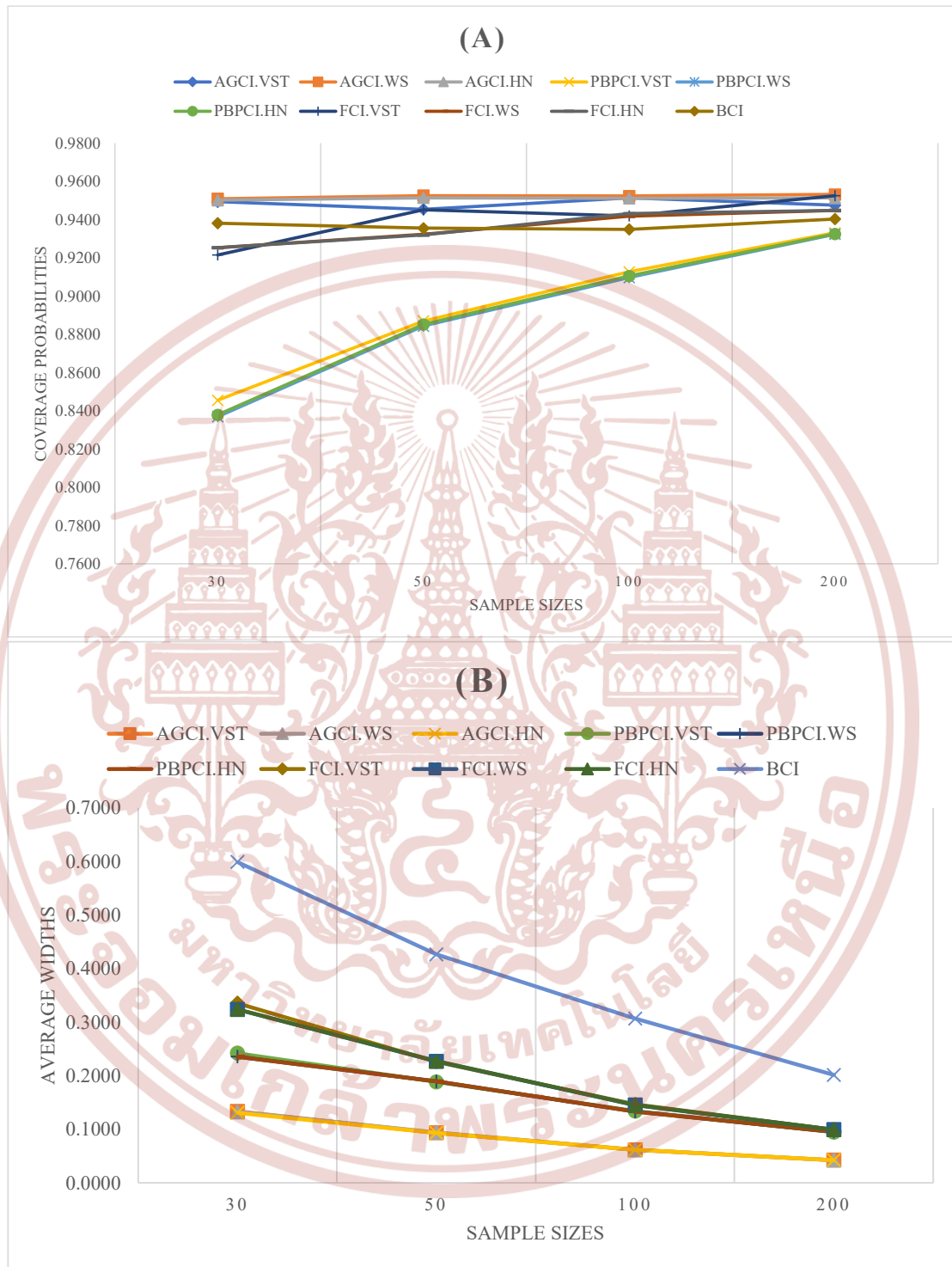


FIGURE 3-1 Comparison of the CPs (A) and AWs (B) for estimating the 95% confidence interval for the single variance of a delta-IG distribution with various sample sizes at $(\mu, \lambda, \delta) = (0.5, 1.0, 0.3)$.

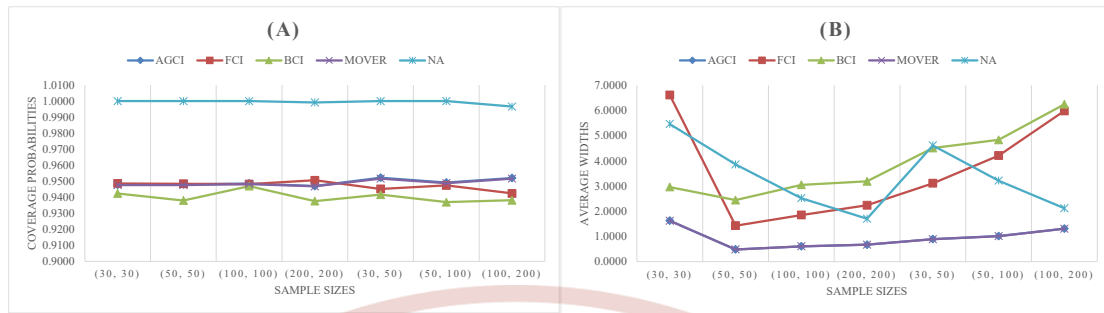


FIGURE 3-2 Comparison of the CPs (A) and AWs (B) for estimating the 95% confidence interval for the difference of two variances of delta-IG distributions with various sample sizes at $(\mu_1, \mu_2, \lambda_1, \lambda_2, \delta_1, \delta_2) = (0.5, 0.5, 1.0, 1.0, 0.3, 0.3)$.

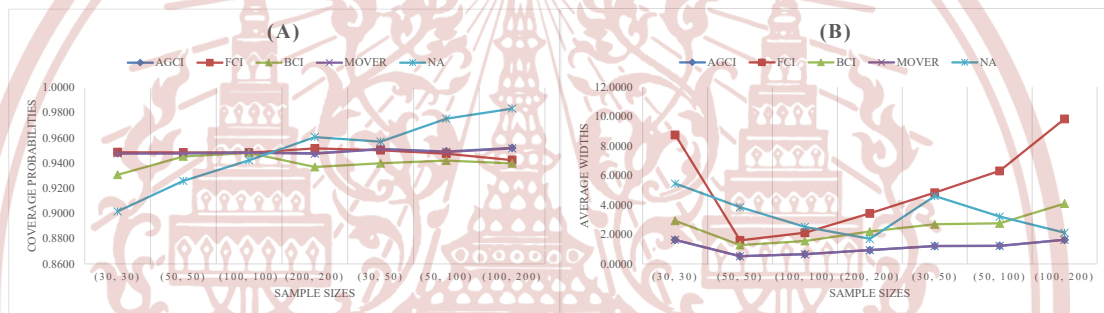


FIGURE 3-3 Comparison of the CPs (A) and AWs (B) for estimating the 95% confidence interval for the ratio of two variances of delta-IG distributions with various sample sizes at $(\mu_1, \mu_2, \lambda_1, \lambda_2, \delta_1, \delta_2) = (0.5, 0.5, 1.0, 1.0, 0.3, 0.3)$.

3.5 An empirical application

To assess the proposed methods for estimating the confidence interval for a single variance, the difference, and the ratio of variances for the delta-IG distributions, traffic fatality data are employed to evaluate their performance.

3.5.1 Application of the single variance of a delta-IG distribution

To demonstrate the effectiveness of the confidence intervals for the variance of a delta-IG distribution constructed using AGCI-VST, AGCI-WS, AGCI-HN, PBPCI-VST, PBPCI-WS, PBPCI-HN, FCI-VST, FCI-WS, FCI-HN, and BCI, traffic fatality data are analyzed. The dataset is obtained from a statistical report on road accident casualties published by the Road Accident Victims Protection Company Limited and is available on the Thai Road Safety Center (<https://www.thairsc.com/bangkok-in-depth-statistics>). Table 3-7 consists of death counts from traffic accidents recorded in December 2023 across 50 districts in Bangkok, Thailand. The aim is to evaluate the performance of the proposed confidence intervals for the variance of the delta-IG distribution based on real data. To compare traffic fatality estimates in Bangkok, Thailand, the normal Q–Q plot and histogram of the data are presented in Figure 3-4. By partitioning the death count data into zero and non-zero observations and focusing

on the positive values, the most appropriate distribution for the positive component was identified. To evaluate the underlying distribution of the data, the positive death count datasets were assessed using the Akaike information criterion (AIC) and the Bayesian information criterion (BIC). Based on the AIC and BIC values reported in Table 3-8, the inverse Gaussian (IG) distribution provided the best fit to the positive death count data.

The descriptive statistics for the death count data from Bangkok, Thailand are as follows: approximate variance is $\hat{\eta} = 3.7716$, with parameters $\hat{\mu} = 2.5001$, $\hat{\lambda} = 4.4976$, and $\hat{\delta} = 0.2653$.

Table 3-9 presents the confidence intervals for the variance of the death count data in the central region of Thailand using the considered methods. The results show that the AGCI-based approaches produced substantially narrower intervals than the other methods. Among them, AGCI.HN yielded the smallest interval width, followed by AGCI.VST and AGCI.WS, indicating higher precision. In contrast, the PBPCI and FCI methods resulted in considerably wider intervals, suggesting lower efficiency in this application. The BCI approach produced the widest interval overall, reflecting the highest level of uncertainty. Overall, the AGCI methods, particularly AGCI.HN, demonstrated the best performance for this dataset by providing the most precise confidence intervals for the variance.

TABLE 3-7 The traffic death count data from Bangkok, Thailand in December 2023.

Death count from each district of Bangkok, Thailand in December 2023																								
9	5	3	2	1	1	0	9	4	2	2	2	1	0	5	5	3	2	1	1	1	0	0	3	2
0	4	2	2	1	1	0	0	0	0	4	3	1	1	1	0	0	0	0	0	0	0	0	0	0

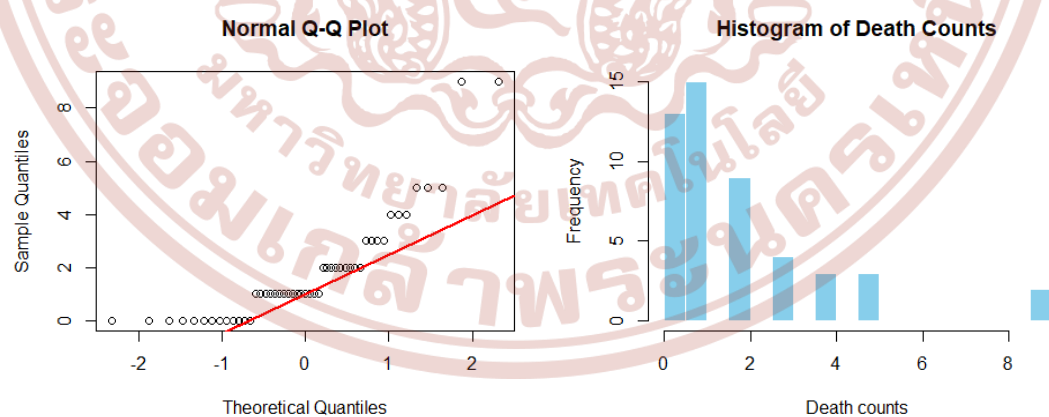


FIGURE 3-4 (left) Normal Q-Q plot and (right) Histogram of death count data from Bangkok.

TABLE 3-8 AIC and BIC values of positive observation of death count data from Bangkok data set.

Distributions	Criterion	
	AIC	BIC
Normal	156.8125	159.9795
Lognormal	125.7218	128.8889
Weibull	135.3959	138.5629
Gamma	131.9867	135.1538
Exponential	139.9729	141.5565
Logistic	150.5875	153.7545
Cauchy	148.3000	151.4670
Inverse Gaussian	124.0850	127.2521

TABLE 3-9 Confidence intervals for the variance of death count data for Bangkok data set.

Methods	CI of variance (η)	
	Interval	Width
AGCI.VST	2.9667–5.6620	2.6953
AGCI.WS	2.9388–5.6557	2.7169
AGCI.HN	2.9495–5.5853	2.6358
PBPCI.VST	1.6030–7.4081	5.8051
PBPCI.WS	1.6322–7.0154	5.3832
PBPCI.HN	1.6342–7.0860	5.4518
FCL.VST	1.9233–8.0938	6.1705
FCL.WS	1.9184–7.7940	5.8757
FCL.HN	1.9205–7.8664	5.9459
BCI	0.5804–8.8329	8.2525

3.5.2 Application of the difference and ratio of variances in delta-IG distributions

To illustrate the effectiveness of confidence intervals for the difference and ratio between two variances under the delta-IG distribution, methods including AGCI, FCI, BCI, MOVER, and NA are applied to traffic fatality data. The dataset is sourced from a statistical report on road accident casualties published by the Road Accident Victims Protection Company Limited and is available via the Thai Road Safety Center (<https://www.thairsc.com/bangkok-in-depth-statistics>). It comprises death counts from traffic accidents recorded in November 2025 across 112 districts in the northern region and 151 districts in the southern region of Thailand which shown in Table 3-10. The normal Q–Q plots and histograms of the death count data are shown in Figures 3-5 and 3-6 for the northern region and the southern region datasets, respectively. After separating the data into zero and non-zero observations and focusing on the positive values, the most suitable distribution for the positive component was identified. To evaluate the underlying distribution of the data, the positive death count datasets were assessed using AIC and BIC. Based on the AIC and BIC values reported in Table 3-11, the IG distribution provided the best fit.

The descriptive statistics for the death count data from the northern and southern region of Thailand are as follows: estimated coefficients of variation are $\hat{\eta}_1 = 3.0565$

for the northern region and $\hat{\eta}_2 = 1.1794$ for the southern region. The corresponding parameter estimates are $(\hat{\mu}_1, \hat{\mu}_2) = (2.3274, 1.7727)$, $(\hat{\lambda}_1, \hat{\lambda}_2) = (3.6356, 5.9946)$ and $(\hat{\delta}_1, \hat{\delta}_2) = (0.5089, 0.5629)$.

Table 3-12 reports the confidence intervals for the difference and ratio of the two variances using the methods under consideration. The results indicate that the MOVER method yields the narrowest interval widths for both the difference and ratio of variances, followed closely by the AGCI method. In contrast, the FCI, BCI, and NA approaches produce relatively wider intervals. Consistent with both simulation and empirical findings, MOVER and AGCI are therefore identified as the most suitable methods for constructing confidence intervals for the difference and ratio of variances in delta-IG populations.

TABLE 3-10 The traffic death count data from the northern and southern region of Thailand in November 2025.

Death count from each district of the northern region of Thailand																													
0	0	1	0	0	0	1	2	8	0	0	0	3	1	0	0	1	1	0	0	2	0	0	2	0	3	2	6	0	1
1	1	0	0	0	0	2	1	1	1	2	0	1	0	0	0	0	1	0	0	5	0	3	0	0	0	0	2	3	
0	1	0	1	0	2	1	1	5	2	2	3	1	0	0	0	0	1	0	0	0	1	1	0	0	0	6	1	1	
0	1	0	0	1	0	0	1	2	7	0	1	0	0	1	1	1	0	3	0	5	16								
Death count from each district of the southern region of Thailand																													
0	0	2	0	3	0	1	2	1	0	5	0	2	3	2	1	3	0	1	4	0	0	0	0	1	0	0	1	1	0
0	0	0	0	2	0	0	1	1	0	1	0	0	0	3	2	2	0	1	0	0	2	2	0	1	0	0	1	0	
1	0	0	0	0	1	0	2	0	0	1	4	0	1	0	0	0	1	1	0	0	2	0	0	0	0	0	2	2	
0	0	0	2	2	3	0	0	0	1	0	2	0	1	0	0	2	0	0	0	0	0	1	1	1	0	0	1	1	0
0	1	0	1	2	0	0	1	0	0	0	0	1	0	8	0	0	0	1	0	0	4	0	1	2	1	2	1	2	0
1																													

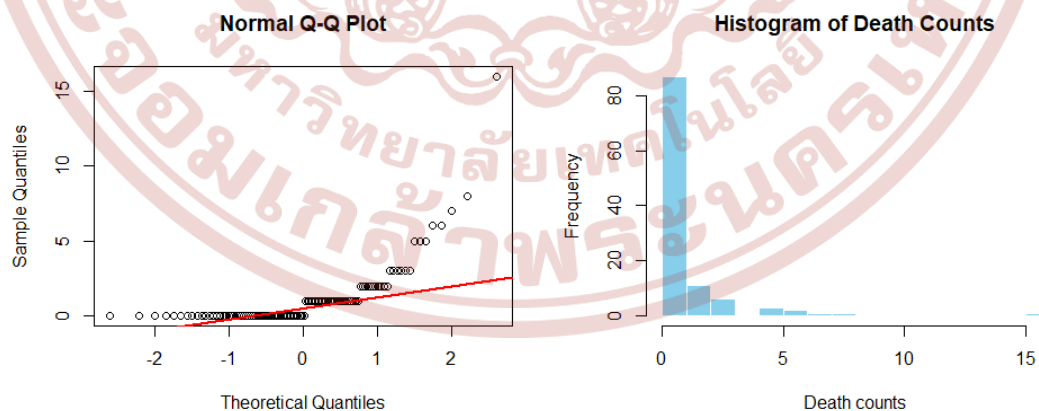


FIGURE 3-5 (left) Normal Q-Q plot and (right) Histogram of death count data from the northern region in November 2025.

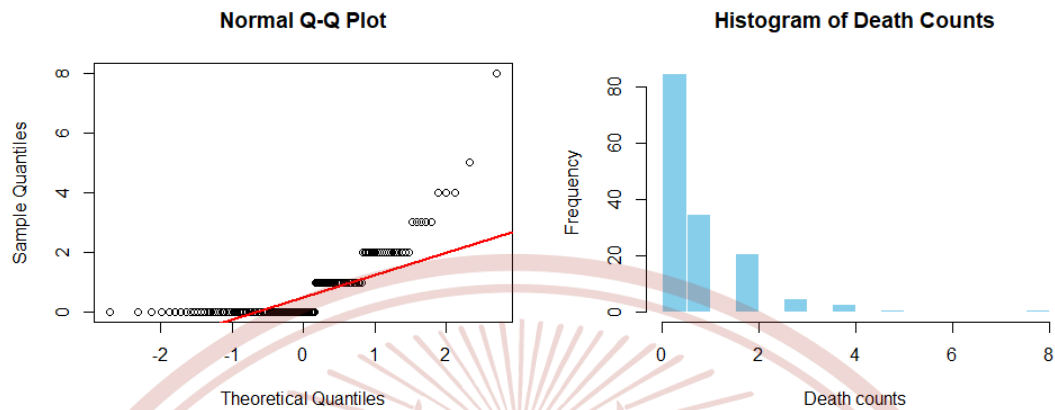


FIGURE 3-6 (left) Normal Q-Q plot and (right) Histogram of death count data from the southern region in November 2025.

TABLE 3-11 AIC and BIC values of positive observation of death count data from the northern and southern region data set.

Distributions	Northern Criterion		Southern Criterion	
	AIC	BIC	AIC	BIC
Normal	261.2514	265.2661	214.3523	218.7316
Lognormal	179.3382	183.3528	157.6174	161.9967
Weibull	203.5997	207.6143	183.9815	188.3609
Gamma	197.9532	201.9678	170.7712	175.1506
Exponential	204.9167	206.9240	209.5725	211.7622
Logistic	235.7683	239.7830	194.5612	198.9405
Inverse Gaussian	177.3787	181.3933	156.7999	161.1792

TABLE 3-12 Confidence intervals for the difference and ratio of two variances of death count data between the northern and southern region data set.

Methods	$\hat{\eta}_{diff} = \hat{\eta}_1 - \hat{\eta}_2 = 1.8771$		Methods	$\hat{\eta}_{ratio} = \hat{\eta}_1 / \hat{\eta}_2 = 2.5915$	
	Interval	Width		Interval	Width
AGCI	(1.3540, 2.8262)	1.4722	AGCI	(2.0610, 3.4610)	1.4000
FCI	(0.4659, 4.3788)	3.9129	FCI	(1.3385, 5.1260)	3.7875
BCI	(-2.7308, 4.4280)	7.1588	BCI	(2.0770, 6.5627)	4.4857
MOVER	(1.3541, 2.8246)	1.4706	MOVER	(2.0613, 3.4605)	1.3992
NA	(-1.2400, 4.8840)	6.1240	NA	(-0.6563, 5.4670)	6.1240

3.6 Conclusions

This study investigated several methods for constructing confidence intervals for the variance, as well as for the difference and ratio of variances under the delta-inverse Gaussian distribution, using both simulation and real-data analyses.

For constructing confidence intervals for a single variance of the delta-IG distribution, AGCI methods performed best overall, with AGCI.VST providing the most consistent balance between coverage probability and interval width. While PBPCI improved with larger samples, it remained less efficient, and BCI produced the widest intervals.

For the difference and ratio of variances, AGCI and MOVER showed the strongest performance across scenarios. AGCI performed well when zero-inflation probabilities differed, whereas MOVER was more effective for larger and unbalanced samples and maintained coverage closest to the nominal level, particularly for ratio-based inference.

The real-data analysis supported these findings. AGCI methods yielded the most precise intervals for a single variance, particularly AGCI based on Hannig method produced the narrowest width of interval, while MOVER produced the narrowest intervals for the difference and ratio of variances. Overall, AGCI and MOVER are recommended as the most reliable methods for variance inference in delta-IG populations.

In Chapters 2 and 3 addressed the confidence intervals for the coefficient of variation and variance of the delta-IG distribution in both single and two population settings, covering estimation of a single parameter as well as the difference and ratio between two independent populations. A natural extension of this framework is the multi-population setting, where a common CV is assumed to hold across several independent groups. This is particularly relevant in practice when comparing relative variability across multiple regions or measurement stations simultaneously. However, constructing reliable confidence intervals for the common CV under delta-IG distributions is more complex than the single- or two-population case, due to the combined effects of skewness, zero inflation, and the need to pool information across multiple samples. Therefore, Chapter 4 extends the preceding framework to address this challenge by developing confidence interval methods for the common CV of multiple independent delta-IG populations.

TABLE 3-13 Summary of the chapter.

Aspect	Key Finding	Recommended Method
Single Variance	Best overall performance	AGCI.VST (best overall)
	Performance improves with larger samples	PBPCI (still less efficient)
	Widest intervals	BCI (not preferred)
Difference of Variances	Strong and consistent across scenarios	AGCI, MOVER
Ratio of Variances	Accurate coverage, especially for ratio inference	MOVER, AGCI

CHAPTER 4

CONFIDENCE INTERVALS FOR THE COMMON COEFFICIENT OF VARIATION OF MULTIPLE DELTA-INVERSE GAUSSIAN DISTRIBUTIONS

This chapter introduces methods for constructing confidence intervals for the common coefficient of variation of multiple delta-IG distributions. These methods are suitable for data consisting of positive values following an inverse Gaussian distribution and zero values distributed by a binomial distribution. For the construction of confidence intervals for the common coefficient of variation, the methods considered include generalized confidence intervals (GCI), adjusted generalized confidence intervals (AGCI), parametric bootstrap percentile confidence intervals (PBPCI), Bayesian credible intervals (BCI), and Bayesian-highest posterior density credible interval (B-HPDCI). A simulation study is proposed to evaluate the coverage probabilities (CPs) and average widths (AWs) of these intervals. The performance of these approaches is illustrated using monthly traffic accident death counts recorded in the second quarter of 2025 across 50 districts in Bangkok, Thailand.

4.1 Introduction

The coefficient of variation (CV) is one of the most widely used measures of relative dispersion in statistical analysis. Defined as the ratio of the standard deviation to the mean, the CV possesses a scale-independent property that makes it particularly advantageous when comparing variability across populations with different units or magnitudes (Hsieh, 1990; Gupta & Akman, 1996). Owing to this characteristic, statistical inference concerning the CV has attracted substantial scholarly attention across a wide range of disciplines, including environmental science, reliability engineering, finance, and biomedical research.

In numerous practical contexts, data are simultaneously collected from multiple populations that are assumed to share a common level of relative variability. This situation gives rise to the concept of the common coefficient of variation, which occupies a central role in comparative studies and meta-analysis (Borenstein et al., 2009; Gelman & Hill, 2007). Estimating a common CV enables the pooling of information across groups, thereby yielding more efficient and statistically reliable inference than would be achievable through separate analyses of individual populations (Graybill & Deal, 1959). Prior research has addressed inference for the common CV under a variety of distributional assumptions, including normal, lognormal, gamma, and Birnbaum–Saunders distributions (Feltz & Miller, 1996; Forkman, 2008; Tian, 2005a, 2005b; Puggard et al., 2022). However, these approaches frequently rest on assumptions that may be violated in practice, particularly when the data exhibit pronounced skewness or contain a substantial proportion of zero observations.

The inverse Gaussian (IG) distribution has been widely adopted for modelling positively skewed data, owing to its theoretical flexibility and sound mathematical foundation (Chhikara & Folks, 1989; Chaubey et al., 2017; Singh et al., 2021). Its applicability spans diverse fields, encompassing survival analysis, reliability studies,

environmental modelling, and insurance data analysis (Bardsley, 1980; Punzo, 2019). In practice, however, real-world data often exhibit not only positive skewness but also a considerable proportion of zero-valued observations. To accommodate this characteristic, extended formulations such as the zero-adjusted and delta-inverse Gaussian (delta-IG) distributions have been developed by incorporating a probability mass at zero (Jana & Gautam, 2024; Khumpasee et al., 2024). These models provide a more realistic and flexible framework for analysing mixed discrete–continuous data structures commonly encountered in applied settings.

Despite the growing interest in delta-IG models, the development of inferential procedures for their variability measures remains relatively limited. Existing studies have focused primarily on the construction of confidence intervals for a single CV or for pairwise differences between CVs. For example, recent contributions have proposed confidence interval procedures for the CV of delta-IG distributions and for simultaneous pairwise comparisons, employing methods such as generalized confidence intervals (GCI), adjusted GCI, and related approaches (Khumpasee et al., 2026). Although these procedures offer useful analytical tools, they do not address the problem of estimating a common coefficient of variation across multiple delta-IG populations simultaneously.

Constructing valid confidence intervals for the common CV under delta-IG distributions presents substantial statistical challenges. The simultaneous presence of skewness, excess zeros, and multiple nuisance parameters gives rise to sampling distributions of considerable complexity, rendering classical methods based on normal approximations unreliable, especially when sample sizes are small. To circumvent these limitations, alternative approaches—including generalized confidence intervals, bootstrap-based procedures, fiducial inference, and Bayesian methods—have been widely employed in complex inferential settings due to their flexibility and robustness (Weerahandi, 1993; Hannig et al., 2006; Efron & Tibshirani, 1986; Zou et al., 2009; Yosboonruang et al., 2022). Furthermore, related work on synthesising information across multiple populations and estimating shared parameters underscores the importance of developing appropriate inferential methods in this context (Ye et al., 2010; Bera & Jana, 2022; Slud et al., 2018). Nonetheless, the inferential challenges become more acute when the objective involves a common coefficient of variation under the delta-IG distributional framework.

Although several methodological contributions have addressed inference for the CV and its extensions, these approaches are generally designed for either single-population settings or specific distributional families and are not directly transferable to the delta-IG framework involving multiple groups. In particular, methods for the common CV are largely confined to standard distributional settings and often rely on assumptions that preclude zero inflation or entail simplified parameter structures. Similarly, advances in delta-IG modelling have focused predominantly on individual CV estimation or pairwise comparisons rather than simultaneous inference across multiple populations. Consequently, existing methodologies do not adequately address the combined inferential challenges posed by zero inflation, positive skewness, and the presence of multiple nuisance parameters in the common CV context. This gap

underscores the need for more flexible and robust inferential procedures specifically designed for the delta-IG setting.

Addressing this gap is of both theoretical and practical significance, as the common CV provides a unified, scale-free measure of relative variability across multiple populations, a quantity of considerable relevance in applications involving heterogeneous yet structurally related data sources. Accordingly, the primary objective of this study is to develop and evaluate inferential procedures for constructing confidence intervals for the common coefficient of variation under delta-inverse Gaussian distributions. Five approaches are considered and systematically compared: generalized confidence intervals (GCI), adjusted GCI (AGCI), parametric bootstrap percentile confidence intervals (PBPCI), Bayesian credible intervals (BCI), and Bayesian-highest posterior density credible intervals (B-HPDCI). Performance is assessed through an extensive Monte Carlo simulation study in terms of coverage probability and average interval width, with particular emphasis on scenarios involving small sample sizes, high skewness, and zero inflation.

4.2 The confidence intervals for the common coefficient of variation of multiple delta-IG distributions

Let $X_{ij} = (X_{i1}, X_{i2}, \dots, X_{in_i})$, for $i = 1, 2, \dots, k, j = 1, 2, \dots, n_i$ denote a non-negative random sample drawn from a delta-IG distribution, written as $X_{ij} \sim \text{delta-IG}(\delta_i, \mu_i, \lambda_i)$. Here, μ_i is the mean, λ_i is the shape parameter, and δ_i represents the probability of zero outcomes. The probability density function is given by

$$f_{\text{delta-IG}}(X_{ij}; \delta_i, \mu_i, \lambda_i) = \delta_i I_{\{X_{ij}=0\}} + (1 - \delta_i) I_{\{X_{ij}>0\}} \sqrt{\frac{\lambda_i}{2\pi X_{ij}^3}} \exp\left[-\frac{\lambda_i(X_{ij}-\mu_i)^2}{2\mu_i^2 X_{ij}}\right], \quad (4-1)$$

where $I(\cdot)$ is an indicator function, $P(X_{ij} = 0) = \delta_i$, and $\mu_i, \lambda_i > 0$. Each sample consists of n_i observations, which can be partitioned into zero observations $n_{i(0)}$ and nonzero observations $n_{i(1)}$, such that $n_i = n_{i(0)} + n_{i(1)}$. The number of zeros follows a binomial distribution, $n_{i(0)} \sim \text{Bin}(n_i, \delta_i)$.

The MLEs of the parameters are $\hat{\delta}_i = \frac{n_{i(0)}}{n_i}$, $\hat{\mu}_i = \bar{X}_i = \frac{1}{n_{i(1)}} \sum_{j=1}^{n_{i(1)}} X_{ij}$, and $n_{i(1)} \hat{\lambda}_i^{-1} = V_i = \sum_{j=1}^{n_{i(1)}} (X_{ij}^{-1} - \bar{X}_i^{-1})$. The mean and variance of X_{ij} are $E(X_{ij}) = (1 - \delta_i)\mu_i$ and $\text{Var}(X_{ij}) = \mu_i^2(1 - \delta_i) \left(\frac{\mu_i}{\lambda_i} + \delta_i \right)$. The CV for the delta-IG distribution is $\text{CV}(X_{ij}) = \theta_i = \sqrt{(\mu_i + \lambda_i \delta_i) / (1 - \delta_i) \lambda_i}$, with the estimator of θ_i is given by

$$\hat{\theta}_i = \sqrt{(\hat{\mu}_i + \hat{\lambda}_i \hat{\delta}_i) / (1 - \hat{\delta}_i) \hat{\lambda}_i}. \quad (4-2)$$

Suppose that $f(\mu_i, \lambda_i, \delta_i) = \theta_i = \sqrt{(\mu_i + \lambda_i \delta_i) / (1 - \delta_i) \lambda_i}$ is the coefficient of variation of the delta-IG distribution. The Delta method was applied to acquire an estimator with an asymptotically normal distribution by using a Taylor series expansion, given by

$$f(\hat{\mu}_i, \hat{\lambda}_i, \hat{\delta}_i) \approx f(\mu_i, \lambda_i, \delta_i) + \frac{\partial f(\mu_i, \lambda_i, \delta_i)}{\partial \mu_i} (\hat{\mu}_i - \mu_i) + \frac{\partial f(\mu_i, \lambda_i, \delta_i)}{\partial \lambda_i} (\hat{\lambda}_i - \lambda_i) + \frac{\partial f(\mu_i, \lambda_i, \delta_i)}{\partial \delta_i} (\hat{\delta}_i - \delta_i). \quad (4-3)$$

The corresponding partial derivatives are given as follows:

$$\begin{aligned} \frac{\partial f(\mu, \lambda, \delta)}{\partial \mu} &= \frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{1}{(1-\delta)\lambda} \right). \\ \frac{\partial f(\mu, \lambda, \delta)}{\partial \lambda} &= \frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1-\delta)\lambda\delta - (\mu+\lambda\delta)(1-\delta)}{((1-\delta)\lambda)^2} \right). \\ \frac{\partial f(\mu, \lambda, \delta)}{\partial \delta} &= \frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1-\delta)\lambda^2 - (\mu+\lambda\delta)(-\lambda)}{((1-\delta)\lambda)^2} \right). \end{aligned} \quad (4-4)$$

Therefore,

$$\begin{aligned} f(\hat{\mu}_i, \hat{\lambda}_i, \hat{\delta}_i) &\approx f(\mu, \lambda, \delta) + \frac{\partial f(\mu, \lambda, \delta)}{\partial \mu} (\hat{\mu} - \mu) + \frac{\partial f(\mu, \lambda, \delta)}{\partial \lambda} (\hat{\lambda} - \lambda) + \frac{\partial f(\mu, \lambda, \delta)}{\partial \delta} (\hat{\delta} - \delta) \\ &= f(\mu, \lambda, \delta) + \left[\frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{1}{(1-\delta)\lambda} \right) \right] (\hat{\mu} - \mu) \\ &\quad + \left[\frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1-\delta)\lambda\delta - (\mu+\lambda\delta)(1-\delta)}{((1-\delta)\lambda)^2} \right) \right] (\hat{\lambda} - \lambda) \\ &\quad + \left[\frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1-\delta)\lambda^2 - (\mu+\lambda\delta)(-\lambda)}{((1-\delta)\lambda)^2} \right) \right] (\hat{\delta} - \delta). \end{aligned} \quad (4-5)$$

Taking the expectation and variance in Equation (4-5), the asymptotic variance for the coefficient of variation estimator is given by

$$\begin{aligned} Var(\hat{\theta}) &\approx Var \left\{ f(\mu, \lambda, \delta) + \left[\frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{1}{(1-\delta)\lambda} \right) \right] (\hat{\mu} - \mu) \right. \\ &\quad + \left[\frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1-\delta)\lambda\delta - (\mu+\lambda\delta)(1-\delta)}{((1-\delta)\lambda)^2} \right) \right] (\hat{\lambda} - \lambda) \\ &\quad + \left. \left[\frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1-\delta)\lambda^2 - (\mu+\lambda\delta)(-\lambda)}{((1-\delta)\lambda)^2} \right) \right] (\hat{\delta} - \delta) \right\} \\ &= \left[\frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{1}{(1-\delta)\lambda} \right) \right]^2 \left(\frac{\mu^3}{\lambda n_{i(1)}} \right) \\ &\quad + \left[\frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1-\delta)\lambda\delta - (\mu+\lambda\delta)(1-\delta)}{((1-\delta)\lambda)^2} \right) \right]^2 \left(\frac{2\lambda^2}{n_{i(1)}} \right) \\ &\quad + \left[\frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1-\delta)\lambda^2 - (\mu+\lambda\delta)(-\lambda)}{((1-\delta)\lambda)^2} \right) \right]^2 \left(\frac{\delta(1-\delta)}{n_i} \right). \end{aligned} \quad (4-6)$$

Thus, the estimate of $Var(\hat{\theta}_i)$ can be expressed as

$$\begin{aligned} \hat{Var}(\hat{\theta}_i) = & \left[\frac{1}{2} \left(\frac{(1-\delta_i)\hat{\lambda}_i}{\hat{\mu}_i + \hat{\lambda}_i\delta_i} \right)^{\frac{1}{2}} \left(\frac{1}{(1-\delta_i)\hat{\lambda}_i} \right) \right]^2 \left(\frac{\hat{\mu}_i^3}{\hat{\lambda}_i n_{i(1)}} \right) \\ & + \left[\frac{1}{2} \left(\frac{(1-\delta_i)\hat{\lambda}_i}{\hat{\mu}_i + \hat{\lambda}_i\delta_i} \right)^{\frac{1}{2}} \left(\frac{(1-\delta_i)\hat{\lambda}_i\delta_i - (\hat{\mu}_i + \hat{\lambda}_i\delta_i)(1-\delta_i)}{((1-\delta_i)\hat{\lambda}_i)^2} \right) \right]^2 \left(\frac{2\hat{\lambda}_i^2}{n_{i(1)}} \right) \\ & + \left[\frac{1}{2} \left(\frac{(1-\delta_i)\hat{\lambda}_i}{\hat{\mu}_i + \hat{\lambda}_i\delta_i} \right)^{\frac{1}{2}} \left(\frac{(1-\delta_i)\hat{\lambda}_i^2 - (\hat{\mu}_i + \hat{\lambda}_i\delta_i)(-\hat{\lambda}_i)}{((1-\delta_i)\hat{\lambda}_i)^2} \right) \right]^2 \left(\frac{\delta_i(1-\delta_i)}{n_i} \right). \end{aligned} \quad (4-7)$$

Consequently, the common coefficient of variation across k delta-IG populations is estimated using a weighted average and is given by

$$\hat{\theta}_{common} = \frac{\sum_{i=1}^k [\hat{\theta}_i / \hat{Var}(\hat{\theta}_i)]}{\sum_{i=1}^k [1 / \hat{Var}(\hat{\theta}_i)]}, \quad (4-8)$$

where $i = 1, 2, \dots, k$.

4.2.1 Generalized Confidence Interval (GCI)

Ye, Ma, and Wang (2010) proposed a GPQ-based method for constructing confidence intervals for the mean of IG distributions across multiple populations. Their approach derives GPQs for the key parameters of the IG distribution, namely the shape parameter λ and the mean parameter μ .

The GPQs for λ_i and μ_i are expressed as follows:

$$R_{\lambda_i} = \frac{n_{i(1)}\lambda_i V_i}{n_{i(1)}v_i} \sim \frac{\chi_{n_{i(1)}-1}^2}{n_{i(1)}v_i}, \quad i = 1, 2, \dots, k, \quad (4-9)$$

where $\chi_{n_{i(1)}-1}^2$ is denoted as a Chi-square distribution with $n_{i(1)} - 1$ degrees of freedom, $i = 1, 2, \dots, k$ and

$$R_{\mu_i} = \frac{\bar{X}_i}{\left| 1 + \frac{\sqrt{n_{i(1)}\lambda_i(\bar{X}_i - \mu_i)}}{\mu_i \sqrt{\bar{X}_i}} \sqrt{\frac{\bar{X}_i}{n_{i(1)}R_{\lambda_i}}} \right|} \sim \frac{\bar{X}_i}{\left| 1 + Z_i \sqrt{\frac{\bar{X}_i}{n_{i(1)}R_{\lambda_i}}} \right|}, \quad (4-10)$$

where Z_i is approximately distributed as a standard normal distribution, denoted by $N(0,1)$.

The estimation for the parameter δ_i is using a variance-stabilizing transformation (VST), introduced by Dasgupta (2008). Wu and Hsieh (2014) later proposed the GPQs for δ_i as follows:

$$R_{\delta_i} = \sin^2 \left[\arcsin \sqrt{\hat{\delta}_i - \frac{T}{2\sqrt{n_i}}} \right], \quad (4-11)$$

where $T = 2\sqrt{n_i}(\arcsin \sqrt{\hat{\delta}_i} - \arcsin \sqrt{\delta_i}) \sim N(0,1)$, $n_i \rightarrow \infty$.

Using the GPQs for λ_i , μ_i , and δ_i from (4-9)-(4-11), the GPQ for the coefficient of variation θ_i is constructed as

$$R_{\theta_i} = \sqrt{(R_{\mu_i} + R_{\lambda_i}R_{\delta_i})/(1 - R_{\delta_i})}R_{\lambda_i}, \quad (4-12)$$

where $i = 1, 2, \dots, k$.

The corresponding GPQ for the variance of $\hat{\theta}_i$ is given by

$$\begin{aligned}
 R_{V\hat{a}r(\hat{\theta}_i)} = & \left[\frac{1}{2} \left(\frac{(1-R_{\delta_i})R_{\lambda_i}}{R_{\mu_i}+R_{\lambda_i}R_{\delta_i}} \right)^{\frac{1}{2}} \left(\frac{1}{(1-R_{\delta_i})R_{\lambda_i}} \right) \right]^2 \left(\frac{R_{\mu_i}^3}{R_{\lambda_i}n_{i(1)}} \right) \\
 & + \left[\frac{1}{2} \left(\frac{(1-R_{\delta_i})R_{\lambda_i}}{R_{\mu_i}+R_{\lambda_i}R_{\delta_i}} \right)^{\frac{1}{2}} \left(\frac{(1-R_{\delta_i})R_{\lambda_i}R_{\delta_i} - (R_{\mu_i}+R_{\lambda_i}R_{\delta_i})(1-R_{\delta_i})}{((1-R_{\delta_i})R_{\delta_i})^2} \right) \right]^2 \left(\frac{2R_{\lambda_i}^2}{n_{i(1)}} \right) \\
 & + \left[\frac{1}{2} \left(\frac{(1-R_{\delta_i})R_{\lambda_i}}{R_{\mu_i}+R_{\lambda_i}R_{\delta_i}} \right)^{\frac{1}{2}} \left(\frac{(1-R_{\delta_i})R_{\lambda_i}^2 - (R_{\mu_i}+R_{\lambda_i}R_{\delta_i})(-R_{\lambda_i})}{((1-R_{\delta_i})R_{\lambda_i})^2} \right) \right]^2 \left(\frac{R_{\delta_i}(1-R_{\delta_i})}{n_i} \right). \quad (4-13)
 \end{aligned}$$

The GPQ for the common coefficient of variation across k populations obtained as a weighted combination of individual GPQs, as defined from (4-12) and (4-13), is

$$R_{\theta_{common}} = \frac{\sum_{i=1}^k [R_{\theta_i}/R_{V\hat{a}r(\hat{\theta}_i)}]}{\sum_{i=1}^k [1/R_{V\hat{a}r(\hat{\theta}_i)}]}. \quad (4-14)$$

Consequently, the $100(1 - \alpha)\%$ two-sided confidence interval for θ_{common} from the GCI method is

$$CI_{\theta_{common}}^{GCI} = [L_{\theta_{common}}^{GCI}, U_{\theta_{common}}^{GCI}] = [R_{\theta_{common}}(\alpha/2), R_{\theta_{common}}(1 - \alpha/2)], \quad (4-15)$$

where $R_{\theta_{common}}(\alpha)$ are the $100(\alpha)^{th}$ percentiles of the distribution of $R_{\theta_{common}}$.

4.2.2 Adjusted generalized confidence interval (AGCI)

The AGCI method employed an alternative GPQ for the parameter μ introduced by Krishnamoorthy and Tian (2008) replacing the GPQ of μ in Equation (4-10) used in the GCI approach.

The GPQ for μ_i , as proposed by Krishnamoorthy and Tian, is defined as

$$R_{\tilde{\mu}_i} = \frac{\bar{X}_i}{\max \left\{ 0, 1 + t_{n_{i(1)}-1} \sqrt{\frac{\bar{X}_i v_i}{n_{i(1)}-1}} \right\}}, \quad (4-16)$$

where $t_{n_{i(1)}-1}$ is Student's t distribution with $n_{i(1)} - 1$ degrees of freedom.

Then, the GPQs for the parameters are R_{λ_i} , $R_{\tilde{\mu}_i}$, and R_{δ_i} , given in equations (4-9), (4-16), and (4-11), respectively. Using these, the GPQ for the coefficient of variation (θ_i) is constructed as

$$R_{\tilde{\theta}_i} = \sqrt{(R_{\tilde{\mu}_i} + R_{\lambda_i}R_{\delta_i})/(1 - R_{\delta_i})R_{\lambda_i}}, \quad (4-17)$$

The corresponding GPQ for the variance of $\hat{\theta}_i$ is given by

$$\begin{aligned} \tilde{R}_{Var}(\hat{\theta}_i) = & \left[\frac{1}{2} \left(\frac{(1-G_{\delta_i})G_{\lambda_i}}{G_{\mu_i}+G_{\lambda_i}G_{\delta_i}} \right)^{\frac{1}{2}} \left(\frac{1}{(1-G_{\delta_i})G_{\lambda_i}} \right) \right]^2 \left(\frac{G_{\mu_i}^3}{G_{\lambda_i}n_{i(1)}} \right) + \\ & \left[\frac{1}{2} \left(\frac{(1-G_{\delta_i})G_{\lambda_i}}{G_{\mu_i}+G_{\lambda_i}G_{\delta_i}} \right)^{\frac{1}{2}} \left(\frac{(1-G_{\delta_i})G_{\lambda_i}G_{\delta_i} - (G_{\mu_i}+G_{\lambda_i}G_{\delta_i})(1-G_{\delta_i})}{((1-G_{\delta_i})G_{\lambda_i})^2} \right) \right]^2 \left(\frac{2G_{\lambda_i}^2}{n_{i(1)}} \right) + \\ & \left[\frac{1}{2} \left(\frac{(1-G_{\delta_i})G_{\lambda_i}}{G_{\mu_i}+G_{\lambda_i}G_{\delta_i}} \right)^{\frac{1}{2}} \left(\frac{(1-G_{\delta_i})G_{\lambda_i}^2 - (G_{\mu_i}+G_{\lambda_i}G_{\delta_i})(-G_{\lambda_i})}{((1-G_{\delta_i})G_{\lambda_i})^2} \right) \right]^2 \left(\frac{G_{\delta_i}(1-G_{\delta_i})}{n_i} \right). \end{aligned} \quad (4-18)$$

The GPQ for the common coefficient of variation across k populations from the GPQs in Equations (4-17) and (4-18) is defined as

$$R_{\tilde{\theta}_{common}} = \frac{\sum_{i=1}^k [R_{\tilde{\theta}_i} / \tilde{R}_{Var}(\hat{\theta}_i)]}{\sum_{i=1}^k [1 / \tilde{R}_{Var}(\hat{\theta}_i)]}. \quad (4-19)$$

Consequently, the $100(1 - \alpha)\%$ two-sided confidence interval for θ_{common} from the AGCI method is given by

$$CI_{\tilde{\theta}_{common}}^{AGCI} = [L_{\tilde{\theta}_{common}}^{AGCI}, U_{\tilde{\theta}_{common}}^{AGCI}] = [R_{\tilde{\theta}_{common}}(\alpha/2), R_{\tilde{\theta}_{common}}(1 - \alpha/2)], \quad (4-20)$$

where $R_{\tilde{\theta}_{common}}(\alpha)$ are the $100(\alpha)^{th}$ percentiles of the distribution of $R_{\tilde{\theta}_{common}}$.

4.2.3 Parametric bootstrap percentile confidence interval (PBPCI)

The PBPCI, introduced by Efron and Tibshirani (1986), is a simulation-based approach for constructing confidence intervals for a parameter of interest. The method involves generating a large number of bootstrap samples from a fitted parametric model, rather than resampling directly from the observed data. For each bootstrap sample, the statistic of interest is calculated, forming an empirical distribution. The confidence interval is then determined using the $\alpha/2$ and $1 - \alpha/2$ quantiles of this distribution.

From $x_{ij} = (x_{i1}, x_{i2}, \dots, x_{in_i})$ be a non-negative random sample of size n_i drawn from a delta-IG($\mu_i, \lambda_i, \delta_i$) distribution. The MLE of μ_i and λ_i denoted as $\hat{\mu}_i$ and $\hat{\lambda}_i$, respectively. Let $x_i^* = (x_{i1}^*, x_{i2}^*, \dots, x_{ib}^*)$ denote the bootstrap samples of size b_i . Define $b_{i(0)}^*$ and $b_{i(1)}^*$ as the numbers of zero and nonzero observations from bootstrap samples, respectively, such that $b_i = b_{i(0)}^* + b_{i(1)}^*$. Subsequently, the unbiased for $\hat{\mu}_i^*$ and $\hat{\lambda}_i^*$, adjusted by applying bias-corrected bootstrap estimators for μ_i and λ_i are given by

$$R_{\mu_i}^{PBPCI} = \hat{\mu}_i^* - 2\hat{B}(\hat{\mu}_i, \mu_i) \text{ and } R_{\lambda_i}^{PBPCI} = \hat{\lambda}_i^* - 2\hat{B}(\hat{\lambda}_i, \lambda_i). \quad (4-21)$$

where $B(\hat{\mu}_i, \mu_i) = \bar{\mu}_i^* - \hat{\mu}_i$ and $B(\hat{\lambda}_i, \lambda_i) = \bar{\lambda}_i^* - \hat{\lambda}_i$, respectively, and $\bar{\mu}_i^* = \frac{1}{b_i} \sum_{j=1}^{b_i} \mu_{ij}^*$, $\bar{\lambda}_i^* = \frac{1}{b_i} \sum_{j=1}^{b_i} \lambda_{ij}^*$, $i = 1, 2, \dots, k$, $j = 1, 2, \dots, b_i$. And the estimating of parameter δ , the VST method will be used as Equation (4-11)

Then, the estimator of CV for the delta-IG distribution is given by

$$\hat{\theta}_i^* = \sqrt{(R_{\mu_i}^{PBPCI} + R_{\lambda_i}^{PBPCI} R_{\delta_i}) / (1 - R_{\delta_i}) R_{\lambda_i}^{PBPCI}}, \quad (4-22)$$

The corresponding variance of $\hat{\theta}_i^*$ is provided by

$$\begin{aligned}
 V\hat{ar}(\hat{\theta}_i^*) = & \left[\frac{1}{2} \left(\frac{(1-R_{\delta_i})R_{\lambda_i}^{PBPCI}}{R_{\mu_i}^{PBPCI} + R_{\lambda_i}^{PBPCI}R_{\delta_i}} \right)^{\frac{1}{2}} \left(\frac{1}{(1-R_{\delta_i})R_{\lambda_i}^{PBPCI}} \right) \right]^2 \left(\frac{R_{\mu_i}^{PBPCI^3}}{R_{\lambda_i}^{PBPCI}b_{i(1)}^*} \right) + \\
 & \left[\frac{1}{2} \left(\frac{(1-R_{\delta_i})R_{\lambda_i}^{PBPCI}}{R_{\mu_i}^{PBPCI} + R_{\lambda_i}^{PBPCI}R_{\delta_i}} \right)^{\frac{1}{2}} \left(\frac{(1-R_{\delta_i})R_{\lambda_i}^{PBPCI}R_{\delta_i} - (R_{\mu_i}^{PBPCI} + R_{\lambda_i}^{PBPCI}R_{\delta_i})(1-R_{\delta_i})}{((1-R_{\delta_i})R_{\lambda_i}^{PBPCI})^2} \right) \right]^2 \left(\frac{2R_{\lambda_i}^{PBPCI^2}}{b_{i(1)}^*} \right) + \\
 & \left[\frac{1}{2} \left(\frac{(1-R_{\delta_i})R_{\lambda_i}^{PBPCI}}{R_{\mu_i}^{PBPCI} + R_{\lambda_i}^{PBPCI}R_{\delta_i}} \right)^{\frac{1}{2}} \left(\frac{(1-R_{\delta_i})R_{\lambda_i}^{PBPCI^2} - (R_{\mu_i}^{PBPCI} + R_{\lambda_i}^{PBPCI}R_{\delta_i})(-R_{\lambda_i}^{PBPCI})}{((1-R_{\delta_i})R_{\lambda_i}^{PBPCI})^2} \right) \right]^2 \left(\frac{R_{\delta_i}(1-R_{\delta_i})}{b_i^*} \right).
 \end{aligned} \tag{4-23}$$

The common coefficient of variation for multiple delta-IG samples is given by

$$\hat{\theta}_{common}^* = \frac{\sum_{i=1}^k [\hat{\theta}_i^* / V\hat{ar}(\hat{\theta}_i^*)]}{\sum_{i=1}^k [1 / V\hat{ar}(\hat{\theta}_i^*)]}. \tag{4-24}$$

Finally, the $100(1-\alpha)\%$ two-sided CI of θ_{common} for the delta-IG distribution based on the PBPCI method is given by

$$CI_{\theta_{common}}^{PBPCI} = [\hat{\theta}_{common}^*(\alpha/2), \hat{\theta}_{common}^*(1 - \alpha/2)], \tag{4-25}$$

where $\hat{\theta}_{common}^*(\alpha/2)$ and $\hat{\theta}_{common}^*(1 - \alpha/2)$ are the $100\frac{\alpha}{2}$ -th and $100(1 - \frac{\alpha}{2})$ -th percentiles of the distribution of $\hat{\theta}_{common}^*$.

4.2.4 Bayesian credible interval (BCI)

Let $X_{ij} = (X_{i1}, X_{i2}, \dots, X_{in_i})$, for $i = 1, 2, j = 1, 2, \dots, n_i$ be a non-negative random sample from a delta-IG distribution stated as $X_{ij} \sim \text{delta-IG}(\delta_i, \mu_i, \lambda_i)$. The likelihood function of the delta-IG distribution is given by

$$L(\theta|X_i) \propto \delta_i^{n_{i(0)}} (1 - \delta_i)^{n_{i(1)}} \lambda_i^{n_{i(1)}/2} \exp\left(-\frac{\lambda_i}{2\mu_i^2} (t_1 - 2n_{i(1)}\mu_i + t_2\mu_i^2)\right), \tag{4-26}$$

where $t_1 = \sum_{j=1}^{n_{i(1)}} X_{ij}$, and $t_2 = \sum_{j=1}^{n_{i(1)}} X_{ij}^{-1}$, $i = 1, 2, \dots, k$.

The posterior density is then proportional to the product of the prior and the likelihood, given by

$$f(\theta|X_i) \propto f(\theta)L(\theta|X_i). \tag{4-27}$$

Since the resulting posterior distributions are analytically intractable, Markov Chain Monte Carlo (MCMC) methods, specifically Gibbs sampling, are used to generate the posterior samples. In this study, non-informative priors are assigned as follows: a Uniform(0, M) prior for μ_i , where M is set sufficiently large relative to the observed data range, a Gamma(0.001, 0.001) prior for λ_i , and a Beta(1,1) prior for δ_i , the latter being equivalent to a Uniform(0,1) distribution. The computation is implemented using the *R2OpenBUGS* package in the R programming language.

Based on the posterior samples, the estimator of the coefficient of variation (θ_i) for the delta-IG distribution are defined as

$$\hat{\theta}_i^{BS} = \sqrt{(\hat{\mu}_i^{BS} + \hat{\lambda}_i^{BS} \hat{\delta}_i^{BS}) / (1 - \hat{\delta}_i^{BS}) \hat{\lambda}_i^{BS}}, \quad (4-28)$$

The corresponding variance of $\hat{\theta}_i^{BS}$ is given by

$$\begin{aligned} V\hat{ar}(\hat{\theta}_i^{BS}) = & \left[\frac{1}{2} \left(\frac{(1-\hat{\delta}_i^{BS}) \hat{\lambda}_i^{BS}}{\hat{\mu}_i^{BS} + \hat{\lambda}_i^{BS} \hat{\delta}_i^{BS}} \right)^{\frac{1}{2}} \left(\frac{1}{(1-\hat{\delta}_i^{BS}) \hat{\lambda}_i^{BS}} \right) \right]^2 \left(\frac{\hat{\mu}_i^{BS^3}}{\hat{\lambda}_i^{BS} n_{i(1)}} \right) + \\ & \left[\frac{1}{2} \left(\frac{(1-\hat{\delta}_i^{BS}) \hat{\lambda}_i^{BS}}{\hat{\mu}_i^{BS} + \hat{\lambda}_i^{BS} \hat{\delta}_i^{BS}} \right)^{\frac{1}{2}} \left(\frac{(1-\hat{\delta}_i^{BS}) \hat{\lambda}_i^{BS} \hat{\delta}_i^{BS} - (\hat{\mu}_i^{BS} + \hat{\lambda}_i^{BS} \hat{\delta}_i^{BS})(1-\hat{\delta}_i^{BS})}{((1-\hat{\delta}_i^{BS}) \hat{\lambda}_i^{BS})^2} \right) \right]^2 \left(\frac{2\hat{\lambda}_i^{BS^2}}{n_{i(1)}} \right) + \\ & \left[\frac{1}{2} \left(\frac{(1-\hat{\delta}_i^{BS}) \hat{\lambda}_i^{BS}}{\hat{\mu}_i^{BS} + \hat{\lambda}_i^{BS} \hat{\delta}_i^{BS}} \right)^{\frac{1}{2}} \left(\frac{(1-\hat{\delta}_i^{BS}) \hat{\lambda}_i^{BS^2} - (\hat{\mu}_i^{BS} + \hat{\lambda}_i^{BS} \hat{\delta}_i^{BS})(-\hat{\lambda}_i^{BS})}{((1-\hat{\delta}_i^{BS}) \hat{\lambda}_i^{BS})^2} \right) \right]^2 \left(\frac{\hat{\delta}_i^{BS}(1-\hat{\delta}_i^{BS})}{n_i} \right). \end{aligned} \quad (4-29)$$

The common CV across multiple populations is defined as

$$\hat{\theta}_{common}^{BS} = \frac{\sum_{i=1}^k [\hat{\theta}_i^{BS} / V\hat{ar}(\hat{\theta}_i^{BS})]}{\sum_{i=1}^k [1 / V\hat{ar}(\hat{\theta}_i^{BS})]}. \quad (4-30)$$

Finally, the $100(1-\alpha)\%$ two-sided CI of θ_{common} for the delta-IG distribution based on the BCI method is given by

$$CI_{\theta_{common}}^{BCI} = [\hat{\theta}_{common}^{BS}(\alpha/2), \hat{\theta}_{common}^{BS}(1-\alpha/2)], \quad (4-31)$$

where $\hat{\theta}_{common}^{BS}(\alpha/2)$ and $\hat{\theta}_{common}^{BS}(1-\alpha/2)$ are the $100\frac{\alpha}{2}$ -th and $100(1-\frac{\alpha}{2})$ -th percentiles of the distribution of $\hat{\theta}_{common}^{BS}$.

4.2.5 Bayesian-highest posterior density credible interval (B-HPDCI)

The HPD method is used to construct credible intervals for an unknown parameter based on its posterior distribution such as equation (4-27). An HPD interval is defined as the shortest interval containing a specified posterior probability, $100(1-\alpha)\%$, such that all points within the interval have higher posterior density than those outside. This property makes the HPD interval particularly suitable for asymmetric or non-normal posterior distributions, where equal-tailed intervals may be less representative.

Accordingly, the $100(1-\alpha)\%$ credible interval for θ_{common} based on the B-HPDCI method is given by

$$CI_{\theta_{common}}^{B-HPD} = [L_{\theta_{common}}^{B-HPD}, U_{\theta_{common}}^{B-HPD}], \quad (4-32)$$

where $L_{\theta_{common}}^{HPD}$ and $U_{\theta_{common}}^{HPD}$ are obtained using the *hdi* function in R.

4.3 Simulation studies and results

This study examines five methods for constructing confidence intervals for the common coefficient of variation of the delta-IG distribution. A Monte Carlo simulation, implemented in R, is conducted to evaluate their performance in terms of CPs and AWs. An effective confidence interval method is expected to achieve coverage probabilities close to the nominal confidence level. Among methods with comparable coverage performance, the preferred method is the one that produces the shortest interval width. The simulation includes 5,000 replications, with data generated from $X_{ij} \sim \text{delta-IG}(\delta_i, \mu_i, \lambda_i)$ for $i = 1, \dots, k$ and $j = 1, \dots, n_i$, under various parameter

settings. Two scenarios are considered ($k = 3$ and $k = 6$), with varied in mean parameters, shape parameters, zero-inflation probabilities, and sample sizes to reflect a wide range of practical conditions.

The results summarized in Tables 4–1 and 4–2 present the CPs and AWs for the case $k = 3$, as illustrated in Figure 4–1. Correspondingly, Tables 4–3 and 4–4 report the CPs and AWs for $k = 6$, with the results visualized in Figure 4–2. Overall, none of the methods consistently achieved the nominal confidence level of 0.95 across all scenarios. A clear trade-off between coverage accuracy and interval efficiency was observed: methods with higher CPs generally produced wider intervals, while those with narrower widths tended to exhibit substantial undercoverage.

Across all settings, the B-HPDCI method provided the highest coverage probability among the competing methods. Although the average coverage probability (approximately 0.944) remained slightly below the nominal confidence level of 0.95, it consistently outperformed the alternative methods in terms of coverage accuracy. The BCI method ranked second, whereas the GCI and AGCI methods exhibited more pronounced undercoverage. The PBPCI method showed the lowest coverage probability, indicating that it did not adequately account for the sampling variability of the CV estimator under the delta-IG distribution. Given a Monte Carlo standard error of approximately 0.003 based on 5,000 replications, the observed average coverage probability of B-HPDCI remains statistically below the nominal level. This undercoverage reflects the inherent difficulty of constructing confidence intervals with exact nominal coverage for the delta-IG distribution, where skewness, zero inflation, and multiple nuisance parameters complicate the sampling distribution of the CV estimator. Nevertheless, across the majority of parameter configurations, B-HPDCI consistently achieved coverage probabilities closer to the nominal level than the competing methods, making it the most reliable procedure among those considered in this study.

When interval width was considered, the PBPCI method yielded the narrowest intervals, followed by AGCI, GCI, HPDCI, and BCI. However, interval width should be interpreted together with coverage probability rather than in isolation. Coverage probability is the primary criterion for evaluating confidence interval methods because reliable statistical inference requires the achieved coverage probability to be close to the nominal confidence level. Interval width serves as a secondary criterion and is used to distinguish among methods with comparable coverage performance, with narrower intervals indicating greater precision. Although PBPCI produced the shortest intervals, its substantial undercoverage limits its practical applicability. In contrast, HPDCI maintained coverage probabilities closest to the nominal level while producing reasonably narrow intervals, thereby providing the best overall balance between reliability and precision among the competing methods.

TABLE 4-1 The CPs of the 95% confidence intervals for the common coefficient of variation of delta-IG distributions when $k = 3$.

n_i^3	μ_i^3	λ_i^3	δ_i^3	Coverage Probabilities				
				GCI	AGCI	PBPCI	BCI	B-HPDCI
30^3	0.5^3	1^3	0.1^3	0.8405	0.8948	0.8048	0.9421	0.9561
			0.3^3	0.8305	0.8826	0.8369	0.9421	0.9521
			0.5^3	0.8485	0.9265	0.8461	0.9361	0.9561
			0.1,0.3,0.5	0.8864	0.9399	0.8228	0.9254	0.9472
		5^3	0.1^3	0.9142	0.7884	0.8271	0.9541	0.9701
			0.3^3	0.9461	0.9082	0.8575	0.9221	0.9381
			0.5^3	0.9401	0.9202	0.8194	0.8842	0.8902
			0.1,0.3,0.5	0.8727	0.8944	0.8271	0.9201	0.9282
	1^3	1^3	0.1^3	0.8126	0.8291	0.8589	0.9501	0.9581
			0.3^3	0.8589	0.8729	0.8499	0.9481	0.9501
			0.5^3	0.8212	0.5908	0.8443	0.9381	0.9501
			0.1,0.3,0.5	0.8050	0.8453	0.8275	0.8846	0.9028
		5^3	0.1^3	0.8603	0.8770	0.8591	0.9321	0.9541
			0.3^3	0.9102	0.9209	0.8425	0.9401	0.9521
			0.5^3	0.9102	0.9202	0.8134	0.9121	0.9142
			0.1,0.3,0.5	0.9381	0.9168	0.8242	0.8363	0.8512
50^3	0.5^3	1^3	0.1^3	0.8405	0.8749	0.7699	0.9281	0.9461
			0.3^3	0.8648	0.8765	0.7647	0.9401	0.9581
			0.5^3	0.8589	0.9009	0.7553	0.9361	0.9461
			0.1,0.3,0.5	0.8806	0.8981	0.8210	0.8287	0.8461
		5^3	0.1^3	0.9281	0.9362	0.8487	0.9381	0.9521
			0.3^3	0.9061	0.9301	0.7293	0.9201	0.9361
			0.5^3	0.9441	0.9222	0.7240	0.9402	0.9516
			0.1,0.3,0.5	0.8870	0.9044	0.7236	0.8722	0.8703
	1^3	1^3	0.1^3	0.8226	0.8423	0.7667	0.9401	0.9681
			0.3^3	0.8792	0.8890	0.7631	0.9261	0.9481
			0.5^3	0.8156	0.8313	0.7543	0.9301	0.9401
			0.1,0.3,0.5	0.8174	0.8341	0.8253	0.8850	0.9068
		5^3	0.1^3	0.8942	0.9092	0.8617	0.9281	0.9461
			0.3^3	0.8802	0.8987	0.8431	0.9241	0.9361
			0.5^3	0.8822	0.9026	0.8417	0.9281	0.9361
			0.1,0.3,0.5	0.9182	0.9256	0.8194	0.9040	0.9285
100^3	0.5^3	1^3	0.1^3	0.8128	0.8788	0.7794	0.9521	0.9741
			0.3^3	0.8774	0.8936	0.7689	0.9421	0.9561
			0.5^3	0.8836	0.9017	0.7653	0.9181	0.9401
			0.1,0.3,0.5	0.8431	0.8629	0.8096	0.9369	0.9373
		5^3	0.1^3	0.9122	0.9102	0.8489	0.9501	0.9621
			0.3^3	0.9062	0.8802	0.8309	0.9201	0.9401
			0.5^3	0.8942	0.8483	0.8285	0.9341	0.9461
			0.1,0.3,0.5	0.8617	0.8204	0.8180	0.8788	0.8884
	1^3	1^3	0.1^3	0.8888	0.8943	0.7774	0.9381	0.9581
			0.3^3	0.8838	0.9016	0.7186	0.9301	0.9541
			0.5^3	0.8060	0.8206	0.7689	0.9161	0.9341
			0.1,0.3,0.5	0.8639	0.8743	0.8216	0.8555	0.8753

TABLE 4-1 (CONTINUED)

n_i^3	μ_i^3	λ_i^3	δ_i^3	Coverage Probabilities				
				GCI	AGCI	PBPCI	BCI	B-HPDCI
30,50,100		5^3	0.1^3	0.8263	0.8379	0.7725	0.9421	0.9561
			0.3^3	0.7764	0.7926	0.8505	0.9301	0.9401
			0.5^3	0.7445	0.7669	0.8395	0.9341	0.9501
			0.1,0.3,0.5	0.9182	0.9207	0.8096	0.9455	0.9461
	0.5^3	1^3	0.1^3	0.8265	0.8495	0.7717	0.9361	0.9521
			0.3^3	0.8607	0.8868	0.7671	0.9421	0.9601
			0.5^3	0.8545	0.8653	0.8577	0.9201	0.9361
			0.1,0.3,0.5	0.8417	0.8612	0.8158	0.9166	0.9267
		5^3	0.1^3	0.9381	0.8743	0.7441	0.9361	0.9541
			0.3^3	0.9361	0.9142	0.8315	0.9441	0.9641
			0.5^3	0.9242	0.8982	0.8261	0.9281	0.9461
			0.1,0.3,0.5	0.9381	0.8904	0.8188	0.9387	0.9488
		1^3	0.1^3	0.8986	0.8914	0.7752	0.9401	0.9561
			0.3^3	0.8972	0.9152	0.7669	0.9381	0.9561
			0.5^3	0.8415	0.8794	0.7603	0.9361	0.9441
			0.1,0.3,0.5	0.8758	0.8953	0.8214	0.9377	0.9476
		5^3	0.1^3	0.8922	0.8924	0.8601	0.9481	0.9661
			0.3^3	0.8902	0.9036	0.7477	0.9361	0.9521
			0.5^3	0.8623	0.8852	0.8399	0.9341	0.9521
			0.1,0.3,0.5	0.8365	0.8680	0.8168	0.9670	0.9745

Note: $n_i^3 = (n_1, n_2, n_3)$ with $30^3 = (30, 30, 30)$. Bold values indicate coverage probabilities (CP) that satisfy the nominal confidence level of 0.95 ($CP \geq 0.945$, rounded to 0.950); in configurations where no method attains this threshold, the bold value denotes the highest CP achieved among the five methods for that parameter setting.

TABLE 4-2 The AWs of the 95% confidence intervals for the common coefficient of variation of delta-IG distributions when $k = 3$.

n_i^3	μ_i^3	λ_i^3	δ_i^3	Average Widths				
				GCI	AGCI	PBPCI	BCI	B-HPDCI
30^3	0.5^3	1^3	0.1^3	0.9774	0.5649	0.2236	0.3252	<i>0.3221</i>
			0.3^3	1.2971	0.9239	0.2333	0.4592	<i>0.4542</i>
			0.5^3	1.7717	1.3932	0.2693	0.7011	<i>0.6889</i>
			0.1,0.3,0.5	1.3343	1.0448	0.3066	0.4686	<i>0.4623</i>
		5^3	0.1^3	0.5174	0.3281	0.0840	0.2141	<i>0.2126</i>
			0.3^3	0.9246	0.5026	0.0634	0.3141	0.3124
			0.5^3	1.2274	0.8146	0.0686	0.4509	<i>0.4482</i>
			0.1,0.3,0.5	0.9474	0.5835	0.0982	0.3419	<i>0.3391</i>
	1^3	1^3	0.1^3	1.6129	1.0290	0.3700	0.5234	<i>0.5119</i>
			0.3^3	2.2880	1.5339	0.3939	0.7283	<i>0.7089</i>
			0.5^3	3.7561	2.2028	0.4698	1.1726	<i>1.1261</i>
			0.1,0.3,0.5	2.9338	1.7207	0.4663	0.7555	<i>0.7325</i>
		5^3	0.1^3	0.6030	0.4001	0.1246	0.2278	<i>0.2260</i>
			0.3^3	0.8927	0.5355	0.1142	0.3350	<i>0.3331</i>
			0.5^3	1.4204	0.7561	0.1282	0.5023	<i>0.4989</i>
			0.1,0.3,0.5	0.9795	0.5812	0.1731	0.3529	0.3497
50^3	0.5^3	1^3	0.1^3	0.6302	0.3895	0.1781	0.2369	<i>0.2352</i>
			0.3^3	0.9266	0.5825	0.1901	0.3311	<i>0.3290</i>
			0.5^3	1.2610	0.8736	0.2243	0.4912	<i>0.4875</i>
			0.1,0.3,0.5	1.1224	0.6479	0.2581	0.3562	<i>0.3535</i>
		5^3	0.1^3	0.3917	0.2920	0.0619	0.1629	<i>0.1620</i>
			0.3^3	0.7223	0.3728	0.0505	0.2404	<i>0.2393</i>
			0.5^3	0.9762	0.5717	0.0564	0.3518	<i>0.3501</i>
			0.1,0.3,0.5	0.7399	0.4013	0.0814	0.2711	0.2694
	1^3	1^3	0.1^3	1.0367	0.6299	0.2906	0.3633	<i>0.3589</i>
			0.3^3	1.5211	0.9598	0.3267	0.4935	<i>0.4874</i>
			0.5^3	2.3653	1.5727	0.4012	0.7714	<i>0.7544</i>
			0.1,0.3,0.5	1.5693	1.0820	0.3867	0.5184	<i>0.5110</i>
		5^3	0.1^3	0.4898	0.2956	0.0969	0.1708	<i>0.1699</i>
			0.3^3	0.6573	0.4294	0.0903	0.2510	<i>0.2498</i>
			0.5^3	1.0631	0.5968	0.1052	0.3741	<i>0.3723</i>
			0.1,0.3,0.5	0.8287	0.4176	0.1441	0.2758	<i>0.2741</i>
100^3	0.5^3	1^3	0.1^3	0.4732	0.2786	0.1278	0.1594	<i>0.1586</i>
			0.3^3	0.6639	0.3700	0.1362	0.2192	<i>0.2182</i>
			0.5^3	0.8848	0.5399	0.1685	0.3266	<i>0.3249</i>
			0.1,0.3,0.5	0.6063	0.4295	0.1956	0.2500	<i>0.2487</i>
		5^3	0.1^3	0.3389	0.2035	0.0416	0.1112	<i>0.1108</i>
			0.3^3	0.4383	0.2875	0.0363	0.1648	<i>0.1642</i>
			0.5^3	0.6934	0.3972	0.0415	0.2428	<i>0.2419</i>
			0.1,0.3,0.5	0.5178	0.3150	0.0586	0.1991	<i>0.1983</i>
	1^3	1^3	0.1^3	0.7275	0.4169	0.2090	0.2402	<i>0.2385</i>
			0.3^3	0.9918	0.5705	0.2370	0.3183	<i>0.3161</i>
			0.5^3	1.3759	0.7884	0.3000	0.4670	<i>0.4633</i>
			0.1,0.3,0.5	1.0621	0.6262	0.2957	0.3552	<i>0.3525</i>

TABLE 4-2 (CONTINUED)

n_i^3	μ_i^3	λ_i^3	δ_i^3	Average Widths				
				GCI	AGCI	PBPCI	BCI	B-HPDCI
100^3	1^3	5^3	0.1^3	0.3576	0.1832	0.0683	0.1166	<i>0.1160</i>
			0.3^3	0.4203	0.2894	0.0656	0.1723	<i>0.1715</i>
			0.5^3	0.6657	0.4035	0.0769	0.2567	<i>0.2556</i>
			0.1,0.3,0.5	0.5349	0.3124	0.1079	0.1973	<i>0.1965</i>
30,50,100	0.5^3	1^3	0.1^3	0.6778	0.4275	0.1679	0.2146	<i>0.2133</i>
			0.3^3	1.0893	0.6528	0.1742	0.2970	<i>0.2953</i>
			0.5^3	1.5720	0.9593	0.2131	0.4452	<i>0.4423</i>
			0.1,0.3,0.5	0.9650	0.5534	0.3032	0.4211	<i>0.4188</i>
		5^3	0.1^3	0.4778	0.2738	0.0560	0.1481	<i>0.1474</i>
			0.3^3	0.6506	0.4085	0.0469	0.2179	<i>0.2170</i>
			0.5^3	0.8505	0.6013	0.0522	0.3190	<i>0.3177</i>
			0.1,0.3,0.5	0.5705	0.3649	0.0941	0.3396	<i>0.3379</i>
	1^3	1^3	0.1^3	1.0967	0.7844	0.2780	0.3271	<i>0.3240</i>
			0.3^3	1.6887	1.0832	0.3046	0.4403	<i>0.4360</i>
			0.5^3	2.8200	1.6292	0.3810	0.6679	<i>0.6582</i>
			0.1,0.3,0.5	1.4042	0.8791	0.4499	0.6012	<i>0.5953</i>
		5^3	0.1^3	0.4255	0.3013	0.0909	0.1540	<i>0.1531</i>
			0.3^3	0.7474	0.4335	0.0840	0.2278	<i>0.2268</i>
			0.5^3	0.9889	0.6357	0.0975	0.3429	<i>0.3412</i>
			0.1,0.3,0.5	0.6482	0.3794	0.1690	0.3383	<i>0.3366</i>

Note: $n_i^3 = (n_1, n_2, n_3)$ with $30^3 = (30, 30, 30)$. The shortest AW values are in italic.

TABLE 4-3 The CPs of the 95% confidence intervals for the common coefficient of variation of delta-IG distributions when $k = 6$.

n_i^6	μ_i^6	λ_i^6	δ_i^6	Coverage Probabilities				
				GCI	AGCI	PBPCI	BCI	B-HPDCI
30^6	0.5^6	1^6	0.1^6	0.8034	0.8489	0.7359	0.9261	0.9361
			0.3^6	0.8178	0.8358	0.7315	0.8982	0.9142
			0.5^6	0.8419	0.8584	0.7242	0.8602	0.8603
			$0.1^2, 0.3^2, 0.5^2$	0.8060	0.8190	0.7030	0.8852	0.9070
		5^6	0.1^6	0.7505	0.7631	0.7265	0.9341	0.9501
			0.3^6	0.8388	0.8503	0.7212	0.8942	0.9142
			0.5^6	0.8623	0.8922	0.7144	0.9024	0.9224
			$0.1^2, 0.3^2, 0.5^2$	0.8571	0.8621	0.7126	0.9323	0.9423
	1^6	1^6	0.1^6	0.8275	0.8331	0.7319	0.8982	0.9122
			0.3^6	0.8038	0.8068	0.7259	0.8822	0.8902
			0.5^6	0.8020	0.8367	0.7196	0.8882	0.8862
			$0.1^2, 0.3^2, 0.5^2$	0.8060	0.8190	0.7028	0.9459	0.9591
		5^6	0.1^6	0.8449	0.8657	0.7335	0.9441	0.9641
			0.3^6	0.8212	0.8523	0.7319	0.9488	0.9502
			0.5^6	0.9134	0.9142	0.7238	0.9437	0.9514
			$0.1^2, 0.3^2, 0.5^2$	0.9088	0.9033	0.7080	0.9090	0.9203
50^6	0.5^6	1^6	0.1^6	0.8029	0.8068	0.7503	0.9421	0.9561
			0.3^6	0.8583	0.8631	0.7445	0.9061	0.9261
			0.5^6	0.8200	0.8531	0.7393	0.8702	0.8922
			$0.1^2, 0.3^2, 0.5^2$	0.8320	0.8756	0.8016	0.9303	0.9541
		5^6	0.1^6	0.7948	0.7904	0.7329	0.9401	0.9541
			0.3^6	0.8643	0.9142	0.7255	0.9348	0.9466
			0.5^6	0.8527	0.9182	0.7180	0.9503	0.9564
			$0.1^2, 0.3^2, 0.5^2$	0.8192	0.8645	0.7130	0.9365	0.9545
	1^6	1^6	0.1^6	0.8234	0.8461	0.7469	0.9241	0.9381
			0.3^6	0.8028	0.8311	0.7357	0.9161	0.9202
			0.5^6	0.8134	0.8515	0.7289	0.8802	0.8882
			$0.1^2, 0.3^2, 0.5^2$	0.8904	0.9198	0.8030	0.9293	0.9381
		5^6	0.1^6	0.7497	0.7824	0.7495	0.9305	0.9406
			0.3^6	0.8401	0.8483	0.7376	0.8810	0.9010
			0.5^6	0.8171	0.8283	0.7267	0.8909	0.8911
			$0.1^2, 0.3^2, 0.5^2$	0.8824	0.9034	0.8020	0.9430	0.9517
100^6	0.5^6	1^6	0.1^6	0.8765	0.8853	0.7675	0.9002	0.9162
			0.3^6	0.9012	0.9122	0.7603	0.9281	0.9461
			0.5^6	0.9015	0.9166	0.7527	0.8982	0.9102
			$0.1^2, 0.3^2, 0.5^2$	0.8523	0.8858	0.8006	0.9456	0.9699
		5^6	0.1^6	0.8447	0.8842	0.7515	0.9404	0.9505
			0.3^6	0.8509	0.8703	0.7307	0.9008	0.9208
			0.5^6	0.8505	0.8683	0.7267	0.9008	0.9210
			$0.1^2, 0.3^2, 0.5^2$	0.8214	0.8549	0.8079	0.9354	0.9574
	1^6	1^6	0.1^6	0.8192	0.8417	0.7733	0.9404	0.9604
			0.3^6	0.8020	0.8898	0.7574	0.9206	0.9208
			0.5^6	0.8567	0.8702	0.7564	0.9305	0.9505
			$0.1^2, 0.3^2, 0.5^2$	0.8753	0.9140	0.7825	0.9099	0.9119

TABLE 4-3 (CONTINUED)

n_i^6	μ_i^6	λ_i^6	δ_i^6	Coverage Probabilities				
				GCI	AGCI	PBPCI	BCI	B-HPDCI
		5^6	0.1^6	0.7810	0.8124	0.7030	0.9305	0.9505
			0.3^6	0.8253	0.8527	0.7554	0.9206	0.9406
			0.5^6	0.8142	0.8467	0.7366	0.9008	0.9109
			$0.1^2, 0.3^2, 0.5^2$	0.8062	0.8637	0.7894	0.9099	0.9119
$30^2, 50^2, 100^2$	0.5^6	1^6	0.1^6	0.7914	0.8059	0.7554	0.8909	0.9010
			0.3^6	0.8399	0.8561	0.7475	0.9206	0.9307
			0.5^6	0.8454	0.8734	0.7406	0.9216	0.9416
			$0.1^2, 0.3^2, 0.5^2$	0.7004	0.7232	0.8013	0.9356	0.9366
		5^6	0.1^6	0.8058	0.8144	0.7396	0.9404	0.9505
			0.3^6	0.8808	0.9162	0.7277	0.9385	0.9486
			0.5^6	0.8547	0.9321	0.7228	0.9281	0.9483
			$0.1^2, 0.3^2, 0.5^2$	0.8349	0.8601	0.7089	0.9279	0.9402
	1^6	1^6	0.1^6	0.8188	0.8425	0.7436	0.9107	0.9208
			0.3^6	0.8006	0.8359	0.7366	0.9305	0.9604
			0.5^6	0.7934	0.8016	0.7416	0.8513	0.8614
			$0.1^2, 0.3^2, 0.5^2$	0.8343	0.8715	0.7141	0.9485	0.9495
		5^6	0.1^6	0.7885	0.8044	0.7465	0.9503	0.9703
			0.3^6	0.8341	0.8723	0.7465	0.9305	0.9505
			0.5^6	0.8246	0.8788	0.7376	0.9008	0.9109
			$0.1^2, 0.3^2, 0.5^2$	0.9026	0.9333	0.8020	0.9327	0.9337

Note: $n_i^6 = (n_1, n_2, n_3, n_4, n_5, n_6)$ with $30^2, 50^2, 100^2 = (30, 30, 50, 50, 100, 100)$. Bold values indicate coverage probabilities (CP) that satisfy the nominal confidence level of 0.95 ($CP \geq 0.945$, rounded to 0.950); in configurations where no method attains this threshold, the bold value denotes the highest CP achieved among the five methods for that parameter setting.

TABLE 4-4 The AWs of the 95% confidence intervals for the common coefficient of variation of delta-IG distributions when $k = 6$.

n_i^6	μ_i^6	λ_i^6	δ_i^6	Average Widths				
				GCI	AGCI	PBPCI	BCI	B-HPDCI
30^6	0.5^6	1^6	0.1^6	0.1278	0.1054	0.1715	0.2343	0.2333
			0.3^6	0.2173	0.2242	0.1698	0.3240	0.3227
			0.5^6	0.3332	0.4092	0.1953	0.4892	0.4866
			$0.1^2, 0.3^2, 0.5^2$	0.1916	0.1692	0.2429	0.3452	0.3436
		5^6	0.1^6	0.1338	0.1316	0.0747	0.1621	0.1616
			0.3^6	0.2356	0.2345	0.0470	0.2322	0.2316
			0.5^6	0.3353	0.3496	0.0505	0.3354	0.3345
			$0.1^2, 0.3^2, 0.5^2$	0.2369	0.2366	0.0854	0.2623	0.2614
	1^6	1^6	0.1^6	0.1747	0.1097	0.2810	0.3649	0.3624
			0.3^6	0.2538	0.2466	0.2931	0.5007	0.4963
			0.5^6	0.3675	0.4794	0.3418	0.7675	0.7561
			$0.1^2, 0.3^2, 0.5^2$	0.2385	0.1737	0.3599	0.5084	0.5030
		5^6	0.1^6	0.1175	0.1123	0.1006	0.1690	0.1684
			0.3^6	0.2203	0.2226	0.0850	0.2462	0.2456
			0.5^6	0.3310	0.3628	0.0931	0.3640	0.3629
			$0.1^2, 0.3^2, 0.5^2$	0.1997	0.1969	0.1407	0.2659	0.2649
50^6	0.5^6	1^6	0.1^6	0.0974	0.0797	0.1308	0.1701	0.1695
			0.3^6	0.1645	0.1690	0.1357	0.2358	0.2350
			0.5^6	0.2498	0.2997	0.1635	0.3541	0.3528
			$0.1^2, 0.3^2, 0.5^2$	0.1493	0.1368	0.1944	0.2597	0.2587
		5^6	0.1^6	0.1100	0.1076	0.0463	0.1206	0.1203
			0.3^6	0.1728	0.1713	0.0369	0.1733	0.1728
			0.5^6	0.2500	0.2589	0.0413	0.2563	0.2557
			$0.1^2, 0.3^2, 0.5^2$	0.1974	0.1991	0.0629	0.2064	0.2059
	1^6	1^6	0.1^6	0.1339	0.0819	0.2145	0.2579	0.2566
			0.3^6	0.1923	0.1864	0.2365	0.3475	0.3457
			0.5^6	0.2754	0.3541	0.2880	0.5186	0.5153
			$0.1^2, 0.3^2, 0.5^2$	0.1836	0.1361	0.2931	0.3699	0.3678
		5^6	0.1^6	0.0932	0.0889	0.0698	0.1251	0.1248
			0.3^6	0.1647	0.1657	0.0657	0.1844	0.1840
			0.5^6	0.2476	0.2676	0.0742	0.2716	0.2709
			$0.1^2, 0.3^2, 0.5^2$	0.1638	0.1648	0.1075	0.2061	0.2055
100^6	0.5^6	1^6	0.1^6	0.0677	0.0557	0.0915	0.1132	0.1128
			0.3^6	0.1123	0.1139	0.0990	0.1578	0.1573
			0.5^6	0.1723	0.2058	0.1199	0.2336	0.2329
			$0.1^2, 0.3^2, 0.5^2$	0.1036	0.0956	0.1429	0.1795	0.1790
		5^6	0.1^6	0.0760	0.0743	0.0293	0.0807	0.0805
			0.3^6	0.1161	0.1149	0.0254	0.1182	0.1179
			0.5^6	0.1706	0.1762	0.0296	0.1737	0.1732
			$0.1^2, 0.3^2, 0.5^2$	0.1416	0.1430	0.0429	0.1479	0.1475
	1^6	1^6	0.1^6	0.0937	0.0565	0.1520	0.1721	0.1714
			0.3^6	0.1307	0.1250	0.1700	0.2259	0.2251
			0.5^6	0.1888	0.2419	0.2219	0.3375	0.3361
			$0.1^2, 0.3^2, 0.5^2$	0.1280	0.0942	0.0784	0.1445	0.1442

TABLE 4-4 (CONTINUED)

n_i^6	μ_i^6	λ_i^6	δ_i^6	Average Widths				
				GCI	AGCI	PBPCI	BCI	B-HPDCI
		5^6	0.1^6	0.0655	0.0628	0.0488	0.0838	<i>0.0836</i>
			0.3^6	0.1122	0.1119	0.0464	0.1230	<i>0.1227</i>
			0.5^6	0.1699	0.1829	0.0554	0.1831	<i>0.1826</i>
			$0.1^2, 0.3^2, 0.5^2$	0.1170	0.1181	0.0784	0.1445	<i>0.1442</i>
$30^2, 50^2, 100^2$	0.5^6	1^6	0.1^6	0.0898	0.0739	0.1204	0.1514	<i>0.1510</i>
			0.3^6	0.1496	0.1537	0.1286	0.2132	<i>0.2126</i>
			0.5^6	0.2290	0.2800	0.1512	0.3147	<i>0.3136</i>
			$0.1^2, 0.3^2, 0.5^2$	0.1754	0.1735	0.2389	0.3154	<i>0.3144</i>
		5^6	0.1^6	0.1019	0.0994	0.0476	0.1113	<i>0.1110</i>
			0.3^6	0.1598	0.1593	0.0334	0.1596	<i>0.1591</i>
			0.5^6	0.2296	0.2387	0.0377	0.2356	<i>0.2349</i>
			$0.1^2, 0.3^2, 0.5^2$	0.2579	0.2624	0.0783	0.2763	<i>0.2753</i>
	1^6	1^6	0.1^6	0.1232	0.0759	0.2040	0.2307	<i>0.2296</i>
			0.3^6	0.1753	0.1677	0.2171	0.3114	<i>0.3099</i>
			0.5^6	0.2528	0.3321	0.2751	0.4710	<i>0.4689</i>
			$0.1^2, 0.3^2, 0.5^2$	0.2006	0.1740	0.3625	0.4448	<i>0.4431</i>
		5^6	0.1^6	0.0864	0.0822	0.0715	0.1128	<i>0.1124</i>
			0.3^6	0.1512	0.1527	0.0611	0.1641	<i>0.1637</i>
			0.5^6	0.2273	0.2482	0.0690	0.2487	<i>0.2480</i>
			$0.1^2, 0.3^2, 0.5^2$	0.2045	0.2109	0.1411	0.2661	<i>0.2654</i>

Note: $n_i^6 = (n_1, n_2, n_3, n_4, n_5, n_6)$ with $30^2, 50^2, 100^2 = (30, 30, 50, 50, 100, 100)$. The shortest AW values are in italic.

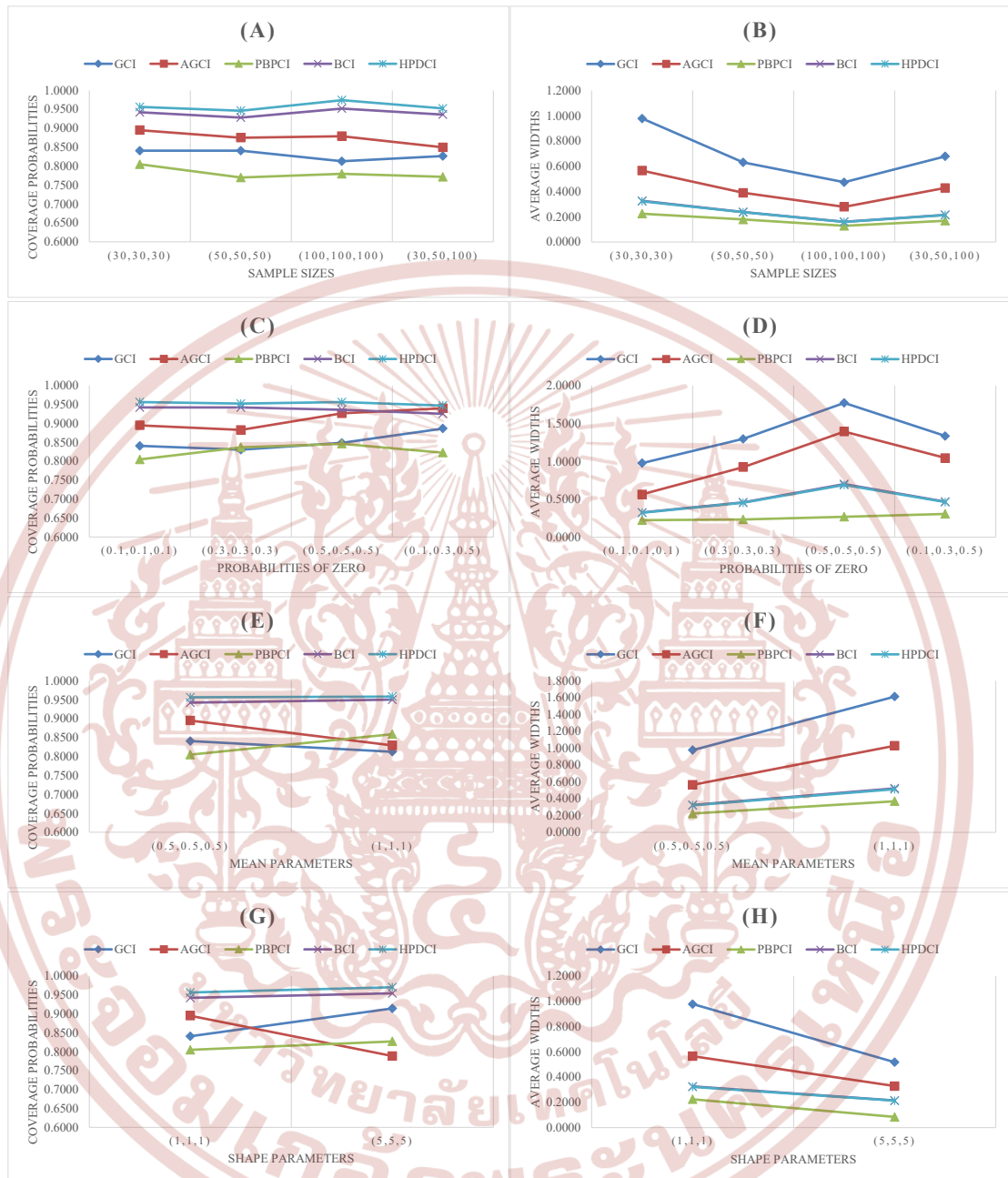


FIGURE 4-1 Comparison of the performance of the proposed methods for estimating the 95% confidence interval for the common coefficient of variation of delta-IG distributions when $k = 3$.

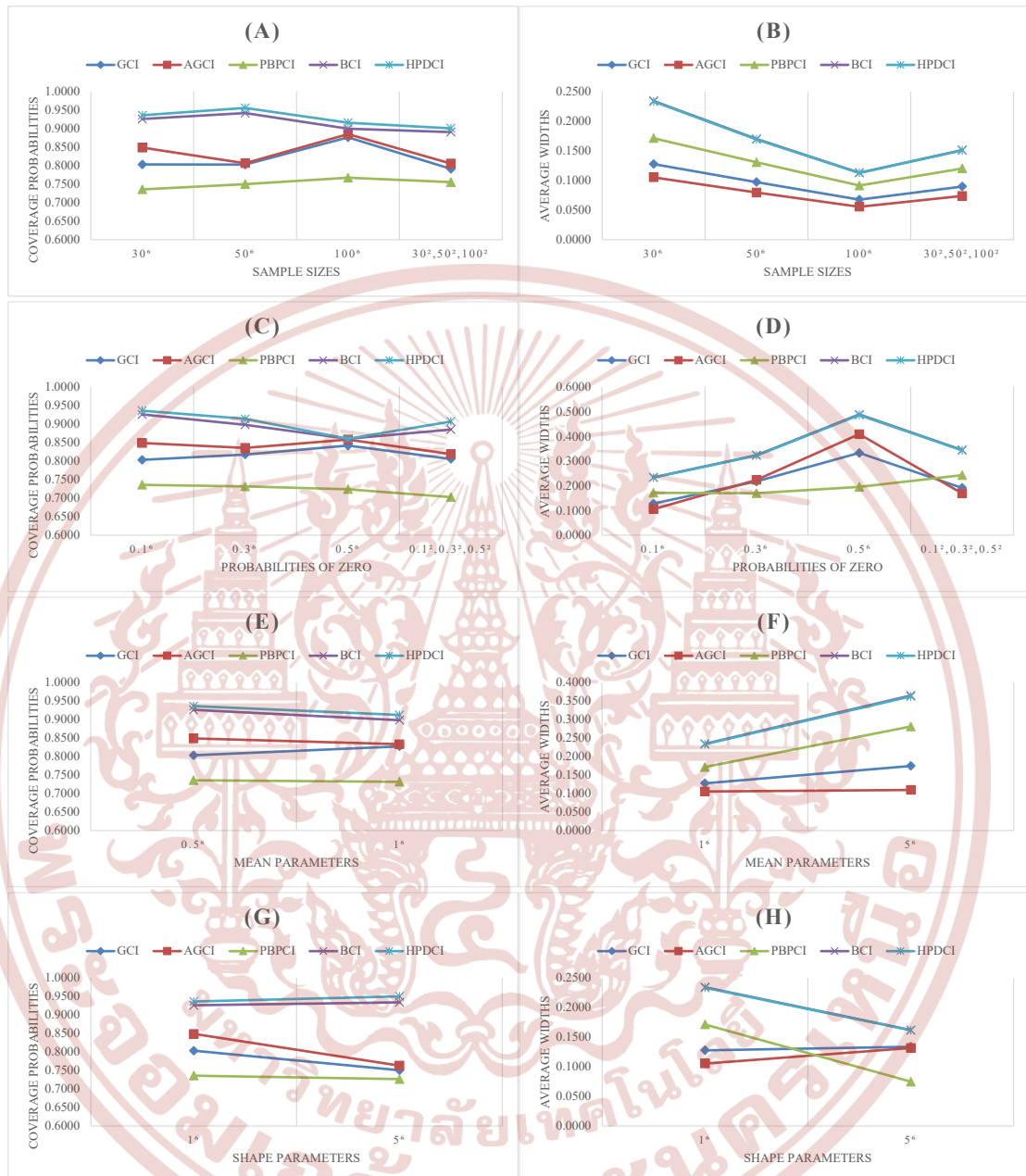


FIGURE 4-2 Comparison of the performance of the proposed methods for estimating the 95% confidence interval for the common coefficient of variation of delta-IG distributions when $k = 6$.

4.4 An empirical application

The data of monthly traffic accident fatality numbers documented in the second quarter of 2025 throughout 50 districts in Bangkok, Thailand were used to assess the performance of the proposed methods for constructing confidence intervals for the common coefficient of variation of multiple delta-IG distributions. These data sourced from the Road Accident Victims Protection Company Limited and accessible via the Thai Road Safety Center (<https://www.thairsc.com/bangkok-in-depth-statistics>). Since

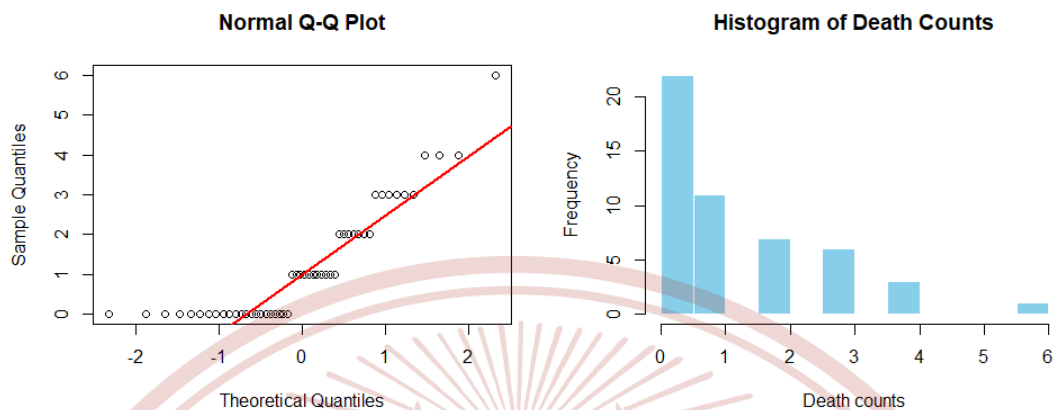


FIGURE 4-3 (left) Normal Q-Q plot and (right) Histogram of death count data from Bangkok in April 2025.

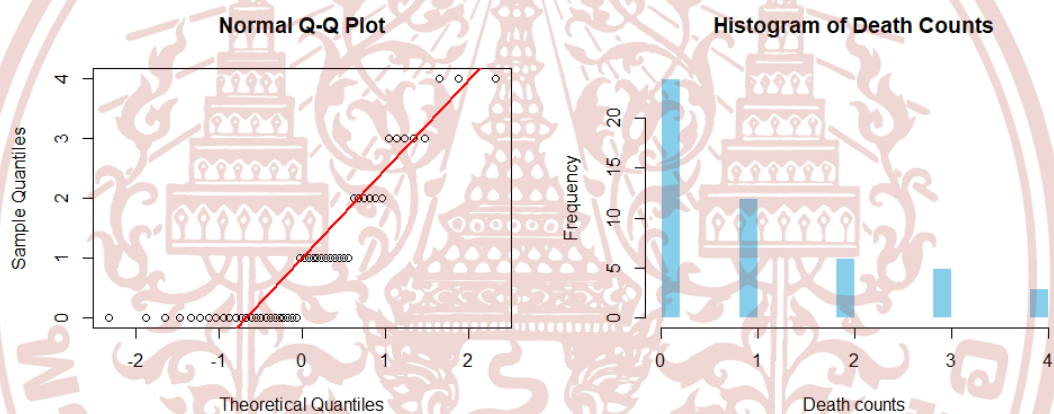


FIGURE 4-4 (left) Normal Q-Q plot and (right) Histogram of death count data from Bangkok in May 2025.

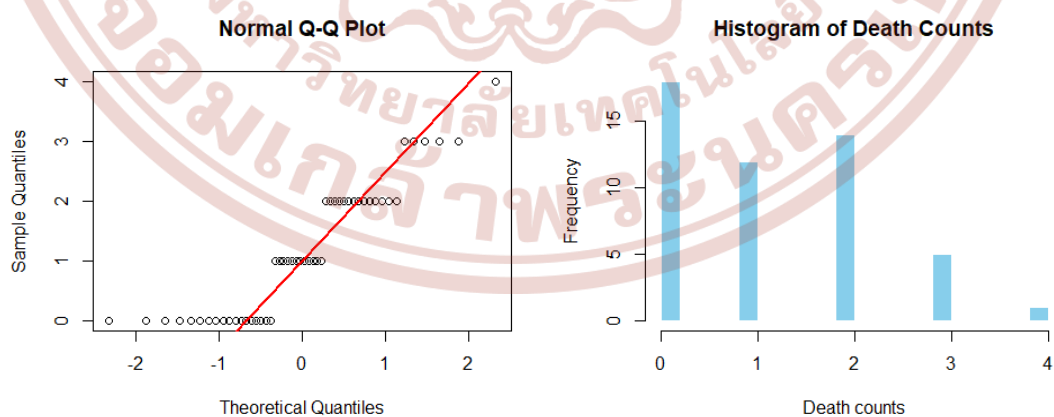


FIGURE 4-5 (left) Normal Q-Q plot and (right) Histogram of death count data from Bangkok in June 2025.

TABLE 4-6 The AIC and BIC values for evaluating the distribution of the traffic fatality data in the second quarter of 2025 from Bangkok, Thailand.

Distributions	April		May		June	
	Criterion		Criterion		Criterion	
	AIC	BIC	AIC	BIC	AIC	BIC
Normal	96.1842	98.8487	80.5895	83.1057	80.1201	83.0515
Lognormal	85.7605	88.4249	72.5468	75.0630	74.8820	77.8135
Weibull	89.4167	92.0811	75.6233	78.1395	77.2555	80.1869
Gamma	87.3232	89.9876	73.9823	76.4985	75.3322	78.2636
Exponential	101.6055	102.9377	89.0339	90.2920	105.1553	106.6210
Logistic	95.8273	98.4917	82.3390	84.8552	81.0436	83.9751
Inverse Gaussian	84.8527	87.5171	71.5812	74.0974	74.2977	77.2292

TABLE 4-7 Descriptive statistics for each month in the second quarter of 2025 from Bangkok, Thailand.

Statistics	Months		
	April	May	June
n_i	50	50	50
$n_{i(0)}$	22	24	18
$n_{i(1)}$	28	26	32
$\hat{\mu}_i$	2.1800	1.9600	1.8400
$\hat{\lambda}_i$	6.1400	6.2500	8.9900
$\hat{\delta}_i$	0.4400	0.4800	0.3600
$\hat{\theta}_{ij}$	1.1900	1.2400	0.9400

TABLE 4-8 Estimated common coefficient of variation and 95% confidence intervals for the common coefficient of variation of the traffic fatality data in the second quarter of 2025 from Bangkok, Thailand.

Point Estimations	Methods	CI of the common CV	
		Interval	Width
$\hat{\theta}_{common} = 1.0188$	GCI	(0.8180, 1.1248)	0.3068
	AGCI	(0.8153, 1.1219)	0.3066
	PBPCI	(0.9815, 1.2797)	0.2982
	BayeCI	(1.0225, 1.3685)	0.3460
	HPDCI	(1.0097, 1.3523)	0.3426

4.5 Conclusions

This study compared several methods for constructing confidence intervals for the common coefficient of variation under the delta-IG distribution using both simulation and real-data analysis. The simulation results show that B-HPDCI provides the most reliable performance, achieving coverage probabilities closest to the nominal level while maintaining reasonable interval widths. BCI also performs well, though slightly less accurate. In contrast, PBPCI produces the narrowest intervals but suffers from severe undercoverage, making it unreliable in practice. The GCI and AGCI methods fail to achieve a satisfactory balance between coverage and efficiency, particularly under skewness and zero inflation.

The real-data application supports these findings. Although PBPCI yields the narrowest interval, its tendency toward undercoverage limits its usefulness. The GCI and AGCI methods offer moderate performance, while Bayesian approaches (BCI and B-HPDCI) produce wider but more reliable intervals by better accounting for uncertainty and distributional complexity. Overall, B-HPDCI is recommended as the most appropriate method when coverage accuracy and robustness are the primary objectives, with BCI serving as a strong alternative.

In Chapters 2 and 3 addressed the confidence intervals for the coefficient of variation and variance of the delta-IG distribution in both single and two population settings, covering estimation of a single parameter as well as the difference and ratio between two independent populations. While Chapter 4 assumed the common coefficient of variation across all populations, this assumption may not always hold in practice. When the CVs differ across groups, it becomes important to identify which specific pairs of populations exhibit significant differences in relative variability. Therefore, Chapter 5 addresses this need by developing simultaneous confidence intervals for all pairwise differences between the CVs of multiple independent delta-IG populations. This approach allows for a comprehensive comparison across all population pairs while controlling the overall error rate, providing a more complete inferential framework for multi-population variability analysis under the delta-IG distribution.

TABLE 4-9 Summary of the chapter.

Aspect	Key Finding	Recommended Method
Common CV	Most reliable coverage with reasonable interval width	B-HPDCI
	Strong performance but slightly less accurate	BCI
	Narrowest intervals but severe undercoverage	PBPCI (not recommended)
Overall Conclusion	Accuracy and robustness prioritized over narrow intervals	B-HPDCI

CHAPTER 5

SIMULTANEOUS CONFIDENCE INTERVALS FOR ALL PAIRWISE DIFFERENCES BETWEEN THE COEFFICIENTS OF VARIATION FOR MULTIPLE DELTA-INVERSE GAUSSIAN DISTRIBUTIONS

This chapter presents methods for estimating simultaneous confidence intervals for all pairwise differences between the coefficients of variation for multiple delta-IG distributions. The methods provide generalized confidence intervals (GCI), adjusted generalized confidence intervals (AGCI), fiducial confidence intervals (FCI), method of variance estimates recovery (MOVER), and normal approximation (NA). A simulation study is conducted to determine the coverage probabilities (CPs) and average widths (AWs) of these intervals. The proposed methods were applied with the traffic fatality data recorded in the second half of 2024 from Bangkok, Thailand.

5.1 Introduction

The quantification of variability constitutes a central objective in statistical analysis across a broad range of scientific disciplines. Although variance and standard deviation are the most commonly employed measures of dispersion, their dependence on the scale of measurement limits their utility when comparisons across heterogeneous populations are required. The coefficient of variation (CV), defined as the ratio of the standard deviation to the mean, overcomes this limitation by providing a scale-independent measure of relative variability. This property renders the CV particularly suited for comparing stochastic behavior across populations that differ in magnitude or unit of measurement. Accordingly, statistical inference for the CV has been extensively investigated in diverse applied fields, including environmental studies, reliability engineering, finance, and biomedical research (Hsieh, 1990; Gupta & Akman, 1996; Chaubey et al., 2017; Chankham et al., 2022).

In numerous applied settings, the primary inferential objective extends beyond the estimation of a single CV to the comparison of relative variability across multiple populations. Over the past several decades, a variety of methodological approaches have been developed for constructing confidence intervals for the CV, encompassing bootstrap-based techniques, generalized inference procedures, and fiducial inference (Efron & Tibshirani, 1986; Weerahandi, 1993; Hannig et al., 2006; Zou et al., 2009). Among these, the generalized confidence interval (GCI) approach has attracted particular attention due to its flexibility in non-normal distributional settings. However, the GCI can be sensitive to sample size and the degree of skewness, often producing coverage distortions in small to moderate samples. Adjusted formulations, such as the adjusted GCI (AGCI), have been proposed to mitigate these deficiencies. Additional approaches, including fiducial confidence intervals (FCI) and the method of variance estimates recovery (MOVER), have also been developed. While the FCI avoids reliance on asymptotic approximations, it can be computationally demanding. The MOVER, by contrast, typically produces shorter interval lengths through the efficient combination of variance estimates, but may suffer from undercoverage when distributional

assumptions are violated. These limitations collectively suggest that no single method performs uniformly well across all inferential scenarios, particularly in the presence of pronounced skewness or distributional complexity.

Despite these methodological advances, most existing contributions focus predominantly on inference for a single CV or on comparisons between two populations. In many practical applications, however, the primary goal is to simultaneously compare relative variability across multiple groups. Conducting multiple pairwise comparisons using separate confidence intervals can inflate the overall type I error rate and lead to misleading inferential conclusions. This concern is especially pronounced when the focus lies on differences in CVs, which provide a direct and interpretable quantification of absolute differences in relative dispersion. Statistical inference for such differences is inherently challenging under non-normal and zero-inflated distributions, owing to the combined effects of skewness, parameter dependence, and the presence of multiple nuisance parameters (Feltz & Miller, 1996; Tian, 2005). These difficulties motivate the adoption of simultaneous confidence intervals (SCIs), which permit valid joint inference across all pairwise comparisons while maintaining control over the overall confidence level (Hochberg & Tamhane, 1987; Hsu, 1996).

At the same time, data arising from many empirical settings frequently exhibit pronounced positive skewness together with a substantial excess of zero observations, rendering classical models premised on normality inadequate. Among skewed continuous distributions, the inverse Gaussian (IG) distribution has received considerable theoretical and applied attention, owing to its flexibility and its derivation from first-passage-time processes (Chhikara & Folks, 1989). The IG distribution has been widely employed for modelling positively skewed continuous data across various domains (Balakrishna & Rahul, 2002; Chatzidiamantis et al., 2011; Bardsley, 1980); however, it cannot accommodate the presence of zero-valued observations. To address this structural limitation, Heller et al. (2006) introduced the delta-inverse Gaussian (delta-IG) distribution as an extension that incorporates a probability mass at zero while preserving the flexibility of the continuous IG component. This model has been successfully applied in fields such as insurance claims analysis, environmental monitoring, and mortality studies (Jana & Gautam, 2024; Khumpasee et al., 2024).

Several prior contributions have developed simultaneous confidence intervals for pairwise differences in CVs under specific distributional assumptions. Thangjai et al. (2021) constructed SCIs for all pairwise differences in CVs arising from multiple normal distributions. Yosboonruang et al. (2021) established SCIs for all pairwise differences in CVs under delta-lognormal distributions. Janthasuwana et al. (2026) proposed and evaluated methods for constructing SCIs for all pairwise differences in CVs under zero-inflated Birnbaum–Saunders distributions. However, these methodological contributions are tailored to specific distributional families and are not directly transferable to more complex data structures that exhibit both strong positive skewness and excess zeros simultaneously. Under such conditions, their coverage and precision may deteriorate substantially, limiting their practical applicability. To the best of the author's knowledge, no comprehensive framework currently exists for

constructing simultaneous confidence intervals for all pairwise differences in CVs under the delta-IG distribution. This gap is non-trivial, as existing procedures may perform poorly due to the combined effects of skewness, zero inflation, and parameter dependence, particularly in small to moderate samples (Dasgupta, 2008). There is therefore a clear need for robust and reliable inferential procedures specifically designed for this class of distributions.

To address the identified gap, this study develops and evaluates simultaneous confidence interval procedures for all pairwise differences of coefficients of variation under independent delta-IG populations. Five competing approaches are considered and systematically compared: the generalized confidence interval (GCI), the adjusted generalized confidence interval (AGCI), the fiducial confidence interval (FCI), the method of variance estimates recovery (MOVER), and the normal approximation (NA) method. Their performance is assessed through a comprehensive Monte Carlo simulation study in terms of coverage probability and average interval width across a broad range of parameter configurations.

5.2 The simultaneous confidence intervals for all pairwise differences between coefficients of variation for multiple delta-IG distributions

Let $X_{ij} = (X_{i1}, X_{i2}, \dots, X_{in_i})$, $i = 1, 2, \dots, k$, $j = 1, 2, \dots, n_i$ be a non-negative random sample from a delta-IG distribution stated as $X_{ij} \sim \text{delta-IG}(\mu_i, \lambda_i, \delta_i)$ where μ_i represents the mean, λ_i denotes the shape parameter, and δ_i is the zero proportion. The probability density function of delta-IG is expressed as

$$f_{\text{delta-IG}}(X_{ij}; \mu_i, \lambda_i, \delta_i) = \delta_i I_{\{X_{ij}=0\}} + (1 - \delta_i) I_{\{X_{ij}>0\}} \sqrt{\frac{\lambda_i}{2\pi X_{ij}^3}} \exp\left[-\frac{\lambda_i(X_{ij}-\mu_i)^2}{2\mu_i^2 X_{ij}}\right], \quad (5-1)$$

where $I(\cdot)$ is an indicator function, $P(X_{ij} = 0) = \delta_i$, and $\mu_i, \lambda_i > 0$. The sample contains n observations, while n_i is separated into zero ($n_{i(0)}$) and nonzero observations ($n_{i(1)}$), such that $n_i = n_{i(0)} + n_{i(1)}$. Moreover, $n_{i(0)}$ follows a binomial distribution denoted as $n_{i(0)} \sim \text{Bin}(n_i, \delta_i)$.

The MLEs of δ_i, μ_i , and λ_i of the distributions are given by: $\hat{\delta}_i = n_{i(0)}/n_i$, $\hat{\mu}_i = \bar{X}_i = \frac{1}{n_{i(1)}} \sum_{j=1}^{n_{i(1)}} X_{ij}$, and $n_{i(1)} \hat{\lambda}_i^{-1} = V_i = \sum_{j=1}^{n_{i(1)}} (X_{ij}^{-1} - \bar{X}_i^{-1})$, respectively. The expected value was $E(X_{ij}) = (1 - \delta_i)\mu_i$. The variance was $\text{Var}(X_{ij}) = \mu_i^2(1 - \delta_i) \left(\frac{\mu_i}{\lambda_i} + \delta_i\right)$. The coefficients of variation of delta-IG distributions were $CV(X_{ij}) = \theta_i = \sqrt{(\mu_i + \lambda_i \delta_i)/(1 - \delta_i)\lambda_i}$. Thus, all pairwise differences between the CVs are parameters that we will compute for their interval are given by

$$\theta_{il} = \theta_i - \theta_l = \sqrt{(\mu_i + \lambda_i \delta_i)/(1 - \delta_i)\lambda_i} - \sqrt{(\mu_l + \lambda_l \delta_l)/(1 - \delta_l)\lambda_l}, \quad (5-2)$$

where $i, l = 1, 2, \dots, k$ and $i \neq l$.

5.2.1 Generalized confidence interval (GCI)

Ye, Ma, and Wang (2010) introduced a GPQ-based approach for constructing confidence intervals for the mean of inverse Gaussian distributions across multiple populations. Their method uses GPQs derived for the unknown parameters of the IG distribution, namely the shape parameter λ and the mean parameter μ .

The GPQs for λ_i and μ_i , as developed by Ye, Ma, and Wang, are expressed as follows:

$$R_{\lambda_i} = \frac{n_{i(1)}\lambda_i v_i}{n_{i(1)}v_i} \sim \frac{\chi_{n_{i(1)}-1}^2}{n_{i(1)}v_i}, i = 1, 2, \dots, k, \quad (5-3)$$

where $\chi_{n_{i(1)}-1}^2$ is denoted as a Chi-square distribution with $n_{i(1)} - 1$ degrees of freedom, $i = 1, 2, \dots, k$ and

$$R_{\mu_i} = \frac{\bar{X}_i}{\left[1 + \frac{n_{i(1)}\lambda_i(\bar{X}_i - \mu_i)}{\mu_i \sqrt{\bar{X}_i}} \sqrt{\frac{\bar{X}_i}{n_{i(1)}R_{\lambda_i}}}\right]} \sim \frac{\bar{X}_i}{\left[1 + Z_i \sqrt{\frac{\bar{X}_i}{n_{i(1)}R_{\lambda_i}}}\right]}, \quad (5-4)$$

where Z_i is approximately distributed as a standard normal distribution, denoted by $N(0,1)$.

The parameter δ was estimated using the VST approach, originally introduced by Dasgupta (2008), based on normal approximation theory. Subsequently, Wu and Hsieh (2014) developed a VST for the binomial distribution using the arcsine square-root transformation to achieve variance stabilization.

Accordingly, the GPQ for δ_i , following Wu and Hsieh, can be expressed as follows:

$$R_{\delta_i} = \sin^2 \left[\arcsin \sqrt{\hat{\delta}_i} - \frac{T}{2\sqrt{n_i}} \right], \quad (5-5)$$

where $T = 2\sqrt{n_i}(\arcsin \sqrt{\hat{\delta}_i} - \arcsin \sqrt{\delta_i}) \sim N(0,1)$, $n_i \rightarrow \infty$.

Hence, from the GPQ of parameters as R_{λ_i} , R_{μ_i} , and R_{δ_i} expressed in Equations (5-3), (5-4), and (5-5), respectively. The GPQ for θ_{il} can be defined as

$$R_{\theta_{il}} = \sqrt{(R_{\mu_i} + R_{\lambda_i}R_{\delta_i})/(1 - R_{\delta_i})R_{\lambda_i}} - \sqrt{(R_{\mu_l} + R_{\lambda_l}R_{\delta_l})/(1 - R_{\delta_l})R_{\lambda_l}}, \quad (5-6)$$

where $i, l = 1, 2, \dots, k$ and $i \neq l$.

The $100(1 - \alpha)\%$ two-sided SCI for θ_{il} using the GCI method is

$$CI_{\theta_{il}}^{GCI} = [L_{\theta_{il}}^{GCI}, U_{\theta_{il}}^{GCI}] = [R_{\theta_{il}}(\alpha/2), R_{\theta_{il}}(1 - \alpha/2)], \quad (5-7)$$

where $R_{\theta_{il}}(\alpha/2)$ and $R_{\theta_{il}}(1 - \alpha/2)$ are the $100\frac{\alpha}{2}$ -th and $100(1 - \frac{\alpha}{2})$ -th percentiles of the distribution of $R_{\theta_{il}}$, respectively, $i, l = 1, 2, \dots, k$ and $i \neq l$.

5.2.2 Adjusted generalized confidence interval (AGCI)

The AGCI method utilizes the modified GPQ of μ , as proposed by Krishnamoorthy and Tian (2008), instead of the GPQ of μ defined in Equation (5-4) under GCI framework. The GPQ for the parameter μ , as introduced by Krishnamoorthy and Tian, is given by:

$$R_{\tilde{\mu}_i} = \frac{\bar{X}_i}{\max \left\{ 0, 1 + t_{n_{i(1)}-1} \sqrt{\frac{\bar{X}_i v_i}{n_{i(1)}-1}} \right\}}, \quad (5-8)$$

where $t_{n_{i(1)}-1}$ is Student's t distribution with $n_{i(1)} - 1$ degrees of freedom.

Hence, from the GPQ of parameters, we have R_{λ_i} , $R_{\tilde{\mu}_i}$, and R_{δ_i} , expressed as Equations (5-3), (5-8), and (5-5), respectively. The GPQ for θ_{il} can be defined as

$$R_{\bar{\theta}_{il}} = \sqrt{(R_{\bar{\mu}_i} + R_{\lambda_i} R_{\delta_i}) / (1 - R_{\delta_i})} R_{\lambda_i} - \sqrt{(R_{\bar{\mu}_l} + R_{\lambda_l} R_{\delta_l}) / (1 - R_{\delta_l})} R_{\lambda_l}, \quad (5-9)$$

where $i, l = 1, 2, \dots, k$ and $i \neq l$.

The $100(1 - \alpha)\%$ two-sided SCI for θ_{il} using the AGCI method is

$$CI_{\theta_{il}}^{AGCI} = [L_{\theta_{il}}^{AGCI}, U_{\theta_{il}}^{AGCI}] = [R_{\bar{\theta}_{il}}(\alpha/2), R_{\bar{\theta}_{il}}(1 - \alpha/2)], \quad (5-10)$$

where $R_{\bar{\theta}_{il}}(\alpha/2)$ and $R_{\bar{\theta}_{il}}(1 - \alpha/2)$ are the $100\frac{\alpha}{2}$ -th and $100(1 - \frac{\alpha}{2})$ -th percentiles of the distribution of $R_{\bar{\theta}_{il}}$, respectively, $i, l = 1, 2, \dots, k$ and $i \neq l$.

5.2.3 Fiducial confidence interval (FCI)

Let $X_i = (X_{i1}, X_{i2}, \dots, X_{in_i})$, $i = 1, 2, \dots, k$ be a random sample from the delta-IG distribution with parameters μ_i , λ_i , and δ_i . All pairwise differences of coefficients of variation are defined as

$$\theta_{il} = p(\mu_i, \lambda_i, \delta_i) = \sqrt{(\mu_i + \lambda_i \delta_i) / (1 - \delta_i)} \lambda_i - \sqrt{(\mu_l + \lambda_l \delta_l) / (1 - \delta_l)} \lambda_l, \quad (5-11)$$

where $i, l = 1, 2, \dots, k$, $i \neq l$ and $p(\cdot)$ is known function depending on the parameterization of the delta-IG distribution.

To construct the fiducial distribution, the sampling distributions of the MLEs are utilized. Let $\hat{\mu}_i$ and $\hat{\lambda}_i$ denote the MLEs of μ_i and λ_i , respectively. Based on the properties of the IG distribution, their sampling distributions can be approximated as: $\mu_i \sim IG(\hat{\mu}_i, n_{i(1)} \hat{\lambda}_i)$ and $\lambda_i \sim \frac{\hat{\lambda}_i}{n_{i(1)}} \chi_{n_{i(1)}-1}^2$. These distributions are treated as fiducial distributions by replacing the unknown parameters with their corresponding MLEs.

Since the $p(\mu_i, \lambda_i, \delta_i)$ is function of μ_i , λ_i , and δ_i , the fiducial distribution of the pairwise differences between CVs can be obtained via simulation with Monte Carlo or Gibbs sampling employed.

The $100(1 - \alpha)\%$ FCI for all pairwise differences between CVs (θ_{il}) are obtained from the empirical quantiles of the simulated values defined as

$$\theta_{ilL} = \text{quantile}_{\alpha/2}(\theta_{il}^{(r)}), \theta_{ilU} = \text{quantile}_{1-\alpha/2}(\theta_{il}^{(r)}). \quad (5-12)$$

Thus, the simultaneous confidence interval of θ_{il} , $i \neq l$ are given by

$$CI_{\theta_{il}}^{FCI} = [L_{\theta_{il}}^{FCI}, U_{\theta_{il}}^{FCI}] = [\theta_{ilL}, \theta_{ilU}]. \quad (5-13)$$

5.2.4 Method of variance estimates recovery (MOVER)

The Method of Variance Estimates Recovery (MOVER), proposed by Zou and Donner (2009), is a flexible and efficient technique for constructing confidence intervals for a wide range of parameters, particularly the difference or ratio of two independent quantities. It is especially useful in situations where standard large-sample methods perform poorly, such as with small sample sizes or skewed distributions.

The MOVER method provides the lower and upper confidence limits for the difference of two parameters or $\theta_{il} = \theta_i - \theta_l$ with $i, l = 1, 2, \dots, k$, $i \neq l$, which are expressed as follows

$$L.diff_{il} = \hat{\theta}_i - \hat{\theta}_l - \sqrt{(\hat{\theta}_i - l_i)^2 + (u_l - \hat{\theta}_l)^2}, \quad (5-14)$$

and

$$U.diff_{il} = \hat{\theta}_i - \hat{\theta}_l + \sqrt{(u_i - \hat{\theta}_i)^2 + (\hat{\theta}_l - l_l)^2}, \quad (5-15)$$

where (l_i, u_i) and (l_l, u_l) are $100(1-\alpha)\%$ confidence limits for the parameters θ_i and θ_l with $i, l = 1, 2, \dots, k, i \neq l$.

By utilizing CI computed from the AGCI method as shown in equations (5-10), we obtain the $100(1-\alpha)\%$ two-sided simultaneous confidence intervals for all pairwise differences between CVs (θ_{il}) as follows:

From the AGCI method, the GPQ for θ_{il} can be defined as

$$R_{\tilde{\theta}_{il}} = \sqrt{(R_{\tilde{\mu}_i} + R_{\lambda_i} R_{\delta_i}) / (1 - R_{\delta_i}) R_{\lambda_i}} - \sqrt{(R_{\tilde{\mu}_l} + R_{\lambda_l} R_{\delta_l}) / (1 - R_{\delta_l}) R_{\lambda_l}}, \quad (5-16)$$

The $100(1-\alpha)\%$ two-sided SCI for θ_{il} using the AGCI method is

$$CI_{\theta_{il}}^{AGCI} = [L_{\theta_{il}}^{AGCI}, U_{\theta_{il}}^{AGCI}] = [R_{\tilde{\theta}_{il}}(\alpha/2), R_{\tilde{\theta}_{il}}(1 - \alpha/2)], \quad (5-17)$$

Therefore, the $100(1-\alpha)\%$ simultaneous confidence intervals for θ_{il} based on the MOVER are

$$CI_{\theta_{il}}^{MOVER} = [L_{\theta_{il}}^{MOVER}, U_{\theta_{il}}^{MOVER}] = [\hat{\theta}_i - \hat{\theta}_l - \sqrt{(\hat{\theta}_i - l_i)^2 + (u_l - \hat{\theta}_l)^2}, \hat{\theta}_i - \hat{\theta}_l + \sqrt{(u_i - \hat{\theta}_i)^2 + (\hat{\theta}_l - l_l)^2}], \quad (5-18)$$

where (l_i, u_i) and (l_l, u_l) are $100(1-\alpha)\%$ confidence limits computed from the AGCI method for the parameters θ_i and θ_l with $i, l = 1, 2, \dots, k, i \neq l$.

5.2.5 Normal approximation (NA)

In the normal approximation approach, the Delta method is applied to obtain the asymptotic normal distribution of the estimator θ . This method is grounded in large-sample theory and is commonly used as a benchmark due to its analytical simplicity and closed-form expression. The procedure can be outlined as follows:

Suppose that $f(\mu_i, \lambda_i, \delta_i) = \theta_i = \sqrt{(\mu_i + \lambda_i \delta_i) / (1 - \delta_i) \lambda_i}$ is the coefficient of variation of the delta-IG distribution. The Delta method was applied to acquire an estimator with an asymptotically normal distribution by using a Taylor series expansion, given by

$$f(\hat{\mu}_i, \hat{\lambda}_i, \hat{\delta}_i) \approx f(\mu_i, \lambda_i, \delta_i) + \frac{\partial f(\mu_i, \lambda_i, \delta_i)}{\partial \mu_i} (\hat{\mu}_i - \mu_i) + \frac{\partial f(\mu_i, \lambda_i, \delta_i)}{\partial \lambda_i} (\hat{\lambda}_i - \lambda_i) + \frac{\partial f(\mu_i, \lambda_i, \delta_i)}{\partial \delta_i} (\hat{\delta}_i - \delta_i). \quad (5-19)$$

The corresponding partial derivatives are given as follows:

$$\begin{aligned} \frac{\partial f(\mu, \lambda, \delta)}{\partial \mu} &= \frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu + \lambda\delta} \right)^{\frac{1}{2}} \left(\frac{1}{(1-\delta)\lambda} \right), \\ \frac{\partial f(\mu, \lambda, \delta)}{\partial \lambda} &= \frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu + \lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1-\delta)\lambda\delta - (\mu + \lambda\delta)(1-\delta)}{((1-\delta)\lambda)^2} \right), \\ \frac{\partial f(\mu, \lambda, \delta)}{\partial \delta} &= \frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu + \lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1-\delta)\lambda^2 - (\mu + \lambda\delta)(-\lambda)}{((1-\delta)\lambda)^2} \right). \end{aligned} \quad (5-20)$$

Therefore,

$$\begin{aligned}
 f(\hat{\mu}_i, \hat{\lambda}_i, \hat{\delta}_i) &\approx f(\mu, \lambda, \delta) + \frac{\partial f(\mu, \lambda, \delta)}{\partial \mu} (\hat{\mu} - \mu) + \frac{\partial f(\mu, \lambda, \delta)}{\partial \lambda} (\hat{\lambda} - \lambda) + \frac{\partial f(\mu, \lambda, \delta)}{\partial \delta} (\hat{\delta} - \delta) \\
 &= f(\mu, \lambda, \delta) + \left[\frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{1}{(1-\delta)\lambda} \right) \right] (\hat{\mu} - \mu) \\
 &\quad + \left[\frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1-\delta)\lambda\delta - (\mu+\lambda\delta)(1-\delta)}{((1-\delta)\lambda)^2} \right) \right] (\hat{\lambda} - \lambda) \\
 &\quad + \left[\frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1-\delta)\lambda^2 - (\mu+\lambda\delta)(-\lambda)}{((1-\delta)\lambda)^2} \right) \right] (\hat{\delta} - \delta).
 \end{aligned} \tag{5-21}$$

Taking the expectation and variance in Equation (4-5), the asymptotic variance for the coefficient of variation estimator is given by

$$\begin{aligned}
 Var(\hat{\theta}) &\approx Var\left\{f(\mu, \lambda, \delta) + \left[\frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{1}{(1-\delta)\lambda} \right) \right] (\hat{\mu} - \mu) \right. \\
 &\quad + \left[\frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1-\delta)\lambda\delta - (\mu+\lambda\delta)(1-\delta)}{((1-\delta)\lambda)^2} \right) \right] (\hat{\lambda} - \lambda) \\
 &\quad + \left. \left[\frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1-\delta)\lambda^2 - (\mu+\lambda\delta)(-\lambda)}{((1-\delta)\lambda)^2} \right) \right] (\hat{\delta} - \delta) \right\} \\
 &= \left[\frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{1}{(1-\delta)\lambda} \right) \right]^2 \left(\frac{\mu^3}{\lambda n_{i(1)}} \right) \\
 &\quad + \left[\frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1-\delta)\lambda\delta - (\mu+\lambda\delta)(1-\delta)}{((1-\delta)\lambda)^2} \right) \right]^2 \left(\frac{2\lambda^2}{n_{i(1)}} \right) \\
 &\quad + \left[\frac{1}{2} \left(\frac{(1-\delta)\lambda}{\mu+\lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1-\delta)\lambda^2 - (\mu+\lambda\delta)(-\lambda)}{((1-\delta)\lambda)^2} \right) \right]^2 \left(\frac{\delta(1-\delta)}{n_i} \right).
 \end{aligned} \tag{5-22}$$

Thus, the estimate of $Var(\hat{\theta}_i)$ can be expressed as

$$\begin{aligned}
 Var(\hat{\theta}_i) &= \left[\frac{1}{2} \left(\frac{(1-\hat{\delta}_i)\hat{\lambda}_i}{\hat{\mu}_i + \hat{\lambda}_i\hat{\delta}_i} \right)^{\frac{1}{2}} \left(\frac{1}{(1-\hat{\delta}_i)\hat{\lambda}_i} \right) \right]^2 \left(\frac{\hat{\mu}_i^3}{\hat{\lambda}_i n_{i(1)}} \right) \\
 &\quad + \left[\frac{1}{2} \left(\frac{(1-\hat{\delta}_i)\hat{\lambda}_i}{\hat{\mu}_i + \hat{\lambda}_i\hat{\delta}_i} \right)^{\frac{1}{2}} \left(\frac{(1-\hat{\delta}_i)\hat{\lambda}_i\hat{\delta}_i - (\hat{\mu}_i + \hat{\lambda}_i\hat{\delta}_i)(1-\hat{\delta}_i)}{((1-\hat{\delta}_i)\hat{\lambda}_i)^2} \right) \right]^2 \left(\frac{2\hat{\lambda}_i^2}{n_{i(1)}} \right) \\
 &\quad + \left[\frac{1}{2} \left(\frac{(1-\hat{\delta}_i)\hat{\lambda}_i}{\hat{\mu}_i + \hat{\lambda}_i\hat{\delta}_i} \right)^{\frac{1}{2}} \left(\frac{(1-\hat{\delta}_i)\hat{\lambda}_i^2 - (\hat{\mu}_i + \hat{\lambda}_i\hat{\delta}_i)(-\hat{\lambda}_i)}{((1-\hat{\delta}_i)\hat{\lambda}_i)^2} \right) \right]^2 \left(\frac{\hat{\delta}_i(1-\hat{\delta}_i)}{n_i} \right)
 \end{aligned} \tag{5-23}$$

The estimated variance of the estimated difference between two coefficients of variation of delta-IG can be expressed as follows:

$$Var(\hat{\theta}_{il}) = Var(\hat{\theta}_i) + Var(\hat{\theta}_l), \tag{5-24}$$

where $cov(\hat{\theta}_i, \hat{\theta}_l) = 0$ with $i, l = 1, 2, \dots, k$ and $i \neq l$.

The $100(1 - \alpha)\%$ simultaneous confidence interval for θ_{il} using the normal approximation is

$$CI_{\theta_{il}}^{NA} = (\hat{\theta}_i - \hat{\theta}_l) \pm z_{1-\frac{\alpha}{2}} \sqrt{Var(\hat{\theta}_{ij})}, \tag{5-25}$$

where $i, l = 1, 2, \dots, k, i \neq l$, and $z_{1-\frac{\alpha}{2}}$ is the $(1 - \frac{\alpha}{2})$ th percentile of the standard normal distribution.

5.3 Simulation studies and results

The performance of the five proposed methods for constructing simultaneous confidence intervals for all pairwise differences of coefficients of variation under the delta-IG distribution was investigated through a Monte Carlo simulation study executed in the R programming language. The competing approaches were evaluated based on coverage probability (CP) and average width (AW) of the resulting confidence intervals. An appropriate method was identified based on two main criteria: An effective confidence interval method is expected to achieve coverage probabilities close to the nominal confidence level. Among methods with comparable coverage performance, the preferred method is the one that produces the shortest interval width. Each simulation scenario was replicated 5,000 times, using 5,000 pivotal quantities for the GCI and AGCI methods. Random samples for comparing all pairwise differences of the CVs were generated as $X_{ij} = (X_{i1}, X_{i2}, \dots, X_{in_i}), i = 1, 2, \dots, k$ where $X_{ij} \sim \text{delta-IG}(\delta_i, \mu_i, \lambda_i)$. We assume that samples from different populations are independent, which is standard in multi-sample inference and appropriate when data are collected from distinct groups. The proposed methods were assessed under various parameter configurations through three sample scenarios with $k = 3, 6$, and 10 . The shape parameters λ_i , the probabilities of zero values δ_i , and the sample sizes n_i were varied, while the mean parameters μ_i were fixed at 1 to isolate the effects of skewness and zero inflation on interval performance.

The results in Tables 5–1 to 5–6 and Figures 5–1 to 5–3 summarize the performance of the methods in terms of coverage probabilities (CPs) and average widths (AWs) for simultaneous confidence intervals across $k = 3, 6, 10$. Overall, most methods achieved CPs close to the nominal level of 0.95, but clear differences emerged when considering both accuracy and efficiency.

For $k = 3$, AGCI and MOVER provided the most reliable coverage across all zero-inflation levels. AGCI's strong performance is due to its bias-correction mechanism, which improves accuracy under skewness, while MOVER achieved comparable coverage with shorter intervals. GCI showed some undercoverage, and FCI, although highly conservative, produced excessively wide intervals.

As the number of populations increased ($k = 6$ and $k = 10$), similar patterns persisted. AGCI maintained stable and accurate coverage across all configurations, demonstrating robustness to increasing complexity. MOVER consistently produced narrower intervals with adequate coverage, though slight undercoverage appeared in some highly skewed settings. In contrast, GCI performance deteriorated with higher dimensions, and FCI remained overly conservative.

Overall, AGCI and MOVER emerged as the most effective methods. AGCI offers strong coverage stability, while MOVER provides superior efficiency through shorter intervals, making both methods suitable for constructing simultaneous confidence intervals under delta-IG distributions.

TABLE 5-1 The CPs of the 95% simultaneous confidence intervals for all pairwise differences between the coefficients of variation of three delta-IG distributions ($k=3$).

n_1, n_2, n_3	$\lambda_1, \lambda_2, \lambda_3$	$\delta_1, \delta_2, \delta_3$	Coverage Probabilities				
			GCI	AGCI	FCI	MOVER	NA
30,30, 30	0.5, 0.5, 0.5	0.1, 0.1, 0.1	0.9400	0.9460	1.0000	0.9503	1.0000
		0.3, 0.3, 0.3	0.9373	0.9570	1.0000	0.9613	1.0000
		0.5, 0.5, 0.5	0.9243	0.9605	0.9987	0.9663	1.0000
	1, 1, 1	0.1, 0.3, 0.5	0.9260	0.9528	0.9747	0.9553	0.9993
		0.1, 0.1, 0.1	0.9325	0.9497	1.0000	0.9537	0.9988
		0.3, 0.3, 0.3	0.9123	0.9565	1.0000	0.9602	1.0000
		0.5, 0.5, 0.5	0.8823	0.9555	0.9987	0.9617	1.0000
	5, 5, 5	0.1, 0.3, 0.5	0.9023	0.9545	0.9953	0.9568	0.9942
		0.1, 0.1, 0.1	0.8042	0.9452	1.0000	0.9472	0.9618
		0.3, 0.3, 0.3	0.6430	0.9542	1.0000	0.9543	0.9782
		0.5, 0.5, 0.5	0.5595	0.9465	0.9987	0.9485	0.9947
	0.5, 1, 5	0.1, 0.3, 0.5	0.6330	0.9488	1.0000	0.9485	0.9715
		0.1, 0.1, 0.1	0.9285	0.9490	0.9441	0.9517	0.9628
		0.3, 0.3, 0.3	0.9102	0.9550	0.9082	0.9587	0.9732
		0.5, 0.5, 0.5	0.8835	0.9548	0.8071	0.9580	0.9830
		0.1, 0.3, 0.5	0.8928	0.9552	0.9681	0.9583	0.9922
50, 50, 50	0.5, 0.5, 0.5	0.1, 0.1, 0.1	0.9440	0.9505	1.0000	0.9525	0.9992
		0.3, 0.3, 0.3	0.9367	0.9555	1.0000	0.9597	0.9998
		0.5, 0.5, 0.5	0.9158	0.9538	1.0000	0.9573	1.0000
	1, 1, 1	0.1, 0.3, 0.5	0.9312	0.9540	0.9967	0.9577	0.9932
		0.1, 0.1, 0.1	0.9317	0.9498	1.0000	0.9532	0.9853
		0.3, 0.3, 0.3	0.9052	0.9523	1.0000	0.9557	0.9952
		0.5, 0.5, 0.5	0.8718	0.9555	1.0000	0.9600	0.9998
	5, 5, 5	0.1, 0.3, 0.5	0.9033	0.9510	1.0000	0.9525	0.9842
		0.1, 0.1, 0.1	0.7872	0.9462	1.0000	0.9485	0.9582
		0.3, 0.3, 0.3	0.6147	0.9492	1.0000	0.9502	0.9632
		0.5, 0.5, 0.5	0.5350	0.9495	1.0000	0.9500	0.9780
	0.5, 1, 5	0.1, 0.3, 0.5	0.6172	0.9513	1.0000	0.9512	0.9675
		0.1, 0.1, 0.1	0.9272	0.9487	0.9927	0.9477	0.9547
		0.3, 0.3, 0.3	0.9038	0.9547	0.9774	0.9557	0.9662
		0.5, 0.5, 0.5	0.8762	0.9510	0.9508	0.9535	0.9740
		0.1, 0.3, 0.5	0.8867	0.9512	0.9947	0.9535	0.9803
100, 100, 100	0.5, 0.5, 0.5	0.1, 0.1, 0.1	0.9428	0.9500	1.0000	0.9510	0.9777
		0.3, 0.3, 0.3	0.9347	0.9558	1.0000	0.9570	0.9865
		0.5, 0.5, 0.5	0.9180	0.9508	1.0000	0.9522	0.9967
	1, 1, 1	0.1, 0.3, 0.5	0.9282	0.9535	1.0000	0.9535	0.9828
		0.1, 0.1, 0.1	0.9327	0.9480	1.0000	0.9503	0.9682
		0.3, 0.3, 0.3	0.9018	0.9542	1.0000	0.9542	0.9770
		0.5, 0.5, 0.5	0.8775	0.9528	1.0000	0.9553	0.9852
	5, 5, 5	0.1, 0.3, 0.5	0.8998	0.9525	1.0000	0.9532	0.9743
		0.1, 0.1, 0.1	0.7905	0.9475	1.0000	0.9475	0.9525
		0.3, 0.3, 0.3	0.5847	0.9487	1.0000	0.9490	0.9567
		0.5, 0.5, 0.5	0.5153	0.9487	1.0000	0.9492	0.9620
	0.5, 1, 5	0.1, 0.3, 0.5	0.6005	0.9453	1.0000	0.9475	0.9575
		0.1, 0.1, 0.1	0.9297	0.9483	0.9980	0.9493	0.9550

TABLE 5-1 (CONTINUED)

n_1, n_2, n_3	$\lambda_1, \lambda_2, \lambda_3$	$\delta_1, \delta_2, \delta_3$	Coverage Probabilities				
			GCI	AGCI	FCI	MOVER	NA
100, 100, 100	0.5, 1, 5	0.3, 0.3, 0.3	0.9005	0.9545	0.9980	0.9543	0.9643
		0.5, 0.5, 0.5	0.8770	0.9543	0.9980	0.9538	0.9672
		0.1, 0.3, 0.5	0.8825	0.9538	1.0000	0.9547	0.9692
30, 50, 100	0.5, 0.5, 0.5	0.1, 0.1, 0.1	0.9372	0.9458	0.9980	0.9483	0.9763
		0.3, 0.3, 0.3	0.9333	0.9573	0.9973	0.9593	0.9852
		0.5, 0.5, 0.5	0.9188	0.9553	0.9907	0.9582	0.9947
	1, 1, 1	0.1, 0.3, 0.5	0.9317	0.9523	1.0000	0.9558	0.9993
		0.1, 0.1, 0.1	0.9308	0.9500	0.9987	0.9512	0.9725
		0.3, 0.3, 0.3	0.8990	0.9527	0.9987	0.9552	0.9778
	5, 5, 5	0.5, 0.5, 0.5	0.8757	0.9542	0.9947	0.9573	0.9865
		0.1, 0.3, 0.5	0.9037	0.9523	1.0000	0.9562	0.9932
		0.1, 0.1, 0.1	0.7852	0.9517	1.0000	0.9522	0.9575
	0.5, 1, 5	0.3, 0.3, 0.3	0.6043	0.9393	1.0000	0.9405	0.9572
		0.5, 0.5, 0.5	0.5292	0.9442	0.9987	0.9463	0.9712
		0.1, 0.3, 0.5	0.6435	0.9477	1.0000	0.9492	0.9643
		0.1, 0.1, 0.1	0.9280	0.9462	0.9581	0.9455	0.9477
		0.3, 0.3, 0.3	0.9145	0.9542	0.9235	0.9553	0.9585
		0.5, 0.5, 0.5	0.8920	0.9515	0.8463	0.9522	0.9687
		0.1, 0.3, 0.5	0.9055	0.9528	0.9228	0.9532	0.9633

Note: Bold values indicate coverage probabilities (CP) that satisfy the nominal confidence level of 0.95 ($CP \geq 0.945$, rounded to 0.950).

TABLE 5-2 The AWs of the 95% simultaneous confidence intervals for all pairwise differences between the coefficients of variation of three delta-IG distributions ($k=3$).

n_1, n_2, n_3	$\lambda_1, \lambda_2, \lambda_3$	$\delta_1, \delta_2, \delta_3$	Average Widths				
			GCI	AGCI	FCI	MOVER	NA
30,30, 30	0.5, 0.5, 0.5	0.1, 0.1, 0.1	0.9400	0.9460	1.0000	0.9503	1.0000
		0.3, 0.3, 0.3	0.9373	0.9570	1.0000	0.9613	1.0000
		0.5, 0.5, 0.5	0.9243	0.9605	0.9987	0.9663	1.0000
	1, 1, 1	0.1, 0.3, 0.5	0.9260	0.9528	0.9747	0.9553	0.9993
		0.1, 0.1, 0.1	0.9325	0.9497	1.0000	0.9537	0.9988
		0.3, 0.3, 0.3	0.9123	0.9565	1.0000	0.9602	1.0000
		0.5, 0.5, 0.5	0.8823	0.9555	0.9987	0.9617	1.0000
	5, 5, 5	0.1, 0.3, 0.5	0.9023	0.9545	0.9953	0.9568	0.9942
		0.1, 0.1, 0.1	0.8042	0.9452	1.0000	0.9472	0.9618
		0.3, 0.3, 0.3	0.6430	0.9542	1.0000	0.9543	0.9782
		0.5, 0.5, 0.5	0.5595	0.9465	0.9987	0.9485	0.9947
	0.5, 1, 5	0.1, 0.3, 0.5	0.6330	0.9488	1.0000	0.9485	0.9715
		0.1, 0.1, 0.1	0.9285	0.9490	0.9441	0.9517	0.9628
		0.3, 0.3, 0.3	0.9102	0.9550	0.9082	0.9587	0.9732
		0.5, 0.5, 0.5	0.8835	0.9548	0.8071	0.9580	0.9830
		0.1, 0.3, 0.5	0.8928	0.9552	0.9681	0.9583	0.9922
50, 50, 50	0.5, 0.5, 0.5	0.1, 0.1, 0.1	0.9440	0.9505	1.0000	0.9525	0.9992
		0.3, 0.3, 0.3	0.9367	0.9555	1.0000	0.9597	0.9998
		0.5, 0.5, 0.5	0.9158	0.9538	1.0000	0.9573	1.0000
	1, 1, 1	0.1, 0.3, 0.5	0.9312	0.9540	0.9967	0.9577	0.9932
		0.1, 0.1, 0.1	0.9317	0.9498	1.0000	0.9532	0.9853
		0.3, 0.3, 0.3	0.9052	0.9523	1.0000	0.9557	0.9952
		0.5, 0.5, 0.5	0.8718	0.9555	1.0000	0.9600	0.9998
	5, 5, 5	0.1, 0.3, 0.5	0.9033	0.9510	1.0000	0.9525	0.9842
		0.1, 0.1, 0.1	0.7872	0.9462	1.0000	0.9485	0.9582
		0.3, 0.3, 0.3	0.6147	0.9492	1.0000	0.9502	0.9632
		0.5, 0.5, 0.5	0.5350	0.9495	1.0000	0.9500	0.9780
	0.5, 1, 5	0.1, 0.3, 0.5	0.6172	0.9513	1.0000	0.9512	0.9675
		0.1, 0.1, 0.1	0.9272	0.9487	0.9927	0.9477	0.9547
		0.3, 0.3, 0.3	0.9038	0.9547	0.9774	0.9557	0.9662
		0.5, 0.5, 0.5	0.8762	0.9510	0.9508	0.9535	0.9740
		0.1, 0.3, 0.5	0.8867	0.9512	0.9947	0.9535	0.9803
100, 100, 100	0.5, 0.5, 0.5	0.1, 0.1, 0.1	0.9428	0.9500	1.0000	0.9510	0.9777
		0.3, 0.3, 0.3	0.9347	0.9558	1.0000	0.9570	0.9865
		0.5, 0.5, 0.5	0.9180	0.9508	1.0000	0.9522	0.9967
	1, 1, 1	0.1, 0.3, 0.5	0.9282	0.9535	1.0000	0.9535	0.9828
		0.1, 0.1, 0.1	0.9327	0.9480	1.0000	0.9503	0.9682
		0.3, 0.3, 0.3	0.9018	0.9542	1.0000	0.9542	0.9770
		0.5, 0.5, 0.5	0.8775	0.9528	1.0000	0.9553	0.9852
	5, 5, 5	0.1, 0.3, 0.5	0.8998	0.9525	1.0000	0.9532	0.9743
		0.1, 0.1, 0.1	0.7905	0.9475	1.0000	0.9475	0.9525
		0.3, 0.3, 0.3	0.5847	0.9487	1.0000	0.9490	0.9567
		0.5, 0.5, 0.5	0.5153	0.9487	1.0000	0.9492	0.9620
	0.5, 1, 5	0.1, 0.3, 0.5	0.6005	0.9453	1.0000	0.9475	0.9575
		0.1, 0.1, 0.1	0.9297	0.9483	0.9980	0.9493	0.9550

TABLE 5-2 (CONTINUED)

n_1, n_2, n_3	$\lambda_1, \lambda_2, \lambda_3$	$\delta_1, \delta_2, \delta_3$	Average Widths				
			GCI	AGCI	FCI	MOVER	NA
100, 100, 100	0.5, 1, 5	0.3, 0.3, 0.3	0.9005	0.9545	0.9980	0.9543	0.9643
		0.5, 0.5, 0.5	0.8770	0.9543	0.9980	0.9538	0.9672
		0.1, 0.3, 0.5	0.8825	0.9538	1.0000	0.9547	0.9692
30, 50, 100	0.5, 0.5, 0.5	0.1, 0.1, 0.1	0.9372	0.9458	0.9980	0.9483	0.9763
		0.3, 0.3, 0.3	0.9333	0.9573	0.9973	0.9593	0.9852
		0.5, 0.5, 0.5	0.9188	0.9553	0.9907	0.9582	0.9947
	1, 1, 1	0.1, 0.3, 0.5	0.9317	0.9523	1.0000	0.9558	0.9993
		0.1, 0.1, 0.1	0.9308	0.9500	0.9987	0.9512	0.9725
		0.3, 0.3, 0.3	0.8990	0.9527	0.9987	0.9552	0.9778
		0.5, 0.5, 0.5	0.8757	0.9542	0.9947	0.9573	0.9865
		0.1, 0.3, 0.5	0.9037	0.9523	1.0000	0.9562	0.9932
	5, 5, 5	0.1, 0.1, 0.1	0.7852	0.9517	1.0000	0.9522	0.9575
		0.3, 0.3, 0.3	0.6043	0.9393	1.0000	0.9405	0.9572
		0.5, 0.5, 0.5	0.5292	0.9442	0.9987	0.9463	0.9712
	0.5, 1, 5	0.1, 0.3, 0.5	0.6435	0.9477	1.0000	0.9492	0.9643
		0.1, 0.1, 0.1	0.9280	0.9462	0.9581	0.9455	0.9477
		0.3, 0.3, 0.3	0.9145	0.9542	0.9235	0.9553	0.9585
		0.5, 0.5, 0.5	0.8920	0.9515	0.8463	0.9522	0.9687
		0.1, 0.3, 0.5	0.9055	0.9528	0.9228	0.9532	0.9633

Note: The shortest AWs are in italic.

TABLE 5-3 The CPs of the 95% simultaneous confidence intervals for all pairwise differences between the coefficients of variation of six delta-IG distributions ($k=6$).

n_i^6	λ_i^6	δ_i^6	Coverage Probabilities				
			GCI	AGCI	FCI	MOVER	NA
30^6	0.5^6	0.1^6	0.9449	0.9519	0.9999	0.9556	1.0000
		0.3^6	0.9334	0.9533	0.9999	0.9574	1.0000
		0.5^6	0.9265	0.9606	0.9988	0.9654	1.0000
	1^6	$0.1^2, 0.3^2, 0.5^2$	0.9324	0.9551	0.9860	0.9595	0.9995
		0.1^6	0.9346	0.9520	0.9999	0.9560	0.9987
		0.3^6	0.9047	0.9535	1.0000	0.9572	1.0000
	5^6	0.5^6	0.8865	0.9586	0.9989	0.9634	1.0000
		$0.1^2, 0.3^2, 0.5^2$	0.9035	0.9562	0.9981	0.9600	0.9959
		0.1^6	0.8001	0.9514	0.9999	0.9534	0.9676
	$0.5^2, 1^2, 5^2$	0.3^6	0.6382	0.9473	1.0000	0.9494	0.9753
		0.5^6	0.5646	0.9496	0.9996	0.9523	0.9942
		$0.1^2, 0.3^2, 0.5^2$	0.6377	0.9495	1.0000	0.9513	0.9756
		0.1^6	0.9242	0.9492	0.9558	0.9522	0.9685
		0.3^6	0.8871	0.9514	0.9321	0.9534	0.9777
		0.5^6	0.8636	0.9559	0.8361	0.9587	0.9868
		$0.1^2, 0.3^2, 0.5^2$	0.8700	0.9526	0.9771	0.9556	0.9912
50^6	0.5^6	0.1^6	0.9432	0.9500	1.0000	0.9528	0.9987
		0.3^6	0.9306	0.9506	1.0000	0.9532	0.9998
		0.5^6	0.9249	0.9578	0.9997	0.9619	1.0000
	1^6	$0.1^2, 0.3^2, 0.5^2$	0.9314	0.9537	0.9983	0.9556	0.9960
		0.1^6	0.9339	0.9507	1.0000	0.9527	0.9825
		0.3^6	0.9026	0.9505	1.0000	0.9532	0.9940
	5^6	0.5^6	0.8833	0.9565	1.0000	0.9598	0.9998
		$0.1^2, 0.3^2, 0.5^2$	0.9001	0.9519	0.9999	0.9545	0.9858
		0.1^6	0.7888	0.9499	1.0000	0.9515	0.9603
	$0.5^2, 1^2, 5^2$	0.3^6	0.6102	0.9441	1.0000	0.9455	0.9604
		0.5^6	0.5317	0.9462	0.9999	0.9482	0.9759
		$0.1^2, 0.3^2, 0.5^2$	0.6177	0.9468	1.0000	0.9482	0.9641
		0.1^6	0.9232	0.9502	0.9902	0.9517	0.9645
		0.3^6	0.8866	0.9500	0.9811	0.9517	0.9693
		0.5^6	0.8583	0.9542	0.9670	0.9561	0.9788
		$0.1^2, 0.3^2, 0.5^2$	0.8682	0.9516	0.9967	0.9537	0.9822
100^6	0.5^6	0.1^6	0.9443	0.9507	1.0000	0.9520	0.9774
		0.3^6	0.9362	0.9565	1.0000	0.9581	0.9896
		0.5^6	0.9216	0.9530	1.0000	0.9553	0.9985
	1^6	$0.1^2, 0.3^2, 0.5^2$	0.9308	0.9517	0.9997	0.9537	0.9816
		0.1^6	0.9340	0.9506	1.0000	0.9517	0.9676
		0.3^6	0.9104	0.9547	1.0000	0.9564	0.9754
	5^6	0.5^6	0.8727	0.9507	1.0000	0.9523	0.9845
		$0.1^2, 0.3^2, 0.5^2$	0.8985	0.9523	1.0000	0.9534	0.9729
		0.1^6	0.7800	0.9488	1.0000	0.9498	0.9541
	$0.5^2, 1^2, 5^2$	0.3^6	0.6030	0.9498	1.0000	0.9508	0.9577
		0.5^6	0.5067	0.9476	1.0000	0.9483	0.9607
		$0.1^2, 0.3^2, 0.5^2$	0.6019	0.9492	1.0000	0.9499	0.9573
		0.1^6	0.9233	0.9502	0.9985	0.9511	0.9592

TABLE 5-3 (CONTINUED)

n_i^6	λ_i^6	δ_i^6	Coverage Probabilities				
			GCI	AGCI	FCI	MOVER	NA
100^6	$0.5^2, 1^2, 5^2$	0.3^6	0.8904	0.9544	0.9977	0.9549	0.9661
		0.5^6	0.8534	0.9509	0.9959	0.9516	0.9685
		$0.1^2, 0.3^2, 0.5^2$	0.8618	0.9494	0.9997	0.9503	0.9668
$30^3, 50^3$	0.5^6	0.1^6	0.9455	0.9530	0.9995	0.9561	0.9970
		0.3^6	0.9332	0.9534	0.9995	0.9561	0.9998
		0.5^6	0.9210	0.9566	0.9977	0.9612	1.0000
		$0.1^2, 0.3^2, 0.5^2$	0.9307	0.9530	0.9997	0.9568	0.9997
	1^6	0.1^6	0.9369	0.9547	0.9996	0.9578	0.9879
		0.3^6	0.9022	0.9517	0.9993	0.9550	0.9936
		0.5^6	0.8823	0.9573	0.9989	0.9615	0.9997
		$0.1^2, 0.3^2, 0.5^2$	0.9021	0.9530	1.0000	0.9566	0.9965
	5^6	0.1^6	0.7970	0.9512	0.9999	0.9526	0.9625
		0.3^6	0.6273	0.9457	0.9999	0.9470	0.9682
		0.5^6	0.5449	0.9502	0.9995	0.9524	0.9841
		$0.1^2, 0.3^2, 0.5^2$	0.6293	0.9485	1.0000	0.9502	0.9716
	$0.5^2, 1^2, 5^2$	0.1^6	0.9263	0.9524	0.9554	0.9536	0.9627
		0.3^6	0.8909	0.9513	0.9248	0.9532	0.9676
		0.5^6	0.8617	0.9555	0.8302	0.9581	0.9769
		$0.1^2, 0.3^2, 0.5^2$	0.8730	0.9499	0.9573	0.9523	0.9765
	0.5^6	0.1^6	0.9454	0.9532	0.9973	0.9556	0.9810
		0.3^6	0.9310	0.9524	0.9953	0.9543	0.9858
		0.5^6	0.9226	0.9557	0.9899	0.9581	0.9948
		$0.1^2, 0.3^2, 0.5^2$	0.9321	0.9530	1.0000	0.9560	0.9993
	1^6	0.1^6	0.9377	0.9544	0.9987	0.9566	0.9744
		0.3^6	0.9005	0.9503	0.9967	0.9525	0.9783
		0.5^6	0.8819	0.9551	0.9944	0.9583	0.9885
		$0.1^2, 0.3^2, 0.5^2$	0.9110	0.9552	0.9993	0.9578	0.9929
	5^6	0.1^6	0.7952	0.9516	0.9997	0.9525	0.9570
		0.3^6	0.6105	0.9465	0.9995	0.9474	0.9628
		0.5^6	0.5319	0.9474	0.9977	0.9485	0.9726
		$0.1^2, 0.3^2, 0.5^2$	0.6462	0.9502	0.9997	0.9512	0.9652
	$0.5^2, 1^2, 5^2$	0.1^6	0.9270	0.9512	0.9659	0.9522	0.9538
		0.3^6	0.8880	0.9477	0.9397	0.9488	0.9597
		0.5^6	0.8655	0.9538	0.8649	0.9551	0.9715
		$0.1^2, 0.3^2, 0.5^2$	0.8865	0.9533	0.9512	0.9538	0.9691

Note: Bold values indicate coverage probabilities (CP) that satisfy the nominal confidence level of 0.95 ($CP \geq 0.945$, rounded to 0.950).

TABLE 5-4 The AWs of the 95% simultaneous confidence intervals for all pairwise differences between the coefficients of variation of six delta-IG distributions ($k=6$).

n_i^6	λ_i^6	δ_i^6	Average Widths				
			GCI	AGCI	FCI	MOVER	NA
30 ⁶	0.5 ⁶	0.1 ⁶	2.4212	2.4684	2.9491	2.5845	4.2278
		0.3 ⁶	3.6697	3.8232	2.7222	4.0154	7.5072
		0.5 ⁶	6.6199	6.9914	2.3427	7.3844	14.1515
	1 ⁶	0.1 ² , 0.3 ² , 0.5 ²	4.2925	4.4921	2.7562	4.6964	9.0359
		0.1 ⁶	1.1849	1.2445	2.9822	1.3009	1.3873
		0.3 ⁶	1.6067	1.8018	2.7605	1.8878	2.4812
	5 ⁶	0.5 ⁶	2.8026	3.2214	2.3832	3.3910	5.8556
		0.1 ² , 0.3 ² , 0.5 ²	1.8869	2.1284	2.8091	2.2182	3.3395
		0.1 ⁶	0.3572	0.5315	3.0465	0.5467	0.5285
	0.5 ² , 1 ² , 5 ²	0.3 ⁶	0.3916	0.8145	2.8388	0.8319	0.8101
		0.5 ⁶	0.5547	1.3002	2.4621	1.3336	1.3258
		0.1 ² , 0.3 ² , 0.5 ²	0.4368	0.9087	2.8933	0.9270	0.9161
		0.1 ⁶	1.4094	1.4852	3.0385	1.5272	2.2396
		0.3 ⁶	1.9971	2.2215	2.8265	2.2879	3.9002
		0.5 ⁶	3.5053	3.9744	2.4503	4.1115	7.7213
		0.1 ² , 0.3 ² , 0.5 ²	1.5956	1.8782	2.7862	1.9494	2.8018
50 ⁶	0.5 ⁶	0.1 ⁶	1.5115	1.5465	3.8852	1.5995	1.7344
		0.3 ⁶	2.0655	2.1845	3.6713	2.2700	3.0099
		0.5 ⁶	3.5082	3.7606	3.5032	3.9325	7.0382
	1 ⁶	0.1 ² , 0.3 ² , 0.5 ²	2.3895	2.5361	3.7603	2.6259	4.0400
		0.1 ⁶	0.8253	0.8712	3.9309	0.8976	0.8638
		0.3 ⁶	1.0425	1.1948	3.7104	1.2345	1.2177
	5 ⁶	0.5 ⁶	1.5626	1.8779	3.5388	1.9519	2.2778
		0.1 ² , 0.3 ² , 0.5 ²	1.1554	1.3393	3.8058	1.3800	1.4978
		0.1 ⁶	0.2595	0.3990	4.0022	0.4059	0.3978
	0.5 ² , 1 ² , 5 ²	0.3 ⁶	0.2731	0.6043	3.7884	0.6122	0.6021
		0.5 ⁶	0.3569	0.9274	3.6219	0.9421	0.9216
		0.1 ² , 0.3 ² , 0.5 ²	0.2972	0.6613	3.9106	0.6695	0.6593
		0.1 ⁶	0.9262	0.9843	3.9912	1.0035	1.0607
		0.3 ⁶	1.2064	1.3772	3.7781	1.4073	1.6919
		0.5 ⁶	1.9369	2.2736	3.6144	2.3326	3.5791
		0.1 ² , 0.3 ² , 0.5 ²	1.0208	1.2309	3.7889	1.2631	1.2988
100 ⁶	0.5 ⁶	0.1 ⁶	0.9405	0.9641	5.2183	0.9829	0.9549
		0.3 ⁶	1.2056	1.2877	5.1719	1.3176	1.2837
		0.5 ⁶	1.7679	1.9470	5.1706	2.0046	2.0843
	1 ⁶	0.1 ² , 0.3 ² , 0.5 ²	1.3249	1.4275	5.2408	1.4585	1.4859
		0.1 ⁶	0.5427	0.5753	5.2270	0.5847	0.5713
		0.3 ⁶	0.6616	0.7697	5.1925	0.7836	0.7641
	5 ⁶	0.5 ⁶	0.9140	1.1384	5.1718	1.1641	1.1325
		0.1 ² , 0.3 ² , 0.5 ²	0.7141	0.8442	5.2906	0.8584	0.8404
		0.1 ⁶	0.1747	0.2762	5.2729	0.2785	0.2759
	0.5 ² , 1 ² , 5 ²	0.3 ⁶	0.1789	0.4142	5.2504	0.4169	0.4137
		0.5 ⁶	0.2220	0.6232	5.2569	0.6281	0.6214
		0.1 ² , 0.3 ² , 0.5 ²	0.1926	0.4500	5.3863	0.4528	0.4495
		0.1 ⁶	0.5944	0.6350	5.2931	0.6418	0.6333

TABLE 5-4 (CONTINUED)

n_i^6	λ_i^6	δ_i^6	Average Widths				
			GCI	AGCI	FCI	MOVER	NA
100^6	$0.5^2, 1^2, 5^2$	0.3^6	0.7430	<i>0.8605</i>	5.2601	0.8709	0.8618
		0.5^6	1.0529	<i>1.2849</i>	5.2668	1.3050	1.3431
		$0.1^2, 0.3^2, 0.5^2$	0.6457	0.7909	5.2703	0.8023	<i>0.7863</i>
$30^3, 50^3$	0.5^6	0.1^6	1.9512	<i>1.9947</i>	3.4944	2.0733	3.0140
		0.3^6	2.8789	<i>3.0146</i>	3.2774	3.1434	5.5913
		0.5^6	5.1375	<i>5.4460</i>	<i>2.9741</i>	5.7120	10.8469
		$0.1^2, 0.3^2, 0.5^2$	2.9330	<i>3.0822</i>	3.2485	3.2180	5.6238
	1^6	0.1^6	1.0062	<i>1.0599</i>	3.5394	1.0988	1.1286
		0.3^6	1.3323	<i>1.5083</i>	3.3255	1.5669	1.8402
		0.5^6	2.2014	<i>2.5747</i>	3.0233	2.6888	4.1214
		$0.1^2, 0.3^2, 0.5^2$	1.3611	<i>1.5491</i>	3.2914	1.6105	1.8274
	5^6	0.1^6	0.3092	0.4671	3.6380	0.4775	<i>0.4653</i>
		0.3^6	0.3354	0.7146	3.4534	0.7265	<i>0.7119</i>
		0.5^6	0.4575	<i>1.1207</i>	3.1392	1.1432	1.1318
		$0.1^2, 0.3^2, 0.5^2$	0.3488	0.7321	3.3708	0.7451	<i>0.7288</i>
	$0.5^2, 1^2, 5^2$	0.1^6	1.3076	<i>1.3741</i>	3.6728	1.4049	2.0913
		0.3^6	1.8777	<i>2.0686</i>	3.4740	2.1179	3.7061
		0.5^6	3.2596	<i>3.6577</i>	<i>3.1517</i>	3.7598	7.1460
		$0.1^2, 0.3^2, 0.5^2$	1.4530	<i>1.6647</i>	3.3242	1.7141	2.4161
	0.5^6	0.1^6	<i>1.6317</i>	<i>1.6685</i>	4.1777	1.7183	2.3624
		0.3^6	2.3444	<i>2.4681</i>	4.0498	2.5497	4.1878
		0.5^6	4.0150	4.2951	3.8385	4.4594	8.4754
		$0.1^2, 0.3^2, 0.5^2$	2.0825	<i>2.1964</i>	4.0586	2.2812	3.1723
	1^6	0.1^6	0.8591	<i>0.9066</i>	4.2449	0.9314	0.9526
		0.3^6	1.1208	<i>1.2774</i>	4.1259	1.3148	1.5239
		0.5^6	1.7929	<i>2.1229</i>	3.9235	2.1949	3.4241
		$0.1^2, 0.3^2, 0.5^2$	1.0478	<i>1.1914</i>	4.1071	1.2317	1.2436
	5^6	0.1^6	0.2673	0.4076	4.4254	0.4142	<i>0.4067</i>
		0.3^6	0.2856	0.6214	4.2933	0.6289	<i>0.6198</i>
		0.5^6	0.3839	<i>0.9664</i>	4.1172	0.9805	0.9769
		$0.1^2, 0.3^2, 0.5^2$	0.2867	0.5877	4.2127	0.5969	<i>0.5854</i>
	$0.5^2, 1^2, 5^2$	0.1^6	1.2378	<i>1.2912</i>	4.4718	1.3134	1.9578
		0.3^6	1.7778	<i>1.9399</i>	4.3462	1.9760	3.4535
		0.5^6	3.0036	<i>3.3484</i>	4.1524	3.4211	6.7533
		$0.1^2, 0.3^2, 0.5^2$	1.3307	<i>1.4830</i>	4.2728	1.5171	2.1928
$30^2, 50^2, 100^2$	0.5^6	0.1^6	<i>1.6317</i>	<i>1.6685</i>	4.1777	1.7183	2.3624
		0.3^6	2.3444	<i>2.4681</i>	4.0498	2.5497	4.1878
		0.5^6	4.0150	4.2951	3.8385	4.4594	8.4754
		$0.1^2, 0.3^2, 0.5^2$	2.0825	<i>2.1964</i>	4.0586	2.2812	3.1723
	1^6	0.1^6	0.8591	<i>0.9066</i>	4.2449	0.9314	0.9526
		0.3^6	1.1208	<i>1.2774</i>	4.1259	1.3148	1.5239
		0.5^6	1.7929	<i>2.1229</i>	3.9235	2.1949	3.4241
		$0.1^2, 0.3^2, 0.5^2$	1.0478	<i>1.1914</i>	4.1071	1.2317	1.2436
	5^6	0.1^6	0.2673	0.4076	4.4254	0.4142	<i>0.4067</i>
		0.3^6	0.2856	0.6214	4.2933	0.6289	<i>0.6198</i>
		0.5^6	0.3839	<i>0.9664</i>	4.1172	0.9805	0.9769
		$0.1^2, 0.3^2, 0.5^2$	0.2867	0.5877	4.2127	0.5969	<i>0.5854</i>
	$0.5^2, 1^2, 5^2$	0.1^6	1.2378	<i>1.2912</i>	4.4718	1.3134	1.9578
		0.3^6	1.7778	<i>1.9399</i>	4.3462	1.9760	3.4535
		0.5^6	3.0036	<i>3.3484</i>	4.1524	3.4211	6.7533
		$0.1^2, 0.3^2, 0.5^2$	1.3307	<i>1.4830</i>	4.2728	1.5171	2.1928

Note: The shortest AWs are in italic.

TABLE 5-5 The CPs of the 95% simultaneous confidence intervals for all pairwise differences between the coefficients of variation of ten delta-IG distributions ($k=10$).

n_i^{10}	λ_i^{10}	δ_i^{10}	Coverage Probabilities				
			GCI	AGCI	FCI	MOVER	NA
$30^5, 50^5$	0.5^{10}	$0.1^5, 0.3^5$	0.9387	0.9522	1.0000	0.9553	0.9999
		$0.3^5, 0.5^5$	0.9257	0.9536	0.9996	0.9577	1.0000
		$0.1^6, 0.3^2, 0.5^2$	0.9388	0.9547	0.9994	0.9581	0.9988
		$0.1^2, 0.3^2, 0.5^6$	0.9277	0.9554	0.9963	0.9586	0.9998
	1^{10}	$0.1^5, 0.3^5$	0.9236	0.9543	0.9998	0.9577	0.9961
		$0.3^5, 0.5^5$	0.8910	0.9538	1.0000	0.9575	0.9999
		$0.1^6, 0.3^2, 0.5^2$	0.9163	0.9531	0.9999	0.9564	0.9939
		$0.1^2, 0.3^2, 0.5^6$	0.8913	0.9525	0.9992	0.9559	0.9981
	5^{10}	$0.1^5, 0.3^5$	0.7116	0.9487	1.0000	0.9505	0.9653
		$0.3^5, 0.5^5$	0.5781	0.9456	0.9999	0.9473	0.9751
		$0.1^6, 0.3^2, 0.5^2$	0.6947	0.9478	1.0000	0.9489	0.9669
		$0.1^2, 0.3^2, 0.5^6$	0.5903	0.9468	0.9999	0.9480	0.9744
	$0.5^4, 1^3, 5^3$	$0.1^5, 0.3^5$	0.9036	0.9522	0.9605	0.9541	0.9696
		$0.3^5, 0.5^5$	0.8719	0.9523	0.9193	0.9548	0.9792
		$0.1^6, 0.3^2, 0.5^2$	0.8910	0.9505	0.9608	0.9526	0.9736
		$0.1^2, 0.3^2, 0.5^6$	0.8725	0.9529	0.9343	0.9551	0.9814
$50^5, 100^5$	0.5^{10}	$0.1^5, 0.3^5$	0.9368	0.9499	0.9999	0.9521	0.9911
		$0.3^5, 0.5^5$	0.9267	0.9535	1.0000	0.9560	0.9992
		$0.1^6, 0.3^2, 0.5^2$	0.9346	0.9506	0.9999	0.9527	0.9898
		$0.1^2, 0.3^2, 0.5^6$	0.9268	0.9542	0.9996	0.9565	0.9969
	1^{10}	$0.1^5, 0.3^5$	0.9190	0.9505	0.9999	0.9524	0.9770
		$0.3^5, 0.5^5$	0.8918	0.9521	0.9999	0.9550	0.9898
		$0.1^6, 0.3^2, 0.5^2$	0.9154	0.9508	1.0000	0.9529	0.9778
		$0.1^2, 0.3^2, 0.5^6$	0.8937	0.9534	0.9998	0.9557	0.9854
	5^{10}	$0.1^5, 0.3^5$	0.7005	0.9474	1.0000	0.9487	0.9564
		$0.3^5, 0.5^5$	0.5650	0.9490	1.0000	0.9498	0.9634
		$0.1^6, 0.3^2, 0.5^2$	0.6878	0.9502	1.0000	0.9511	0.9587
		$0.1^2, 0.3^2, 0.5^6$	0.5808	0.9513	0.9999	0.9521	0.9649
	$0.5^4, 1^3, 5^3$	$0.1^5, 0.3^5$	0.8999	0.9495	0.9883	0.9503	0.9627
		$0.3^5, 0.5^5$	0.8739	0.9513	0.9752	0.9525	0.9715
		$0.1^6, 0.3^2, 0.5^2$	0.8905	0.9510	0.9874	0.9522	0.9648
		$0.1^2, 0.3^2, 0.5^6$	0.8737	0.9530	0.9830	0.9543	0.9713
$30^6, 50^2, 100^2$	0.5^{10}	$0.1^5, 0.3^5$	0.9391	0.9526	0.9978	0.9559	0.9919
		$0.3^5, 0.5^5$	0.9287	0.9553	0.9968	0.9579	0.9977
		$0.1^6, 0.3^2, 0.5^2$	0.9371	0.9517	1.0000	0.9551	0.9996
		$0.1^2, 0.3^2, 0.5^6$	0.9272	0.9548	0.9914	0.9578	0.9972
	1^{10}	$0.1^5, 0.3^5$	0.9188	0.9509	0.9986	0.9540	0.9867
		$0.3^5, 0.5^5$	0.8945	0.9559	0.9984	0.9592	0.9936
		$0.1^6, 0.3^2, 0.5^2$	0.9173	0.9504	0.9996	0.9539	0.9937
		$0.1^2, 0.3^2, 0.5^6$	0.8918	0.9547	0.9966	0.9579	0.9914
	5^{10}	$0.1^5, 0.3^5$	0.7117	0.9479	0.9996	0.9493	0.9631
		$0.3^5, 0.5^5$	0.5832	0.9489	0.9995	0.9505	0.9739
		$0.1^6, 0.3^2, 0.5^2$	0.7013	0.9467	0.9997	0.9485	0.9632
		$0.1^2, 0.3^2, 0.5^6$	0.5958	0.9489	0.9992	0.9503	0.9723
	$0.5^4, 1^3, 5^3$	$0.1^5, 0.3^5$	0.9034	0.9516	0.9591	0.9532	0.9698

TABLE 5-5 (CONTINUED)

n_i^{10}	λ_i^{10}	δ_i^{10}	Coverage Probabilities				
			GCI	AGCI	FCI	MOVER	NA
$30^6, 50^2, 100^2$	$0.5^4, 1^3, 5^3$	$0.3^5, 0.5^5$	0.8784	0.9537	0.9181	0.9559	0.9767
		$0.1^6, 0.3^2, 0.5^2$	0.8978	0.9498	0.9498	0.9518	0.9707
		$0.1^2, 0.3^2, 0.5^6$	0.8756	0.9525	0.9303	0.9550	0.9761
$30^2, 50^2, 100^6$	0.5^{10}	$0.1^5, 0.3^5$	0.9397	0.9524	0.9986	0.9543	0.9854
		$0.3^5, 0.5^5$	0.9256	0.9528	0.9990	0.9550	0.9945
		$0.1^6, 0.3^2, 0.5^2$	0.9367	0.9522	0.9986	0.9540	0.9846
	1^{10}	$0.1^2, 0.3^2, 0.5^6$	0.9281	0.9530	1.0000	0.9554	0.9992
		$0.1^5, 0.3^5$	0.9211	0.9521	0.9991	0.9541	0.9755
		$0.3^5, 0.5^5$	0.8885	0.9512	0.9991	0.9534	0.9840
	5^{10}	$0.1^6, 0.3^2, 0.5^2$	0.9178	0.9527	0.9993	0.9547	0.9759
		$0.1^2, 0.3^2, 0.5^6$	0.8973	0.9544	0.9995	0.9567	0.9888
		$0.1^5, 0.3^5$	0.7009	0.9492	0.9997	0.9505	0.9572
	$0.5^4, 1^3, 5^3$	$0.3^5, 0.5^5$	0.5613	0.9474	0.9996	0.9486	0.9634
		$0.1^6, 0.3^2, 0.5^2$	0.6899	0.9488	0.9998	0.9498	0.9580
		$0.1^2, 0.3^2, 0.5^6$	0.5942	0.9493	0.9999	0.9503	0.9638
		$0.1^5, 0.3^5$	0.9008	0.9506	0.9733	0.9518	0.9605
		$0.3^5, 0.5^5$	0.8718	0.9527	0.9440	0.9536	0.9671
		$0.1^6, 0.3^2, 0.5^2$	0.8902	0.9519	0.9720	0.9530	0.9611
		$0.1^2, 0.3^2, 0.5^6$	0.8765	0.9533	0.9591	0.9547	0.9692

Note: Bold values indicate coverage probabilities (CP) that satisfy the nominal confidence level of 0.95 ($CP \geq 0.945$, rounded to 0.950).

TABLE 5-6 The AWs of the 95% simultaneous confidence intervals for all pairwise differences between the coefficients of variation of ten delta-IG distributions ($k=10$).

n_i^{10}	λ_i^{10}	δ_i^{10}	Average Widths				
			GCI	AGCI	FCI	MOVER	NA
$30^5, 50^5$	0.5^{10}	$0.1^5, 0.3^5$	2.2293	2.3117	3.3804	2.4109	3.6063
		$0.3^5, 0.5^5$	3.6188	3.8242	3.0758	4.0074	7.2603
		$0.1^6, 0.3^2, 0.5^2$	2.4892	2.5945	3.3565	2.7030	4.4187
		$0.1^2, 0.3^2, 0.5^6$	3.6646	3.8763	3.1320	4.0518	7.5293
	1^{10}	$0.1^5, 0.3^5$	1.1115	1.2164	3.4085	1.2639	1.3012
		$0.3^5, 0.5^5$	1.5937	1.8507	3.1248	1.9312	2.4062
		$0.1^6, 0.3^2, 0.5^2$	1.2009	1.3347	3.3951	1.3853	1.5019
		$0.1^2, 0.3^2, 0.5^6$	1.6366	1.8945	3.1768	1.9719	2.5802
	5^{10}	$0.1^5, 0.3^5$	0.3164	0.5682	3.4991	0.5796	0.5656
		$0.3^5, 0.5^5$	0.3755	0.8749	3.2120	0.8909	0.8698
		$0.1^6, 0.3^2, 0.5^2$	0.3317	0.6226	3.4798	0.6344	0.6200
		$0.1^2, 0.3^2, 0.5^6$	0.3854	0.8757	3.2716	0.8913	0.8751
	$0.5^4, 1^3, 5^3$	$0.1^5, 0.3^5$	1.4488	1.5703	3.5041	1.6130	2.3519
		$0.3^5, 0.5^5$	2.1702	2.4424	3.2122	2.5188	4.3157
		$0.1^6, 0.3^2, 0.5^2$	1.4554	1.6088	3.4503	1.6541	2.4750
		$0.1^2, 0.3^2, 0.5^6$	2.0259	2.2984	3.2404	2.3726	3.8888
$50^5, 100^5$	0.5^{10}	$0.1^5, 0.3^5$	1.3567	1.4144	4.6549	1.4548	1.5359
		$0.3^5, 0.5^5$	1.9238	2.0714	4.4563	2.1423	2.5944
		$0.1^6, 0.3^2, 0.5^2$	1.4460	1.5204	4.5980	1.5636	1.6485
		$0.1^2, 0.3^2, 0.5^6$	1.9454	2.0936	4.5021	2.1611	2.7670
	1^{10}	$0.1^5, 0.3^5$	0.7436	0.8196	4.6878	0.8392	0.8136
		$0.3^5, 0.5^5$	0.9815	1.1691	4.5033	1.2015	1.1809
		$0.1^6, 0.3^2, 0.5^2$	0.7839	0.8794	4.6390	0.9003	0.8737
		$0.1^2, 0.3^2, 0.5^6$	0.9862	1.1736	4.5502	1.2045	1.2124
	5^{10}	$0.1^5, 0.3^5$	0.2207	0.4067	4.7890	0.4114	0.4058
		$0.3^5, 0.5^5$	0.2484	0.6139	4.6027	0.6203	0.6119
		$0.1^6, 0.3^2, 0.5^2$	0.2281	0.4397	4.7342	0.4446	0.4388
		$0.1^2, 0.3^2, 0.5^6$	0.2533	0.6108	4.6465	0.6170	0.6090
	$0.5^4, 1^3, 5^3$	$0.1^5, 0.3^5$	0.9400	1.0227	4.8101	1.0405	1.1376
		$0.3^5, 0.5^5$	1.2740	1.4632	4.6125	1.4940	1.8571
		$0.1^6, 0.3^2, 0.5^2$	0.9343	1.0379	4.7290	1.0564	1.1252
		$0.1^2, 0.3^2, 0.5^6$	1.1955	1.3850	4.6474	1.4146	1.6898
$30^6, 50^2, 100^2$	0.5^{10}	$0.1^5, 0.3^5$	2.2405	2.3228	3.6416	2.4135	3.8249
		$0.3^5, 0.5^5$	3.5792	3.7801	3.4234	3.9436	7.3199
		$0.1^6, 0.3^2, 0.5^2$	2.2185	2.3063	3.6213	2.4020	3.5283
		$0.1^2, 0.3^2, 0.5^6$	3.6231	3.8292	3.4300	3.9861	7.1745
	1^{10}	$0.1^5, 0.3^5$	1.1020	1.2045	3.6962	1.2479	1.3769
		$0.3^5, 0.5^5$	1.5885	1.8387	3.4781	1.9111	2.5410
		$0.1^6, 0.3^2, 0.5^2$	1.1060	1.2148	3.6734	1.2611	1.3100
		$0.1^2, 0.3^2, 0.5^6$	1.6303	1.8836	3.4963	1.9534	2.7389
	5^{10}	$0.1^5, 0.3^5$	0.3118	0.5554	3.8120	0.5659	0.5533
		$0.3^5, 0.5^5$	0.3715	0.8547	3.6193	0.8692	0.8548
		$0.1^6, 0.3^2, 0.5^2$	0.3158	0.5645	3.7686	0.5759	0.5619
		$0.1^2, 0.3^2, 0.5^6$	0.3792	0.8546	3.6381	0.8688	0.8587
	$0.5^4, 1^3, 5^3$	$0.1^5, 0.3^5$	1.5054	1.6166	3.8511	1.6610	2.5381

TABLE 5-6 (CONTINUED)

n_i^{10}	λ_i^{10}	δ_i^{10}	Average Widths				
			GCI	AGCI	FCI	MOVER	NA
$30^6, 50^2, 100^2$	$0.5^4, 1^3, 5^3$	$0.3^5, 0.5^5$	2.2789	<i>2.5334</i>	3.6390	2.6117	4.6823
		$0.1^6, 0.3^2, 0.5^2$	1.4734	<i>1.5918</i>	3.8142	1.6355	2.3917
		$0.1^2, 0.3^2, 0.5^6$	2.1362	<i>2.3937</i>	3.6477	2.4692	4.1759
$30^2, 50^2, 100^6$	0.5^{10}	$0.1^5, 0.3^5$	1.4928	<i>1.5518</i>	4.6209	1.5954	2.0027
		$0.3^5, 0.5^5$	2.1731	<i>2.3256</i>	4.4763	2.4023	3.4094
		$0.1^6, 0.3^2, 0.5^2$	1.5790	<i>1.6552</i>	4.6459	1.7020	2.1070
	1^{10}	$0.1^2, 0.3^2, 0.5^6$	1.9587	<i>2.0982</i>	4.4910	2.1714	2.7846
		$0.1^5, 0.3^5$	0.7958	<i>0.8730</i>	4.6782	0.8943	0.8993
		$0.3^5, 0.5^5$	1.0620	<i>1.2550</i>	4.5188	1.2900	1.4010
		$0.1^6, 0.3^2, 0.5^2$	0.8323	<i>0.9296</i>	4.6995	0.9523	0.9522
		$0.1^2, 0.3^2, 0.5^6$	0.9958	<i>1.1713</i>	4.5389	1.2055	1.2017
	5^{10}	$0.1^5, 0.3^5$	0.2344	0.4235	4.8124	0.4287	<i>0.4226</i>
		$0.3^5, 0.5^5$	0.2646	0.6407	4.6580	0.6477	<i>0.6387</i>
		$0.1^6, 0.3^2, 0.5^2$	0.2414	0.4564	4.8293	0.4619	<i>0.4554</i>
		$0.1^2, 0.3^2, 0.5^6$	0.2613	0.6017	4.6304	0.6091	<i>0.5996</i>
		$0.1^5, 0.3^5$	1.1160	<i>1.2000</i>	4.7983	1.2200	1.6750
	$0.5^4, 1^3, 5^3$	$0.3^5, 0.5^5$	1.5870	<i>1.7830</i>	4.6446	1.8180	2.8380
		$0.1^6, 0.3^2, 0.5^2$	1.1090	<i>1.2150</i>	4.7969	1.2350	1.6630
		$0.1^2, 0.3^2, 0.5^6$	1.3330	<i>1.5120</i>	4.6501	1.5440	2.1520

Note: The shortest AWs are in italic.

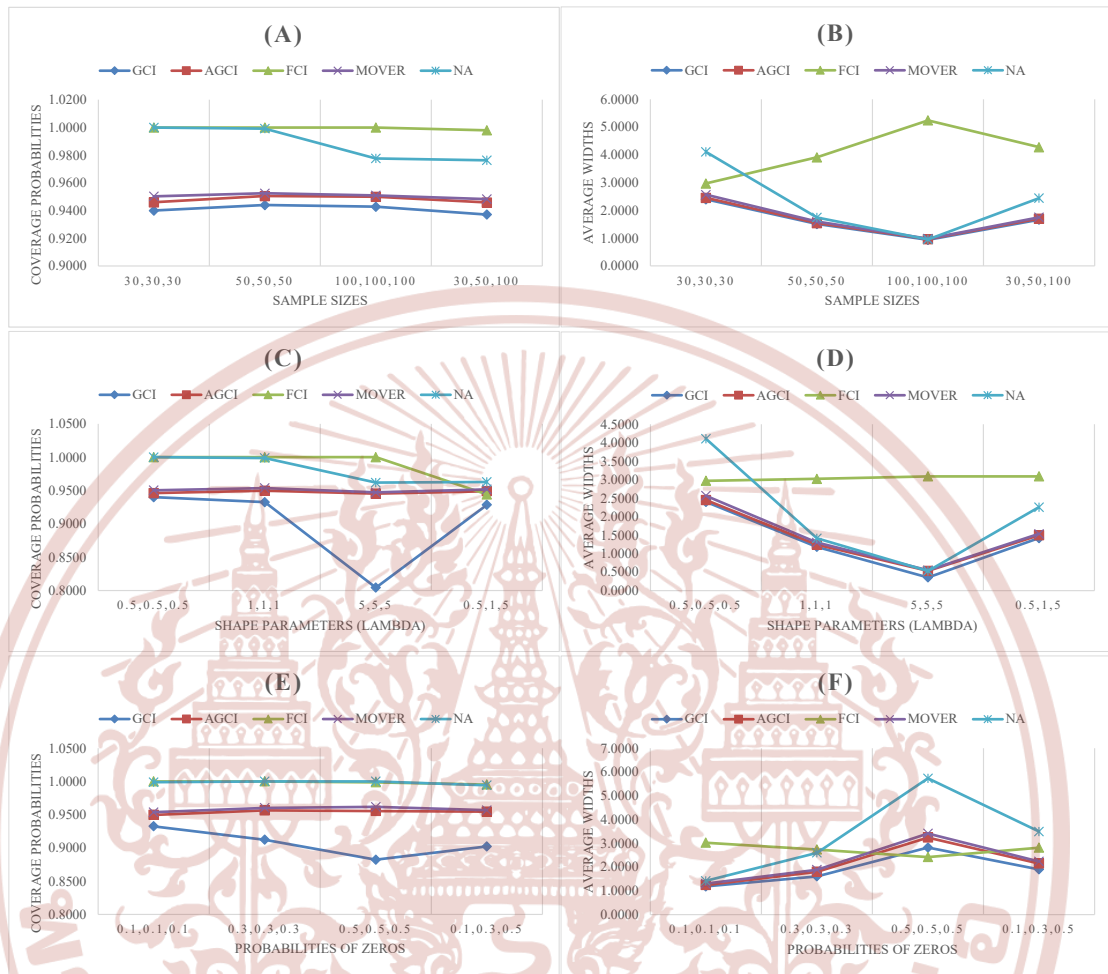


FIGURE 5-1 Comparison of the performance of the proposed methods for estimating the 95% simultaneous confidence interval for all pairwise differences of the coefficients of variation of delta-IG distributions when $k = 3$.

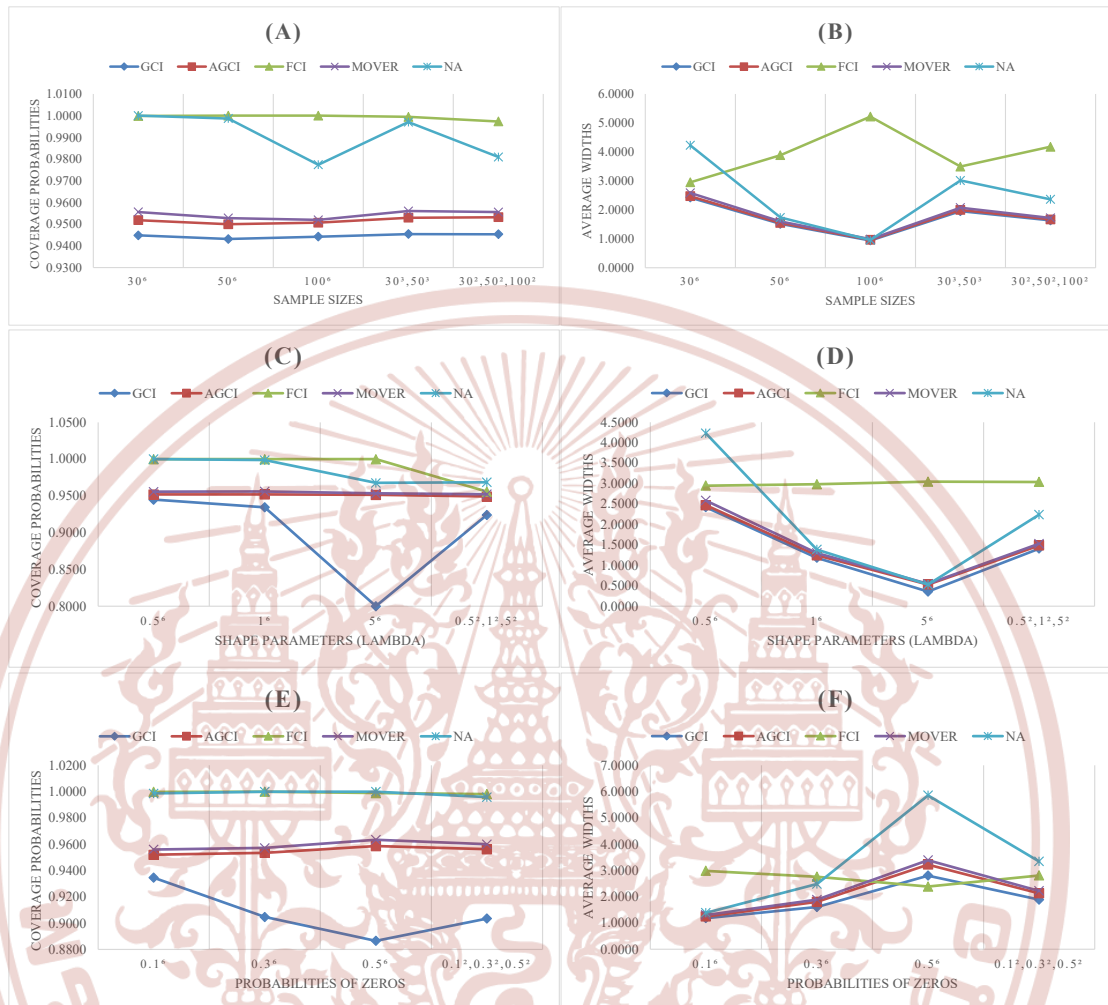


FIGURE 5-2 Comparison of the performance of the proposed methods for estimating the 95% simultaneous confidence interval for all pairwise differences of the coefficients of variation of delta-IG distributions when $k = 6$.



FIGURE 5-3 Comparison of the performance of the proposed methods for estimating the 95% simultaneous confidence interval for all pairwise differences of the coefficients of variation of delta-IG distributions when $k = 10$.

5.4 An empirical application

The data of monthly traffic accident fatality numbers documented in the second half of 2024 throughout 50 districts in Bangkok, Thailand were used to assess the performance of the proposed methods for constructing simultaneous confidence intervals for all pairwise differences of the coefficients of variation for delta-IG distributions. These data sourced from the Road Accident Victims Protection Company Limited and accessible via the Thai Road Safety Center (<https://www.thairsc.com/bangkok-in-depth-statistics>). Since the traffic fatality data collected from Bangkok, Thailand includes both zero and positive values, as shown in Table 5-7. The normal Q-Q plot and histogram of the data are visualized in Figure 5-4 to Figure 5-9 for each month of the second half of 2025. The AIC and BIC values were used to identify the most suitable model for only positive data. Table 5-8 compares AIC and BIC values across various distributions and found that the traffic fatality data of the second half of 2024 from Bangkok, Thailand were suitable with IG distribution, which

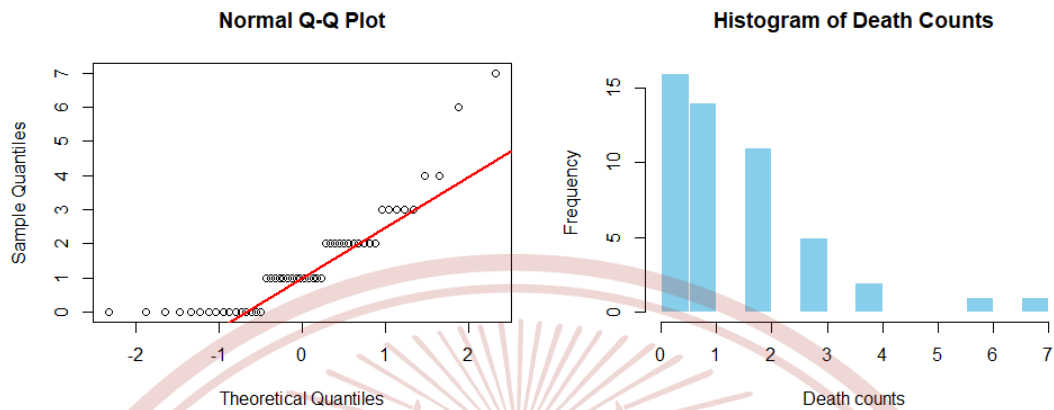


FIGURE 5-4 (left) Normal Q-Q plot and (right) Histogram of death count data from Bangkok in December 2024.

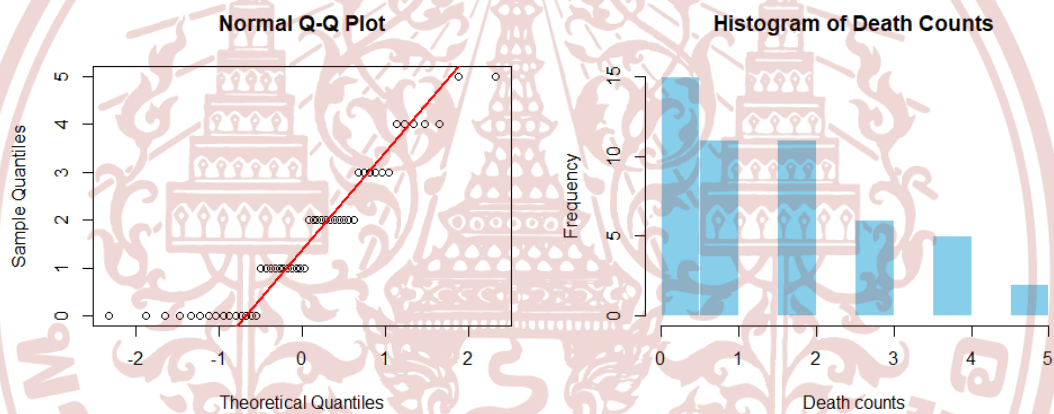


FIGURE 5-5 (left) Normal Q-Q plot and (right) Histogram of death count data from Bangkok in November 2024.

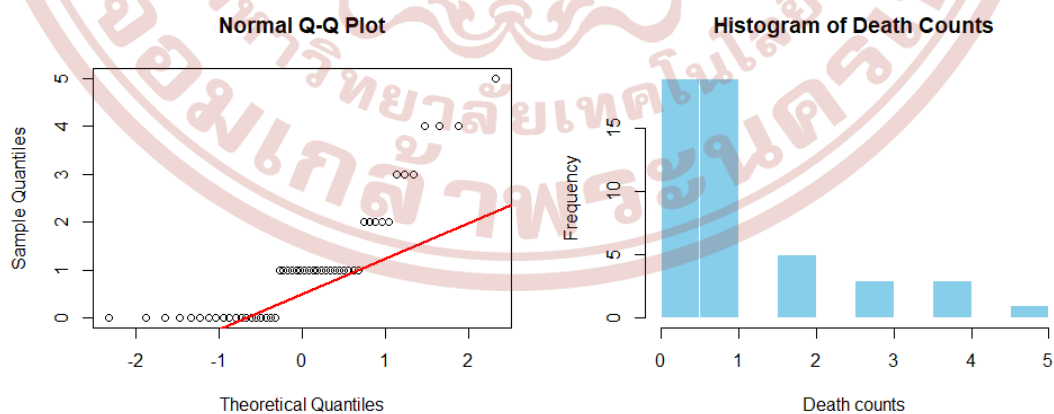


FIGURE 5-6 (left) Normal Q-Q plot and (right) Histogram of death count data from Bangkok in October 2024.

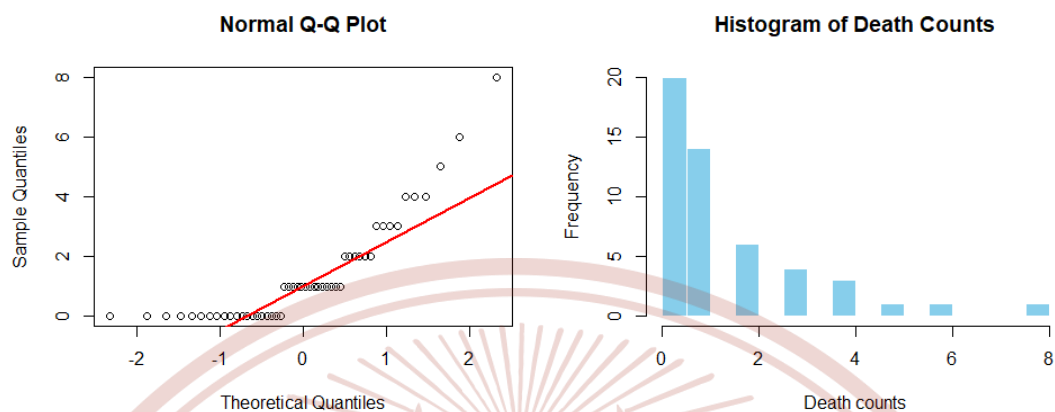


FIGURE 5-7 (left) Normal Q-Q plot and (right) Histogram of death count data from Bangkok in September 2024.

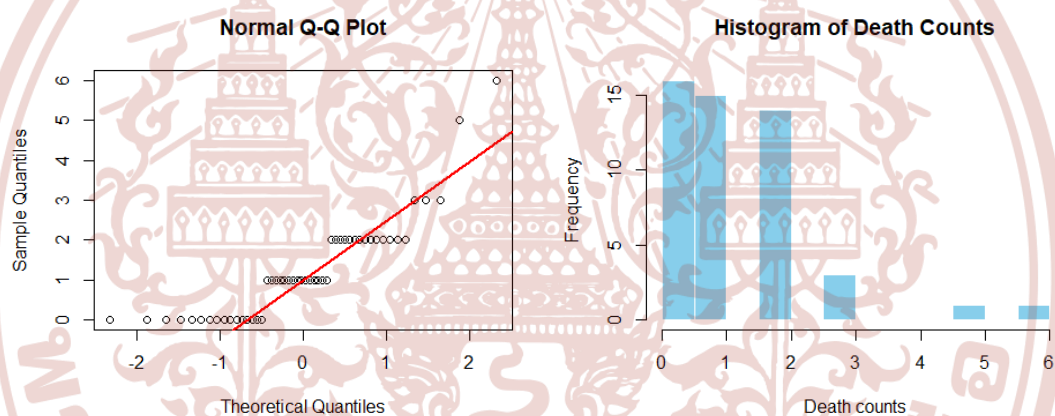


FIGURE 5-8 (left) Normal Q-Q plot and (right) Histogram of death count data from Bangkok in August 2024.

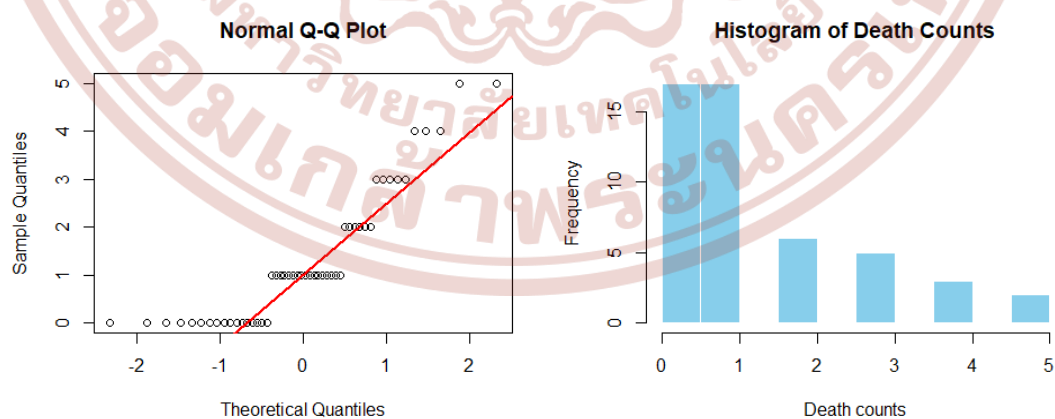


FIGURE 5-9 (left) Normal Q-Q plot and (right) Histogram of death count data from Bangkok in July 2024.

TABLE 5-8 The AIC and BIC values for evaluating the distribution of the traffic fatality data in the second half of 2024 from Bangkok, Thailand.

Criteria	Months	Distributions						
		Normal	Lognormal	Weibull	Gamma	Exponential	Logistic	Inverse Gaussian
AIC	December	123.8187	100.8440	110.0505	105.5633	121.0208	118.2031	99.9425
	November	116.8730	109.4250	111.1910	109.8520	130.7370	118.6540	108.504
	October	100.9568	79.7587	89.2526	85.1330	99.5475	99.6527	78.5372
	September	121.5226	99.4208	106.7473	103.9384	111.9745	118.4413	97.9782
	August	107.8859	85.0842	96.2964	90.1061	111.9406	100.0647	84.6343
	July	112.6562	95.6802	102.3020	99.3460	113.7477	113.1824	94.1796
BIC	December	126.8710	103.8970	113.1030	108.6160	122.5470	121.2560	102.9950
	November	119.9840	112.5360	114.3020	112.9620	132.2920	121.7650	111.6150
	October	103.8248	82.6267	92.1206	88.0010	100.9814	102.5206	81.4051
	September	124.3250	102.2230	109.5500	106.7410	113.3760	121.2440	100.7810
	August	110.9387	88.1370	99.3492	93.1589	113.4670	103.1175	87.6870
	July	115.6493	98.6732	105.2950	102.3390	115.2442	116.1754	97.1726

TABLE 5-9 Descriptive statistics for each month in the second half of 2024 from Bangkok, Thailand.

Statistics	Months					
	December (1)	November (2)	October (3)	September (4)	August (5)	July (6)
n_i	50	50	50	50	50	50
$n_{i(0)}$	16	15	19	20	16	17
$n_{i(1)}$	34	35	31	30	34	33
$\hat{\mu}_i$	2.1200	2.3100	1.7700	2.300	1.8500	2.0000
$\hat{\lambda}_i$	5.7400	6.9600	5.1900	4.5900	6.7800	5.2200
$\hat{\delta}_i$	0.3200	0.3000	0.3800	0.4000	0.3200	0.3400
$\hat{\theta}_i$	1.0100	0.9500	1.0800	1.2300	0.9340	1.0500

TABLE 5-10 The estimated differences between the coefficients of variation with the 95% simultaneous confidence intervals of the traffic fatality data in the second half of 2024 from Bangkok, Thailand.

Months	GCI			AGCI			FCI		
	Lower	Upper	Width	Lower	Upper	Width	Lower	Upper	Width
$\theta_1 - \theta_2$	-0.1848	0.2664	0.4512	-0.2931	0.4179	0.7110	-0.9302	3.2570	4.1872
$\theta_1 - \theta_3$	-0.3037	0.1468	0.4505	-0.4889	0.3064	0.7953	-2.2831	2.3638	4.6469
$\theta_1 - \theta_4$	-0.5120	0.0185	0.5306	-0.6967	0.1621	0.8588	-1.9986	2.2722	4.2708
$\theta_1 - \theta_5$	-0.1271	0.2811	0.4082	-0.2426	0.4417	0.6843	-1.3618	2.3661	3.7279
$\theta_1 - \theta_6$	-0.3169	0.1874	0.5043	-0.3835	0.3377	0.7212	-3.3376	2.6905	6.0281
$\theta_2 - \theta_3$	-0.3668	0.1035	0.4703	-0.5396	0.2593	0.7989	-2.5963	1.0361	3.6324
$\theta_2 - \theta_4$	-0.5678	-0.0335	0.5342	-0.7292	0.0960	0.8252	-2.0523	0.6992	2.7515
$\theta_2 - \theta_5$	-0.1782	0.2553	0.4334	-0.3146	0.3884	0.7030	-1.6720	1.0469	2.7189
$\theta_2 - \theta_6$	-0.3419	0.1427	0.4847	-0.4826	0.2663	0.7489	-3.9301	0.9891	4.9192
$\theta_3 - \theta_4$	-0.4359	0.1020	0.5379	-0.6383	0.2715	0.9098	-1.6290	1.8667	3.4956
$\theta_3 - \theta_5$	-0.0526	0.3934	0.4459	-0.2041	0.5675	0.7716	-1.1658	2.1179	3.2837
$\theta_3 - \theta_6$	-0.2459	0.2730	0.5189	-0.3724	0.4528	0.8252	-3.9894	2.4744	6.4638

TABLE 5-10 (CONTINUED)

Months	GCI			AGCI			FCI		
	Lower	Upper	Width	Lower	Upper	Width	Lower	Upper	Width
$\theta_4 - \theta_5$	0.0687	0.5827	0.5140	-0.0664	0.7716	0.8379	-1.2182	1.8760	3.0942
$\theta_4 - \theta_6$	-0.0995	0.4597	0.5592	-0.2317	0.6634	0.8951	-3.4453	1.6246	5.0699
$\theta_5 - \theta_6$	-0.3641	0.0720	0.4360	-0.5043	0.2129	0.7172	-3.5239	1.3830	4.9069
Months	MOVER			NA					
	Lower	Upper	Width	Lower	Upper	Width			
$\theta_1 - \theta_2$	-0.3096	0.4221	0.7317	-0.2674	0.3793	0.6467			
$\theta_1 - \theta_3$	-0.5048	0.3077	0.8125	-0.4219	0.2764	0.6983			
$\theta_1 - \theta_4$	-0.6902	0.1734	0.8636	-0.6010	0.1634	0.7644			
$\theta_1 - \theta_5$	-0.2555	0.4413	0.6968	-0.2500	0.3944	0.6444			
$\theta_1 - \theta_6$	-0.4196	0.3269	0.7464	-0.3815	0.3013	0.6828			
$\theta_2 - \theta_3$	-0.5679	0.2592	0.8270	-0.4683	0.2108	0.6792			
$\theta_2 - \theta_4$	-0.7527	0.1246	0.8773	-0.6483	0.0987	0.7470			
$\theta_2 - \theta_5$	-0.3207	0.3930	0.7137	-0.2955	0.3281	0.6236			
$\theta_2 - \theta_6$	-0.4836	0.2786	0.7622	-0.4276	0.2355	0.6632			
$\theta_3 - \theta_4$	-0.6350	0.3150	0.9501	-0.5421	0.2500	0.7920			
$\theta_3 - \theta_5$	-0.2075	0.5866	0.7941	-0.1934	0.4835	0.6769			
$\theta_3 - \theta_6$	-0.3684	0.4725	0.8409	-0.3241	0.3895	0.7136			
$\theta_4 - \theta_5$	-0.0741	0.7712	0.8453	-0.0814	0.6635	0.7449			
$\theta_4 - \theta_6$	-0.2335	0.6572	0.8908	-0.2105	0.5679	0.7784			
$\theta_5 - \theta_6$	-0.5026	0.2255	0.7282	-0.4428	0.2181	0.6609			

5.5 Conclusions

This research focuses on evaluating simultaneous confidence intervals for pairwise differences between the coefficients of variation of multiple delta-IG distributions using five approaches: GCI, AGCI, FCI, MOVER, and NA. Performance was measured through CP and AW. For simulation results, AGCI and MOVER consistently demonstrate the best overall performance. AGCI provides the most stable and reliable coverage across all scenarios, while MOVER offers greater efficiency through shorter interval widths. In contrast, GCI suffers from undercoverage, particularly as the number of populations increases, and FCI is overly conservative, producing excessively wide intervals that limit practical usefulness. The NA method shows moderate performance but does not outperform AGCI or MOVER.

The real-data application supports the simulation findings, confirming that AGCI and MOVER are the most suitable methods in practice, with AGCI preferred for coverage accuracy and MOVER favored for interval efficiency.

TABLE 5-11 Summary of the chapter.

Aspect	Key Finding	Recommended Method
Pairwise CV Differences	Most stable and reliable coverage across scenarios	AGCI (best for accuracy)
	More efficient with shorter interval widths	MOVER (best for efficiency)
Overall Conclusion	Most suitable methods with accuracy and efficiency	AGCI, MOVER



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APPENDIX A

THE R PROGRAM USED IN THIS THESIS

A1 Functions are used to calculate the coverage probability and the average width of confidence interval for the coefficient of variation of delta-IG distribution.

The following R code is used to compute the confidence intervals, the coverage probability, and the average width of the confidence intervals for the coefficient of variation of delta-IG distribution in Chapter 2.

#- This code was written by Wasurat Khumpasee -#

#####

```
library(VGAM)
library(MASS)
library(HDIInterval)
library(coda)
library(boot)
library(R2OpenBUGS)
library(statmod)
library(EnvStats)
library(beepr)
```

```
# M = number of simulation runs (iteration about 15,000 times)
# m = number of generalized computation (in GCI, AGCI about 5,000 times)
# n = number of sample size are (50,100, and 200)
# mu be 0.5 and 1
# lambda be 1,5, and 10
# delta be 0.3, and 0.5
# B = number of bootstrap sample
```

#Generating data

```
rZAIG <- function(x,mu,lambda,prob0){
  x_0 <- rbinom(x,size = 1,prob = 1-prob0)
  x0 <- subset(x_0,x_0==0)
  nzero <- length(x0)
  nNonzero <- x-nzero
  x1 <- rinvgauss(nNonzero,mu,lambda)
  x <- c(x0,x1)
  return(x)
}
```

#model from your distribution please change in phi[i] put in likelihood function

```
IGmodel <- function(){
  for(i in 1:n){
    one[i] <- 1
```

```

p[i]<-(sqrt(1/(2*3.14*pow(x[i],3)))*exp(-1*pow(x[i]-m,2)/2*pow(m,2)*x[i]))/10000
one[i] ~ dbern(p[i])
}
m ~ dunif(0.5,3) #define uniform for parameter mu
l ~ dgamma(12,3); #define gamma for shape parameter lambda
}

#define p as ratio
Binomialmodel <- function() {
  for (i in 1:n)
  {
    x[i] ~ dbern(p);
  }
  p ~ dbeta(12,3); #define beta for probability parameter
}

CI.CV.Bayes <- function(M,n,m,l,p){
  #M = loop for montecarlo
  #n = number of sample size
  #m = mean parameter of ZAIG
  #l = shape parameter of ZAIG
  #p = probability of zero for ZAIG
  set.seed(999)
  CP.CV.bayesian <- rep(0,M) # coverage probability of CI based on bayesianMCMC
  CP.BAYE2.CV <- rep(0,M) # coverage probability of CI based on bayesianHPTD
  AW.CV.bayesian <- rep(0,M)
  AW.BAYE2.CV <- rep(0,M)
  mcoda <- rep(0,M)
  lcoda <- rep(0,M)
  pcoda <- rep(0,M)
  mean <- (1-p)*m
  var <- m^2*(1-p)*((m/l)+p)
  cv <- sqrt((m+(1*p))/((1-p)*l))
  ###stat iteration
  for(i in 1:M){
    x <- rZAIG(n,m,l,p)
    x0 <- subset(x,x==0)
    x1 <- subset(x,x>0)
    x01 <- ifelse(x>0,1,0)
    n0 <- length(x0)
    n1 <- length(x1)

    #Bayesian
    #for p

```

```

zerodata <- list(x = x0,n=length(x0))
inits1<-function(){ list( p=0) }
outputp <- bugs(data = zerodata, inits = inits1,parameters.to.save = c("p"),model.file
= Binomialmodel, n.chains = 1,n.iter = 20000,n.burnin =
5000,debug=FALSE,codaPkg=TRUE)
out.codap <- read.bugs(outputp)
codp <- as.vector(out.codap)
pcoda <- codp[[1]][,2]

#for m,l
ZAIGdata <- list(x = x1,n=length(x1))
inits2<-function(){ list(l=2, m=3) }
outputm <- bugs(data = ZAIGdata, inits = inits2, parameters.to.save =
c("l","m"),model.file = IGmodel, n.chains = 1,n.iter = 20000,n.burnin =
5000,debug=FALSE,codaPkg=TRUE)
out.codam <- read.bugs(outputm)
codm <- as.vector(out.codam)
mcoda <- codm[[1]][,3]
lcoda <- codm[[1]][,2]

baye.mean <- (1-pcoda)*mcoda
baye.var <- mcoda^2*(1-pcoda)*((mcoda/lcoda)+pcoda)
baye.cv <- sqrt((mcoda+(lcoda*pcoda))/((1-pcoda)*lcoda))
L.Baye.cv <- quantile(baye.cv,0.025,type = 8,na.rm = TRUE)
U.Baye.cv <- quantile(baye.cv,0.975,type = 8,na.rm = TRUE)
CP.CV.bayesian[i] <- ifelse(L.Baye.cv<cv&&cv<U.Baye.cv,1,0)
AW.CV.bayesian[i] <- U.Baye.cv-L.Baye.cv

#HPTD
HPD.baye.cv =hdi(baye.cv,0.95)
L.baye2.cv =HPD.baye.cv [1]
U.baye2.cv =HPD.baye.cv [2]
CP.BAYE2.CV[i] = ifelse(L.baye2.cv<cv&&cv<U.baye2.cv,1,0) # compute
probability of CI based on BAYE of Baye
AW.BAYE2.CV[i] = U.baye2.cv- L.baye2.cv # compute length of CI based on
BAYE of Baye

}

ACP.CV.Baye <- mean(CP.CV.bayesian)
AAW.CV.Baye <- mean(AW.CV.bayesian)
ACP.CV.HPDBaye <- mean(CP.BAYE2.CV)
AAW.CV.HPDBaye <- mean(AW.BAYE2.CV)
cat("CP CV BayesianMCMC=",ACP.CV.Baye,"n")

```

```

cat("AW CV BayesianMCMC=",AAW.CV.Baye,"\n")
cat("CP CV BayesianHPTD=",ACP.CV.HPDBaye,"\n")
cat("AW CV BayesianHPTD=",AAW.CV.HPDBaye,"\n")
cat("Lower Bayes =",L.Baye.cv,"Upper Bayes=",U.Baye.cv,"\n")
cat("Lower HPTD =",L.baye2.cv,"Upper HPTD=",U.baye2.cv,"\n")

beep()
}

CI.SINGIG=function(M,m,n,mu,lambda,delta)
{
# Start

starttime = proc.time()
set.seed(999)
B=1000 # number of bootstrap sample

CP.GCI = c() # coverage probability of CI based on GCI
Length.GCI = c() # average length of CI based on GCI
R_theta_GCI = c()

CP.AGCI = c() # coverage probability of CI based on AGCI
Length.AGCI = c() # average length of CI based on AGCI
R_theta_AGCI = c()

delta_GPQ <- c()
Z.VST <- c()
L.GCI <- c()
U.GCI <- c()
AV.GCI <- c()
CP.GCI <- c()

L.AGCI <- c()
U.AGCI <- c()
AV.AGCI <- c()
CP.AGCI <- c()

CP.BPCI = rep(0,M) # coverage probability of CI based on PBPCI
Length.BPCI = rep(0,M) # average length of CI based on BPCI
theta.star = rep(0,B)

CP.FCI = rep(0,M) # coverage probability of CI based on Fiducial
Length.FCI = rep(0,M) # average length of CI based on Fiducial

CP.FHPD = rep(0,M) # coverage probability of CI based on Fiducial HPD
Length.FHPD = rep(0,M) # average length of CI based on Fiducial HPD

```

```

mu.t = c()
lambda.t = c()
mu.gibbs = c()
lambda.gibbs = c()

muhat.t = c()
lambdahat.t = c()

##### Estimation of parameter of interest #####
#delta <- n0/(n0+n1)
theta <- sqrt((mu+(lambda*delta))/((1-delta)*lambda)) # single CV
####Start Iteration####
for (i in 1:M)
{
  # begin loop i
  # _____ GENERATING X _____ #
  x_0 <- rbinom(n, size=1, prob=delta) #Choose n_zero from bi(n,delta) #generate
x0
  x0 <- x_0[which(x_0!=0)]
  n_zero <- length(x0)
  n_nonzero <- n-n_zero

  x1 <- rinvgauss(n_nonzero, mu, lambda) # generate x1
  x <- c(x0,x1) #combine x0,x1

  nonzero_value <- x[which(x!=0)]
  n1 <- n_nonzero
  n0 <- n_zero
  delta_hat <- n0/n

  muhat <- mean(x1) # compute mean of x (muhat)
  lambdahat_inverse <- (1/n1)*sum((1/x1)-(1/muhat))
  lambdahat <- 1/lambdahat_inverse # compute scale of x (lambdahat)

#####Compute CI based on GCI& AGCI#####
for (j in 1:m)
{
  # begin loop j
  xbar = rinvgauss(1, mu, n1*lambda)
  A=rchisq(1,n1-1)
  Z=rnorm(1,0,1)
  V=A/(n1*lambda)
  R_lambda = A/(n1*lambdahat_inverse) # compute GPQ for scale of x
  R_mu_GCI =xbar/abs(1+Z*(sqrt(xbar/(n1*R_lambda)))) # compute GPQ for mean
of x

```

```

#generate Z from standard normal in Population
#Z.VST = rnorm(1,0,1)
Z.VST = 2*(sqrt(n))*(asin(sqrt(delta_hat))-asin(sqrt(delta)))

#Calculate GPQ for delta
delta_GPQ = (sin(asin(sqrt(delta_hat))-(Z.VST/(2*sqrt(n))))^2
R_theta_GCI[j]=sqrt((R_mu_GCI+(R_lambda*delta_GPQ))/((1-
delta_GPQ)*R_lambda)) # compute GPQ for single CV

W = (sqrt(n1-1)*(xbar/mu-1))/sqrt(V*xbar)
R_mu_AGCI = xbar/max(0,1+(W*sqrt((V*xbar)/(n1-1)))) # compute GPQ
for mean of x
R_theta_AGCI[j]=sqrt((R_mu_AGCI+(R_lambda*delta_GPQ))/((1-
delta_GPQ)*R_lambda)) # compute GPQ for single CV

} # end loop j
L.GCI <- quantile(R_theta_GCI,0.025,type=8,na.rm = TRUE) # compute lower
based on GCI of theta
U.GCI <- quantile(R_theta_GCI,0.975,type=8,na.rm = TRUE) # compute upper
based on GCI of theta

L.AGCI <- quantile(R_theta_AGCI,0.025,type=8,na.rm = TRUE) # compute
lower based on AGCI of theta
U.AGCI <- quantile(R_theta_AGCI,0.975,type=8,na.rm = TRUE) # compute
upper based on AGCI of theta

CP.GCI[i] <- ifelse(L.GCI<theta&&theta<U.GCI,1,0) # compute probability of
CI based on GCI of theta
Length.GCI[i] <- U.GCI - L.GCI # compute length of CI based on GCI of theta

CP.AGCI[i] <- ifelse(L.AGCI<theta&&theta<U.AGCI,1,0) # compute probability of
CI based on AGCI of theta
Length.AGCI[i] <- U.AGCI - L.AGCI # compute length of CI based on AGCI of
theta

#####Compute CI based on BPCI#####

for (k in 1:B)
{ # begin loop k
x.star= sample(x1,n1,replace=T) # generate bootstrap sample of x
xbar.star= mean(x.star) # compute mean of bootstrap sample of x
lambdahat_inv_bt = (1/n1) * sum((1/x.star)-(1/xbar.star))
lambdahat_bt = 1/lambdahat_inv_bt # compute scale of bootstrap sample of x
#generate Z from standard normal in Population

```

```

Z.B = 2*(sqrt(n))*(asin(sqrt(delta_hat))-asin(sqrt(delta)))

#Calculate for delta
delta_B = (sin(asin(sqrt(delta_hat))-(Z.B/(2*sqrt(n))))^2
theta.star[k]=sqrt((xbar.star+(lambdahat_bt*delta_B))/((1-
delta_B)*lambdahat_bt)) # compute single CV of bootstrap sample
} # end loop k

L.BPCI <- quantile(theta.star,0.025,type=8,na.rm = TRUE) # compute lower based
on PBCI of theta
U.BPCI <- quantile(theta.star,0.975,type=8,na.rm = TRUE) # compute lower based
on PBCI of theta
CP.BPCI[i] <- ifelse( L.BPCI <theta&&theta< U.BPCI,1,0) # compute probability
of CI based on PBCI of theta
Length.BPCI[i] <- U.BPCI-L.BPCI # compute length of CI based on PBCI of theta

#####Compute CI based on FCI#####
muhat.t[1] = muhat # MLE of mean x
lambdahat.t[1] = lambdahat # MLE of scale x

#### set t for Gibbs ####
for (t in 2:20000)
{ # begin loop t
muhat.t[t]=muhat.t[t-1]
lambdahat.t[t] =lambdahat.t[t-1]
lambda.t[t] = (lambdahat.t[t]/n1)*rchisq(1,n1-1)
mu.t[t] = rinvgauss(1, muhat.t[t], n1*lambdahat.t[t])
} # end loop t

#generate Z from standard normal in Population
Z.F = 2*(sqrt(n))*(asin(sqrt(delta_hat))-asin(sqrt(delta)))

#Calculate for delta
delta_F = (sin(asin(sqrt(delta_hat))-(Z.B/(2*sqrt(n))))^2

mu.gibbs = mu.t
lambda.gibbs = lambda.t

## Gibbs sampling for fiducial distribution#

thetahat.FCI=sqrt((mu.gibbs+(lambda.gibbs*delta_F))/((1-delta_F)*lambda.gibbs))
# compute CV base on MCMC
theta.FCI = thetahat.FCI[-1:-1000] # burn in 1000

```

```
L.CI.FCI <- quantile(theta.FCI,0.025,type=8, na.rm=TRUE) # compute lower based
on Fiducial of theta
```

```
U.CI.FCI <- quantile(theta.FCI,0.975,type=8, na.rm=TRUE) # compute upper based
on Fiducial of theta
```

```
CP.FCI[i] <- ifelse(L.CI.FCI<theta&&theta< U.CI.FCI,1,0) # compute probability
of CI based on Fiducial of theta
```

```
Length.FCI[i] <- U.CI.FCI-L.CI.FCI # compute length of CI based on Fiducial of
theta
```

```
#####Compute CI based on F-HPDCI#####
```

```
FHPD = hdi(theta.FCI,0.95)
```

```
L.FHPD = FHPD[1] # compute lower based on Fiducial HPD of theta
```

```
U.FHPD = FHPD[2] # compute upper based on Fiducial HPD of theta
```

```
CP.FHPD[i] = ifelse(L.FHPD<theta&&theta<U.FHPD,1,0) #compute probability of
CI based on Fiducial HPD of theta
```

```
Length.FHPD[i] <- U.FHPD-L.FHPD #compute length of CI based on Fiducial
HPD of theta
```

```
} # end loop i
```

```
elapsedTime = proc.time()-starttime
```

```
ACP.GCI <- mean(CP.GCI) # compute coverage probability based on GCI of theta
```

```
Ex.length.GCI <- mean(Length.GCI) # compute expected length based on GCI of
theta
```

```
cat("CP|GCI=", ACP.GCI,"Length|GCI=", Ex.length.GCI ,"\n")
```

```
ACP.AGCI <- mean(CP.AGCI) # compute coverage probability based on AGCI of
theta
```

```
Ex.length.AGCI <- mean(Length.AGCI) # compute expected length based on AGCI
of theta
```

```
cat("CP|AGCI=", ACP.AGCI,"Length|AGCI=", Ex.length.AGCI ,"\n")
```

```
ACP.BPCI <- mean(CP.BPCI) # compute coverage probability based on BPCI of
theta
```

```
Ex.length.BPCI <- mean(Length.BPCI) # compute expected length based on BPCI of
theta
```

```
cat("CP|BPCI=", ACP.BPCI,"Length|BPCI=", Ex.length.BPCI ,"\n")
```

```
ACP.FCI <- mean(CP.FCI) # compute coverage probability based on Fiducial of
theta
```

```
Ex.length.FCI <- mean(Length.FCI) # compute expected length based on Fiducial of
theta
```

```
cat("CP|FCI=", ACP.FCI,"Length|FCI=", Ex.length.FCI,"\n")
```

```
ACP.FHPD <- mean(CP.FHPD) # compute coverage probability based on Fiducial
HPD of theta
```

```
Ex.length.FHPD <- mean(Length.FHPD) # compute expected length based on
Fiducial HPD of theta
```

```
cat("CP|FHPD=", ACP.FHPD,"Length|FHPD=", Ex.length.FHPD,"\n")
```

```
cat("Time =", elapsedTime["elapsed"],"\n")
```

```
} #end process
```



A2 Functions are used to calculate the coverage probability and the average width of confidence interval for the difference and ratio between coefficients of variation of delta-IG distributions.

The following R code is used to compute the confidence intervals, the coverage probability, and the average width of the confidence intervals for the difference and ratio between coefficients of variation of delta-IG distributions in Chapter 2.

```
#- This code was written by Wasurat Khumpasee -#

#####

library(VGAM)
library(MASS)
library(HDIInterval)
library(coda)
library(boot)
library(R2OpenBUGS)
library(statmod)
library(EnvStats)
library(beepr)

# M = number of simulation runs (iteration about 15,000 times)
# m = number of generalized computation (in GCI, AGCI about 5,000 times)
# n = number of sample size are (50,100, and 200)
# mu be 0.5 and 1
# lambda be 1,5, and 10
# delta be 0.3, and 0.5
# B = number of bootstrap sample

#Generating data
rZAIG <- function(x,mu,lambda,prob0){
  x_0 <- rbinom(x,size = 1,prob = 1-prob0)
  x0 <- subset(x_0,x_0 ==0)
  nzero <- length(x0)
  nNonzero <- x-nzero
  x1 <- rinvgauss(nNonzero,mu,lambda)
  x <- c(x0,x1)
  return(x)
}

#model from your distribution please change in phi[i] put in likelihood function
IGmodel <- function(){
  for(i in 1:n){
```

```

one[i] <- 1
p[i]<-(sqrt(1/(2*3.14*pow(x[i],3)))*exp(-1*pow(x[i]-m,2)/2*pow(m,2)*x[i]))/10000
one[i] ~ dbern(p[i])
}
m ~ dunif(1,3) #define uniform for parameter mu
l ~ dgamma(90,375); #define gamma for shape parameter lambda
}

#define p as ratio
Binomialmodel <- function() {
  for (i in 1:n)
  {
    x[i] ~ dbern(p);
  }
  p ~ dbeta(8,6); #define beta for probability parameter
}

CI.CV.Bayes <- function(M,n1,n2,m1,m2,l1,l2,p1,p2){
  #M = loop for montecarlo
  #n = number of sample size
  #m = mean parameter of ZAIG
  #l = shape parameter of ZAIG
  #p = probability of zero for ZAIG
  set.seed(999)
  CP.theta.bayesian <- rep(0,M) # coverage probability of CI based on bayesianMCMC
  CP.eta.bayesian <- rep(0,M)
  CP.BAYE2.theta <- rep(0,M) # coverage probability of CI based on bayesianHPTD
  CP.BAYE2.eta <- rep(0,M)
  AW.theta.bayesian <- rep(0,M)
  AW.eta.bayesian <- rep(0,M)
  AW.BAYE2.theta <- rep(0,M)
  AW.BAYE2.eta <- rep(0,M)
  mcoda1 <- rep(0,M)
  lcoda1 <- rep(0,M)
  pcoda1 <- rep(0,M)
  mcoda2 <- rep(0,M)
  lcoda2 <- rep(0,M)
  pcoda2 <- rep(0,M)
  mean1 <- (1-p1)*m1
  mean2 <- (1-p2)*m2
  var1 <- m1^2*(1-p1)*((m1/l1)+p1)
  var2 <- m2^2*(1-p2)*((m2/l2)+p2)
  cv1 <- sqrt((m1+(l1*p1))/((1-p1)*l1))
  cv2 <- sqrt((m2+(l2*p2))/((1-p2)*l2))

```

```

theta1 <- cv1 - cv2
eta1 <- cv1 / cv2
####stat iteration
for(i in 1:M){
  x <- rZAIG(n1,m1,l1,p1)
  x0 <- subset(x,x==0)
  x1 <- subset(x,x>0)
  x0l <- ifelse(x>0,1,0)
  n0_x <- length(x0)
  n1_x <- length(x1)

  y <- rZAIG(n2,m2,l2,p2)
  y0 <- subset(y,y==0)
  y1 <- subset(y,y>0)
  y0l <- ifelse(y>0,1,0)
  n0_y <- length(y0)
  n1_y <- length(y1)

  #Bayesian
  #for p1
  zerodata <- list(x = x0,n =length(x0))
  inits1<-function(){ list( p=0) }
  outputp <- bugs(data = zerodata, inits = inits1,parameters.to.save = c("p"),model.file
= Binomialmodel, n.chains = 1,n.iter = 20000,n.burnin =
5000,debug=FALSE,codaPkg=TRUE)
  out.codap <- read.bugs(outputp)
  codp <- as.vector(out.codap)
  pcoda1 <- codp[[1]][,2]

  #for m1,l1
  ZAIGdata <- list(x = x1,n =length(x1))
  inits2<-function(){ list(l=2, m=3) }
  outputm <- bugs(data = ZAIGdata, inits = inits2, parameters.to.save =
c("l","m"),model.file = IGmodel, n.chains = 1,n.iter = 20000,n.burnin =
5000,debug=FALSE,codaPkg=TRUE)
  out.codam <- read.bugs(outputm)
  codm <- as.vector(out.codam)
  mcodal <- codm[[1]][,3]
  lcodal <- codm[[1]][,2]

  #for p2
  zerodata <- list(x = y0,n =length(y0))
  inits1<-function(){ list( p=0) }

```

```

outputp <- bugs(data = zerodata, inits = inits1, parameters.to.save = c("p"), model.file
= Binomialmodel, n.chains = 1, n.iter = 20000, n.burnin =
5000, debug=FALSE, codaPkg=TRUE)
out.codap <- read.bugs(outputp)
codp <- as.vector(out.codap)
pcoda2 <- codp[[1]][,2]

#for m2,l2
ZAIGdata <- list(x = y1, n = length(y1))
inits2 <- function() { list(l=2, m=3) }
outputm <- bugs(data = ZAIGdata, inits = inits2, parameters.to.save =
c("l", "m"), model.file = IGmodel, n.chains = 1, n.iter = 20000, n.burnin =
5000, debug=FALSE, codaPkg=TRUE)
out.codam <- read.bugs(outputm)
codm <- as.vector(out.codam)
mcoda2 <- codm[[1]][,3]
lcoda2 <- codm[[1]][,2]

baye.mean1 <- (1-pcoda1)*mcoda1
baye.var1 <- mcoda1^2*(1-pcoda1)*((mcoda1/lcoda1)+pcoda1)
baye.cv1 <- sqrt((mcoda1+(lcoda1*pcoda1))/((1-pcoda1)*lcoda1))
baye.mean2 <- (1-pcoda2)*mcoda2
baye.var2 <- mcoda2^2*(1-pcoda2)*((mcoda2/lcoda2)+pcoda2)
baye.cv2 <- sqrt((mcoda2+(lcoda2*pcoda2))/((1-pcoda2)*lcoda2))
baye.theta <- baye.cv1 - baye.cv2
baye.eta <- baye.cv1 / baye.cv2
L.Baye.theta <- quantile(baye.theta, 0.025, type = 8, na.rm = TRUE)
U.Baye.theta <- quantile(baye.theta, 0.975, type = 8, na.rm = TRUE)
L.Baye.eta <- quantile(baye.eta, 0.025, type = 8, na.rm = TRUE)
U.Baye.eta <- quantile(baye.eta, 0.975, type = 8, na.rm = TRUE)
CP.theta.bayesian[i] <- ifelse(L.Baye.theta < theta1 && theta1 < U.Baye.theta, 1, 0)
AW.theta.bayesian[i] <- U.Baye.theta - L.Baye.theta
CP.eta.bayesian[i] <- ifelse(L.Baye.eta < eta1 && eta1 < U.Baye.eta, 1, 0)
AW.eta.bayesian[i] <- U.Baye.eta - L.Baye.eta

#HPTD
HPD.baye.theta = hdi(baye.theta, 0.95)
L.baye2.theta = HPD.baye.theta [1]
U.baye2.theta = HPD.baye.theta [2]
CP.BAYE2.theta[i] = ifelse(L.baye2.theta < theta1 && theta1 < U.baye2.theta, 1, 0) #
compute probability of CI based on BAYE of Baye
AW.BAYE2.theta[i] = U.baye2.theta - L.baye2.theta # compute length of CI
based on BAYE of Baye

```

```

HPD.baye.eta      =hdi(baye.eta,0.95)
L.baye2.eta       =HPD.baye.eta [1]
U.baye2.eta       =HPD.baye.eta [2]
CP.BAYE2.eta[i]   = ifelse(L.baye2.eta<eta1&&eta1<U.baye2.eta,1,0) # compute
probability of CI based on BAYE of Baye
  AW.BAYE2.eta[i] = U.baye2.eta- L.baye2.eta    # compute length of CI based on
BAYE of Baye

}

ACP.theta.Baye <- mean(CP.theta.bayesian)
AAW.theta.Baye <- mean(AW.theta.bayesian)
ACP.theta.HPDBaye <- mean(CP.BAYE2.theta)
AAW.theta.HPDBaye <- mean(AW.BAYE2.theta)
ACP.eta.Baye <- mean(CP.eta.bayesian)
AAW.eta.Baye <- mean(AW.eta.bayesian)
ACP.eta.HPDBaye <- mean(CP.BAYE2.eta)
AAW.eta.HPDBaye <- mean(AW.BAYE2.eta)
cat("CP theta BayesianMCMC=",ACP.theta.Baye,"\n")
cat("AW theta BayesianMCMC=",AAW.theta.Baye,"\n")
cat("CP theta BayesianHPTD=",ACP.theta.HPDBaye,"\n")
cat("AW theta BayesianHPTD=",AAW.theta.HPDBaye,"\n")
cat("CP eta BayesianMCMC=",ACP.eta.Baye,"\n")
cat("AW eta BayesianMCMC=",AAW.eta.Baye,"\n")
cat("CP eta BayesianHPTD=",ACP.eta.HPDBaye,"\n")
cat("AW eta BayesianHPTD=",AAW.eta.HPDBaye,"\n")

beep()
}

CI.DiffRatio.CV <- function(M, m, nx, ny, mu_x, mu_y, lambda_x, lambda_y, delta_x,
delta_y) {
  # --- Dependencies ---
  if (!requireNamespace("statmod", quietly = TRUE)) stop("Package 'statmod' is
required.")
  if (!requireNamespace("HDInterval", quietly = TRUE)) stop("Package 'HDInterval' is
required.")
  rinvgauss <- statmod::rinvgauss
  hdi <- HDInterval::hdi

  starttime <- proc.time()
  set.seed(999)
  B <- 1000 # bootstrap replications

```

```

# --- Storage ---
# theta = CVx - CVy, eta = CVx / CVy
CP.GCI_theta <- numeric(M); Length.GCI_theta <- numeric(M)
CP.AGCI_theta <- numeric(M); Length.AGCI_theta <- numeric(M)
CP.BPCI_theta <- numeric(M); Length.BPCI_theta <- numeric(M)
CP.FCI_theta <- numeric(M); Length.FCI_theta <- numeric(M)
CP.FHPD_theta <- numeric(M); Length.FHPD_theta <- numeric(M)

CP.GCI_eta <- numeric(M); Length.GCI_eta <- numeric(M)
CP.AGCI_eta <- numeric(M); Length.AGCI_eta <- numeric(M)
CP.BPCI_eta <- numeric(M); Length.BPCI_eta <- numeric(M)
CP.FCI_eta <- numeric(M); Length.FCI_eta <- numeric(M)
CP.FHPD_eta <- numeric(M); Length.FHPD_eta <- numeric(M)

# CIMV (new method) for difference and ratio
CP.MV_diff <- numeric(M); AW.MV_diff <- numeric(M)
CP.MV_ratio <- numeric(M); AW.MV_ratio <- numeric(M)

# true CVs and targets
cv_x <- sqrt((mu_x + (lambda_x * delta_x)) / ((1 - delta_x) * lambda_x))
cv_y <- sqrt((mu_y + (lambda_y * delta_y)) / ((1 - delta_y) * lambda_y))
theta <- cv_x - cv_y
eta <- cv_x / cv_y

# --- Simulation loop ---
for (i in 1:M) {
  #----- Generate X -----#
  x0 <- rbinom(nx, 1, delta_x) # 1 = zero, 0 = nonzero
  n0_x <- sum(x0 == 1)
  n1_x <- nx - n0_x
  if (n1_x <= 1) stop("n1_x <= 1 occurred; increase nx or reduce delta_x.")
  x1 <- rinvgauss(n1_x, mu_x, lambda_x)
  delta_hat_x <- n0_x / nx
  muhat_x <- mean(x1)
  lambdahat_inverse_x <- (1 / n1_x) * sum((1 / x1) - (1 / muhat_x))
  lambdahat_x <- 1 / lambdahat_inverse_x

  #----- Generate Y -----#
  y0 <- rbinom(ny, 1, delta_y)
  n0_y <- sum(y0 == 1)
  n1_y <- ny - n0_y
  if (n1_y <= 1) stop("n1_y <= 1 occurred; increase ny or reduce delta_y.")
  y1 <- rinvgauss(n1_y, mu_y, lambda_y)
  delta_hat_y <- n0_y / ny

```

```

muhat_y <- mean(y1)
lambdahat_inverse_y <- (1 / n1_y) * sum((1 / y1) - (1 / muhat_y))
lambdahat_y <- 1 / lambdahat_inverse_y

#----- GPQ Replications -----#
R_theta_GCI <- numeric(m)
R_eta_GCI <- numeric(m)
R_theta_AGCI <- numeric(m)
R_eta_AGCI <- numeric(m)

# also store per-rep CVs for AGCI to build CIMV bounds
R_CV_AGCI_x <- numeric(m)
R_CV_AGCI_y <- numeric(m)

for (j in 1:m) {
  # ===== X: GCI pieces =====
  xbar <- rinvgauss(1, mu_x, n1_x * lambda_x)
  A_x <- rchisq(1, n1_x - 1)
  V_x <- A_x / (n1_x * lambda_x)
  Z.VST_x <- 2 * sqrt(nx) * (asin(sqrt(delta_hat_x)) - asin(sqrt(delta_x)))
  delta_GPQ_x <- (sin(asin(sqrt(delta_hat_x)) - Z.VST_x / (2 * sqrt(nx))))^2
  R_lambda_x <- A_x / (n1_x * lambdahat_inverse_x)
  # GCI mean GPQ
  Z_x <- rnorm(1)
  R_mu_GCI_x <- xbar / abs(1 + Z_x * sqrt(xbar / (n1_x * R_lambda_x)))
  R_CV_GCI_x <- sqrt((R_mu_GCI_x + (R_lambda_x * delta_GPQ_x)) / ((1 -
delta_GPQ_x) * R_lambda_x))

  # ===== Y: GCI pieces =====
  ybar <- rinvgauss(1, mu_y, n1_y * lambda_y)
  A_y <- rchisq(1, n1_y - 1)
  V_y <- A_y / (n1_y * lambda_y)
  Z.VST_y <- 2 * sqrt(ny) * (asin(sqrt(delta_hat_y)) - asin(sqrt(delta_y)))
  delta_GPQ_y <- (sin(asin(sqrt(delta_hat_y)) - Z.VST_y / (2 * sqrt(ny))))^2
  R_lambda_y <- A_y / (n1_y * lambdahat_inverse_y)
  Z_y <- rnorm(1)
  R_mu_GCI_y <- ybar / abs(1 + Z_y * sqrt(ybar / (n1_y * R_lambda_y)))
  R_CV_GCI_y <- sqrt((R_mu_GCI_y + (R_lambda_y * delta_GPQ_y)) / ((1 -
delta_GPQ_y) * R_lambda_y))

  # GCI combos
  R_theta_GCI[j] <- R_CV_GCI_x - R_CV_GCI_y
  R_eta_GCI[j] <- R_CV_GCI_x / R_CV_GCI_y

```

```

# ===== AGCI pieces (adjusted) =====
W_x <- sqrt(n1_x - 1) * (xbar / mu_x - 1) / sqrt(V_x * xbar)
R_mu_AGCI_x <- xbar / max(0, 1 + W_x * sqrt((V_x * xbar) / (n1_x - 1)))
R_CV_AGCI_x[j] <- sqrt((R_mu_AGCI_x + (R_lambda_x * delta_GPQ_x)) / ((1
- delta_GPQ_x) * R_lambda_x))

W_y <- sqrt(n1_y - 1) * (ybar / mu_y - 1) / sqrt(V_y * ybar)
R_mu_AGCI_y <- ybar / max(0, 1 + W_y * sqrt((V_y * ybar) / (n1_y - 1)))
R_CV_AGCI_y[j] <- sqrt((R_mu_AGCI_y + (R_lambda_y * delta_GPQ_y)) / ((1
- delta_GPQ_y) * R_lambda_y))

R_theta_AGCI[j] <- R_CV_AGCI_x[j] - R_CV_AGCI_y[j]
R_eta_AGCI[j] <- R_CV_AGCI_x[j] / R_CV_AGCI_y[j]
}

# GCI bounds (theta, eta)
L.GCI_theta <- quantile(R_theta_GCI, 0.025, type = 8, na.rm = TRUE)
U.GCI_theta <- quantile(R_theta_GCI, 0.975, type = 8, na.rm = TRUE)
CP.GCI_theta[i] <- as.integer(L.GCI_theta < theta & theta < U.GCI_theta)
Length.GCI_theta[i] <- U.GCI_theta - L.GCI_theta

L.GCI_eta <- quantile(R_eta_GCI, 0.025, type = 8, na.rm = TRUE)
U.GCI_eta <- quantile(R_eta_GCI, 0.975, type = 8, na.rm = TRUE)
CP.GCI_eta[i] <- as.integer(L.GCI_eta < eta & eta < U.GCI_eta)
Length.GCI_eta[i] <- U.GCI_eta - L.GCI_eta

# AGCI bounds (theta, eta)
L.AGCI_theta <- quantile(R_theta_AGCI, 0.025, type = 8, na.rm = TRUE)
U.AGCI_theta <- quantile(R_theta_AGCI, 0.975, type = 8, na.rm = TRUE)
CP.AGCI_theta[i] <- as.integer(L.AGCI_theta < theta & theta < U.AGCI_theta)
Length.AGCI_theta[i] <- U.AGCI_theta - L.AGCI_theta

L.AGCI_eta <- quantile(R_eta_AGCI, 0.025, type = 8, na.rm = TRUE)
U.AGCI_eta <- quantile(R_eta_AGCI, 0.975, type = 8, na.rm = TRUE)
CP.AGCI_eta[i] <- as.integer(L.AGCI_eta < eta & eta < U.AGCI_eta)
Length.AGCI_eta[i] <- U.AGCI_eta - L.AGCI_eta

# ===== AGCI marginal bounds for each CV (needed for CIMV) =====
L.CV.AGCI_x <- quantile(R_CV_AGCI_x, 0.025, type = 8, na.rm = TRUE)
U.CV.AGCI_x <- quantile(R_CV_AGCI_x, 0.975, type = 8, na.rm = TRUE)
L.CV.AGCI_y <- quantile(R_CV_AGCI_y, 0.025, type = 8, na.rm = TRUE)
U.CV.AGCI_y <- quantile(R_CV_AGCI_y, 0.975, type = 8, na.rm = TRUE)

# ===== CIMV (Difference) =====

```

```

L.CIMV_diff.vec <- (R_CV_AGCI_x - R_CV_AGCI_y) -
  sqrt(((R_CV_AGCI_x - L.CV.AGCI_x)^2) + ((U.CV.AGCI_y -
R_CV_AGCI_y)^2))
U.CIMV_diff.vec <- (R_CV_AGCI_x - R_CV_AGCI_y) +
  sqrt(((U.CV.AGCI_x - R_CV_AGCI_x)^2) + ((R_CV_AGCI_y -
L.CV.AGCI_y)^2))
L.CIMV_diff <- quantile(L.CIMV_diff.vec, 0.025, na.rm = TRUE)
U.CIMV_diff <- quantile(U.CIMV_diff.vec, 0.975, na.rm = TRUE)
CP.MV_diff[i] <- as.integer(theta > L.CIMV_diff & theta < U.CIMV_diff)
AW.MV_diff[i] <- U.CIMV_diff - L.CIMV_diff

# ===== CIMV (Ratio) =====
L.CIMV_ratio.vec <- ((R_CV_AGCI_x * R_CV_AGCI_y) -
  sqrt(((R_CV_AGCI_x * R_CV_AGCI_y)^2) -
  (L.CV.AGCI_x * U.CV.AGCI_y) *
  (2 * R_CV_AGCI_x - L.CV.AGCI_x) *
  (2 * R_CV_AGCI_y - U.CV.AGCI_y)))) /
  (U.CV.AGCI_y * (2 * R_CV_AGCI_y - U.CV.AGCI_y))
U.CIMV_ratio.vec <- ((R_CV_AGCI_x * R_CV_AGCI_y) +
  sqrt(((R_CV_AGCI_x * R_CV_AGCI_y)^2) -
  (U.CV.AGCI_x * L.CV.AGCI_y) *
  (2 * R_CV_AGCI_x - U.CV.AGCI_x) *
  (2 * R_CV_AGCI_y - L.CV.AGCI_y)))) /
  (L.CV.AGCI_y * (2 * R_CV_AGCI_y - L.CV.AGCI_y))
L.CIMV_ratio <- quantile(L.CIMV_ratio.vec, 0.025, na.rm = TRUE)
U.CIMV_ratio <- quantile(U.CIMV_ratio.vec, 0.975, na.rm = TRUE)
CP.MV_ratio[i] <- as.integer(eta > L.CIMV_ratio & eta < U.CIMV_ratio)
AW.MV_ratio[i] <- U.CIMV_ratio - L.CIMV_ratio

# ===== BPCI (bootstrap percentile) =====
theta.star <- numeric(B)
eta.star <- numeric(B)
for (k in 1:B) {
  x.star <- sample(x1, n1_x, replace = TRUE)
  xbar.star <- mean(x.star)
  lambdahat_inv_bt_x <- (1 / n1_x) * sum((1 / x.star) - (1 / xbar.star))
  lambdahat_bt_x <- 1 / lambdahat_inv_bt_x

  y.star <- sample(y1, n1_y, replace = TRUE)
  ybar.star <- mean(y.star)
  lambdahat_inv_bt_y <- (1 / n1_y) * sum((1 / y.star) - (1 / ybar.star))
  lambdahat_bt_y <- 1 / lambdahat_inv_bt_y
}

```

```

# plug-in delta via variance-stabilizing approach
Z.B_x <- 2 * sqrt(nx) * (asin(sqrt(delta_hat_x)) - asin(sqrt(delta_x)))
delta_B_x <- (sin(asin(sqrt(delta_hat_x)) - Z.B_x / (2 * sqrt(nx))))^2

Z.B_y <- 2 * sqrt(ny) * (asin(sqrt(delta_hat_y)) - asin(sqrt(delta_y)))
delta_B_y <- (sin(asin(sqrt(delta_hat_y)) - Z.B_y / (2 * sqrt(ny))))^2

CV.star_x <- sqrt((xbar.star + (lambdahat_bt_x * delta_B_x)) / ((1 - delta_B_x) *
lambdahat_bt_x))
CV.star_y <- sqrt((ybar.star + (lambdahat_bt_y * delta_B_y)) / ((1 - delta_B_y) *
lambdahat_bt_y))

theta.star[k] <- CV.star_x - CV.star_y
eta.star[k] <- CV.star_x / CV.star_y
}
L.BPCI_theta <- quantile(theta.star, 0.025, type = 8, na.rm = TRUE)
U.BPCI_theta <- quantile(theta.star, 0.975, type = 8, na.rm = TRUE)
CP.BPCI_theta[i] <- as.integer(L.BPCI_theta < theta & theta < U.BPCI_theta)
Length.BPCI_theta[i] <- U.BPCI_theta - L.BPCI_theta

L.BPCI_eta <- quantile(eta.star, 0.025, type = 8, na.rm = TRUE)
U.BPCI_eta <- quantile(eta.star, 0.975, type = 8, na.rm = TRUE)
CP.BPCI_eta[i] <- as.integer(L.BPCI_eta < eta & eta < U.BPCI_eta)
Length.BPCI_eta[i] <- U.BPCI_eta - L.BPCI_eta

# ===== Fiducial (Gibbs) =====
T_iter <- 20000
mu.t_x <- numeric(T_iter); lambda.t_x <- numeric(T_iter)
mu.t_y <- numeric(T_iter); lambda.t_y <- numeric(T_iter)
mu.t_x[1] <- muhat_x; lambda.t_x[1] <- lambdahat_x
mu.t_y[1] <- muhat_y; lambda.t_y[1] <- lambdahat_y

for (t in 2:T_iter) {
  # keep last MLEs fixed as centers (randomization via sampling steps)
  lambda.t_x[t] <- (lambda.t_x[t-1] / n1_x) * rchisq(1, n1_x - 1)
  mu.t_x[t] <- rinvgauss(1, mu.t_x[t-1], n1_x * lambda.t_x[t])

  lambda.t_y[t] <- (lambda.t_y[t-1] / n1_y) * rchisq(1, n1_y - 1)
  mu.t_y[t] <- rinvgauss(1, mu.t_y[t-1], n1_y * lambda.t_y[t])
}

Z.F_x <- 2 * sqrt(nx) * (asin(sqrt(delta_hat_x)) - asin(sqrt(delta_x)))
delta_F_x <- (sin(asin(sqrt(delta_hat_x)) - Z.F_x / (2 * sqrt(nx))))^2

```

```

Z.F_y <- 2 * sqrt(ny) * (asin(sqrt(delta_hat_y)) - asin(sqrt(delta_y)))
delta_F_y <- (sin(asin(sqrt(delta_hat_y)) - Z.F_y / (2 * sqrt(ny))))^2

thetahat.FCI_x <- sqrt((mu.t_x + (lambda.t_x * delta_F_x)) / ((1 - delta_F_x) *
lambda.t_x))
thetahat.FCI_y <- sqrt((mu.t_y + (lambda.t_y * delta_F_y)) / ((1 - delta_F_y) *
lambda.t_y))
theta.FCI <- thetahat.FCI_x - thetahat.FCI_y
eta.FCI <- thetahat.FCI_x / thetahat.FCI_y

# burn-in 1000
if (length(theta.FCI) <= 1000) stop("Not enough Gibbs iterations for burn-in.")
theta.FCI <- theta.FCI[-(1:1000)]
eta.FCI <- eta.FCI[-(1:1000)]

L.CI.FCI_theta <- quantile(theta.FCI, 0.025, type = 8, na.rm = TRUE)
U.CI.FCI_theta <- quantile(theta.FCI, 0.975, type = 8, na.rm = TRUE)
CP.FCI_theta[i] <- as.integer(L.CI.FCI_theta < theta & theta < U.CI.FCI_theta)
Length.FCI_theta[i] <- U.CI.FCI_theta - L.CI.FCI_theta

L.CI.FCI_eta <- quantile(eta.FCI, 0.025, type = 8, na.rm = TRUE)
U.CI.FCI_eta <- quantile(eta.FCI, 0.975, type = 8, na.rm = TRUE)
CP.FCI_eta[i] <- as.integer(L.CI.FCI_eta < eta & eta < U.CI.FCI_eta)
Length.FCI_eta[i] <- U.CI.FCI_eta - L.CI.FCI_eta

# ===== Fiducial HPD =====
FHPD_theta <- hdi(theta.FCI, credMass = 0.95)
L.FHPD_theta <- FHPD_theta[1]; U.FHPD_theta <- FHPD_theta[2]
CP.FHPD_theta[i] <- as.integer(L.FHPD_theta < theta & theta < U.FHPD_theta)
Length.FHPD_theta[i] <- U.FHPD_theta - L.FHPD_theta

FHPD_eta <- hdi(eta.FCI, credMass = 0.95)
L.FHPD_eta <- FHPD_eta[1]; U.FHPD_eta <- FHPD_eta[2]
CP.FHPD_eta[i] <- as.integer(L.FHPD_eta < eta & eta < U.FHPD_eta)
Length.FHPD_eta[i] <- U.FHPD_eta - L.FHPD_eta
}

# --- Summaries ---
elapsedTime <- (proc.time() - starttime)[["elapsed"]]

ACP.GCI_theta <- mean(CP.GCI_theta); Ex.length.GCI_theta <-
mean(Length.GCI_theta)

```

```

ACP.AGCI_theta <- mean(CP.AGCI_theta);    Ex.length.AGCI_theta <-
mean(Length.AGCI_theta)
ACP.BPCI_theta <- mean(CP.BPCI_theta);    Ex.length.BPCI_theta <-
mean(Length.BPCI_theta)
ACP.FCI_theta <- mean(CP.FCI_theta);      Ex.length.FCI_theta <-
mean(Length.FCI_theta)
ACP.FHPD_theta <- mean(CP.FHPD_theta);    Ex.length.FHPD_theta <-
mean(Length.FHPD_theta)

ACP.GCI_eta <- mean(CP.GCI_eta);          Ex.length.GCI_eta <-
mean(Length.GCI_eta)
ACP.AGCI_eta <- mean(CP.AGCI_eta);        Ex.length.AGCI_eta <-
mean(Length.AGCI_eta)
ACP.BPCI_eta <- mean(CP.BPCI_eta);        Ex.length.BPCI_eta <-
mean(Length.BPCI_eta)
ACP.FCI_eta <- mean(CP.FCI_eta);          Ex.length.FCI_eta <-
mean(Length.FCI_eta)
ACP.FHPD_eta <- mean(CP.FHPD_eta);        Ex.length.FHPD_eta <-
mean(Length.FHPD_eta)

ACP.MV_diff <- mean(CP.MV_diff, na.rm = TRUE)
AWbar.MV_diff <- mean(AW.MV_diff, na.rm = TRUE)
ACP.MV_ratio <- mean(CP.MV_ratio, na.rm = TRUE)
AWbar.MV_ratio <- mean(AW.MV_ratio, na.rm = TRUE)

# --- Print (like your original style) ---
cat("CP|GCI_theta=", ACP.GCI_theta, "Length|GCI_theta=", Ex.length.GCI_theta,
"\n")
cat("CP|AGCI_theta=", ACP.AGCI_theta, "Length|AGCI_theta=",
Ex.length.AGCI_theta, "\n")
cat("CP|BPCI_theta=", ACP.BPCI_theta, "Length|BPCI_theta=",
Ex.length.BPCI_theta, "\n")
cat("CP|FCI_theta=", ACP.FCI_theta, "Length|FCI_theta=", Ex.length.FCI_theta,
"\n")
cat("CP|FHPD_theta=", ACP.FHPD_theta, "Length|FHPD_theta=",
Ex.length.FHPD_theta, "\n")

cat("CP|GCI_eta=", ACP.GCI_eta, "Length|GCI_eta=", Ex.length.GCI_eta,
"\n")
cat("CP|AGCI_eta=", ACP.AGCI_eta, "Length|AGCI_eta=", Ex.length.AGCI_eta,
"\n")
cat("CP|BPCI_eta=", ACP.BPCI_eta, "Length|BPCI_eta=", Ex.length.BPCI_eta,
"\n")
cat("CP|FCI_eta=", ACP.FCI_eta, "Length|FCI_eta=", Ex.length.FCI_eta, "\n")

```

```

cat("CP|FHPD_eta=",          ACP.FHPD_eta,          "Length|FHPD_eta=",
Ex.length.FHPD_eta, "\n")

cat("CP|CIMV_diff=", ACP.MV_diff, "AW|CIMV_diff=", AWbar.MV_diff,
"\n")
cat("CP|CIMV_ratio=", ACP.MV_ratio, "AW|CIMV_ratio=", AWbar.MV_ratio,
"\n")
cat("Time =", elapsedTime, "\n")

# --- Return all summaries as a list ---
invisible(list(
  theta = theta, eta = eta,
  GCI_theta_CP = ACP.GCI_theta, GCI_theta_AW = Ex.length.GCI_theta,
  AGCI_theta_CP = ACP.AGCI_theta, AGCI_theta_AW = Ex.length.AGCI_theta,
  BPCI_theta_CP = ACP.BPCI_theta, BPCI_theta_AW = Ex.length.BPCI_theta,
  FCI_theta_CP = ACP.FCI_theta, FCI_theta_AW = Ex.length.FCI_theta,
  FHPD_theta_CP = ACP.FHPD_theta, FHPD_theta_AW = Ex.length.FHPD_theta,
  GCI_eta_CP = ACP.GCI_eta, GCI_eta_AW = Ex.length.GCI_eta,
  AGCI_eta_CP = ACP.AGCI_eta, AGCI_eta_AW = Ex.length.AGCI_eta,
  BPCI_eta_CP = ACP.BPCI_eta, BPCI_eta_AW = Ex.length.BPCI_eta,
  FCI_eta_CP = ACP.FCI_eta, FCI_eta_AW = Ex.length.FCI_eta,
  FHPD_eta_CP = ACP.FHPD_eta, FHPD_eta_AW = Ex.length.FHPD_eta,
  CIMV_diff_CP = ACP.MV_diff, CIMV_diff_AW = AWbar.MV_diff,
  CIMV_ratio_CP = ACP.MV_ratio, CIMV_ratio_AW = AWbar.MV_ratio,
  elapsed_time = elapsedTime
))
}

#####NA#####
tic()

set.seed(95)

# Single

# begin process
starttime <- proc.time()

# x = data
# M = number of simulation runs
# n0 = size of zero in population
# n1 = size of non-zero in population
# tau = mean of delta-IG dist
# alpha_sig = level of significance

```

```
M <- 5000
```

```
n_x=50 #,30,50,100,200
```

```
n_y=50 #,30,50,100,200
```

```
delta_x=0.3
```

```
delta_y=0.3
```

```
mu_x=0.5
```

```
mu_y=0.5
```

```
lambda_x=1
```

```
lambda_y=1
```

```
for (mu_x in c(0.5,1)) #0.3,0.5
```

```
{
```

```
  for (mu_y in c(0.5,1))
```

```
  {
```

```
    for (lambda_x in c(1,5,10)) #0.5,1
```

```
    {
```

```
      for (lambda_y in c(1,5,10)) #0.5,1
```

```
      {
```

```
        for(delta_x in c(0.3,0.5)) # <- 1,2,5,10
```

```
        {
```

```
          for(delta_y in c(0.3,0.5)) # <- 1,2,5,10
```

```
          {
```

```
            alpha_sig <-0.05
```

```
            deltaprime_x <- 1-delta_x
```

```
            deltaprime_y <- 1-delta_y
```

```
            #estimator Variance
```

```
            cv_x <- sqrt((mu_x+(lambda_x*delta_x))/((1-delta_x)*lambda_x))
```

```
            cv_y <- sqrt((mu_y+(lambda_y*delta_y))/((1-delta_y)*lambda_y))
```

```
            theta <- cv_x-cv_y
```

```
            eta <- cv_x/cv_y
```

```
            #-----NA-----#
```

```
            L.NA_theta=c()
```

```
            U.NA_theta=c()
```

```
            L.NA_eta=c()
```

```
            U.NA_eta=c()
```

```
            AV.NA_theta=c()
```

```
            CP.NA_theta=c()
```

```

AverLengthNAVar_theta=c()
CoveragePNAVar_theta=c()
AV.NA_eta=c()
CP.NA_eta=c()
AverLengthNAVar_eta=c()
CoveragePNAVar_eta=c()

```

```

mle_x=c()
mle_mu_x=c()
mle_lambda_x=c()
mle_delta_x=c()
mle_y=c()
mle_mu_y=c()
mle_lambda_y=c()
mle_delta_y=c()
theta_NA=c()
se_theta_NA=c()
eta_NA=c()
se_eta_NA=c()

options(digits = 4)

#####
###      Start Iteration      ###
#####

NAfunc=function(x1_1,x1_2){
  mle_x=fitdist(x1_1, "invgauss", lower=c(0,0), start=list(mean=1,
shape=1))$estimate
  mle_y=fitdist(x1_2, "invgauss", lower=c(0,0), start=list(mean=1,
shape=1))$estimate
  mle_mu_x=mle_x[1]
  mle_mu_y=mle_y[1]
  mle_lambda_x=mle_x[2]
  mle_lambda_y=mle_y[2]
  mle_delta_x=n_zero_1/n_x
  mle_delta_y=n_zero_2/n_y
  z.alpha=qnorm(1-alpha_sig/2)

  cvhat_x      <-      sqrt((mle_mu_x+(mle_lambda_x*mle_delta_x))/((1-
mle_delta_x)*mle_lambda_x))
  cvhat_y      <-      sqrt((mle_mu_y+(mle_lambda_y*mle_delta_y))/((1-
mle_delta_y)*mle_lambda_y))

```

```
thetahat <- cvhat_x - cvhat_y
etahat <- cvhat_x / cvhat_y
```

```
var_cvhat_x <- (((1/2)*(((1-
mle_delta_x)*mle_lambda_x)/(mle_mu_x+(mle_lambda_x*mle_delta_x)))^1/2)*(1/(
1-mle_delta_x)*mle_lambda_x)^2)*(mle_mu_x^3/(mle_lambda_x*n1_1))+
(((1/2)*(((1-
mle_delta_x)*mle_lambda_x)/(mle_mu_x+(mle_lambda_x*mle_delta_x)))^1/2)*(((
1-mle_delta_x)*mle_lambda_x*mle_delta_x)-
(mle_mu_x+(mle_lambda_x*mle_delta_x))*(1-mle_delta_x))/((1-
mle_delta_x)*mle_lambda_x^2)^2)*((2*mle_lambda_x^2)/n1_1))+
(((1/2)*(((1-
mle_delta_x)*mle_lambda_x)/(mle_mu_x+(mle_lambda_x*mle_delta_x)))^1/2)*(((
1-mle_delta_x)*mle_lambda_x*mle_lambda_x)-
(mle_mu_x+(mle_lambda_x*mle_delta_x))*(-mle_lambda_x))/((1-
mle_delta_x)*mle_lambda_x^2)^2)*((mle_delta_x*(1-mle_delta_x))/n_x)
```

```
var_cvhat_y <- (((1/2)*(((1-
mle_delta_y)*mle_lambda_y)/(mle_mu_y+(mle_lambda_y*mle_delta_y)))^1/2)*(1/(
1-mle_delta_y)*mle_lambda_y)^2)*(mle_mu_y^3/(mle_lambda_y*n1_1))+
(((1/2)*(((1-
mle_delta_y)*mle_lambda_y)/(mle_mu_y+(mle_lambda_y*mle_delta_y)))^1/2)*(((
1-mle_delta_y)*mle_lambda_y*mle_delta_y)-
(mle_mu_y+(mle_lambda_y*mle_delta_y))*(1-mle_delta_y))/((1-
mle_delta_y)*mle_lambda_y^2)^2)*((2*mle_lambda_y^2)/n1_1))+
(((1/2)*(((1-
mle_delta_y)*mle_lambda_y)/(mle_mu_y+(mle_lambda_y*mle_delta_y)))^1/2)*(((
1-mle_delta_y)*mle_lambda_y*mle_lambda_y)-
(mle_mu_y+(mle_lambda_y*mle_delta_y))*(-mle_lambda_y))/((1-
mle_delta_y)*mle_lambda_y^2)^2)*((mle_delta_y*(1-mle_delta_y))/n_y)
```

```
var_cv_diff <- var_cvhat_x + var_cvhat_y
```

```
se_theta_NA <- sqrt(var_cv_diff)
se_eta_NA <- sqrt(var_cv_diff)
```

```
L.NA_theta=thetahat-z.alpha*se_theta_NA
U.NA_theta=thetahat+z.alpha*se_theta_NA
```

```
L.NA_eta=etahat-z.alpha*se_eta_NA
U.NA_eta=etahat+z.alpha*se_eta_NA
return(data.frame(L.NA_theta,U.NA_theta,L.NA_eta,U.NA_eta))
```

```

}

for (i in 1:M)
{ #begin loop i

#####
###      Generate x      ###
#####

  x_0_1 <- rbinom(n_x, size=1 , prob=delta_x) #Choose n_zero from
bi(n,delta)
  x0_1 <- x_0_1[which(x_0_1!=0)]
  n_zero_1 <- length(x0_1)
  n_nonzero_1 <- n_x-n_zero_1

  x_0_2 <- rbinom(n_y, size=1 , prob=delta_y) #Choose n_zero from
bi(n,delta)
  x0_2 <- x_0_2[which(x_0_2!=0)]
  n_zero_2 <- length(x0_2)
  n_nonzero_2 <- n_y-n_zero_2

  x1_1 <- rinvgauss(n_nonzero_1, mu_x, lambda_x)
  x1_2 <- rinvgauss(n_nonzero_2, mu_y, lambda_y)
  x1 <- c(x0_1-1,x1_1)
  x2 <- c(x0_2-1,x1_2)

  n0_1 <- n_zero_1
  n1_1 <- n_nonzero_1
  nonzero_value_1 <- x1[which(x1!=0)]
  delta_hat_1 <- n_zero_1/n_x

  n0_2 <- n_zero_2
  n1_2 <- n_nonzero_2
  nonzero_value_2 <- x2[which(x2!=0)]
  delta_hat_2 <- n_zero_2/n_y

#-----Begin NA-----#
#MME of alpha and beta from the original sample
#Call NAFunc
NAVar<-NAfunc(x1_1,x1_2)
L.NA_theta[i]=NAVar[,1]
U.NA_theta[i]=NAVar[,2]
L.NA_eta[i]=NAVar[,3]

```

```

U.NA_eta[i]=NAVar[,4]

#-----END NA-----#

#calculate lower, upper bound and length for NA
AV.NA_theta[i]=U.NA_theta[i]-L.NA_theta[i]
AV.NA_eta[i]=U.NA_eta[i]-L.NA_eta[i]

if ( L.NA_theta[i]<=theta && theta<=U.NA_theta[i] ) { #for NA
  CP.NA_theta[i]=1 } else {CP.NA_theta[i]=0}

if ( L.NA_eta[i]<=eta && eta<=U.NA_eta[i] ) { #for NA
  CP.NA_eta[i]=1 } else {CP.NA_eta[i]=0}
}

AverLengthNAVar_theta=mean(AV.NA_theta) #calculate the average length for
NA
CoveragePNAVar_theta=mean(CP.NA_theta)

AverLengthNAVar_eta=mean(AV.NA_eta) #calculate the average length for
NA
CoveragePNAVar_eta=mean(CP.NA_eta)

theta
eta

alpbe=c(n_x,n_y,mu_x,mu_y,lambda_x,lambda_y,delta_x,delta_y)
print(alpbe)
print(CoveragePNAVar_theta)
print(AverLengthNAVar_theta)
print(CoveragePNAVar_eta)
print(AverLengthNAVar_eta)
print("-----")
}
}
}
}
}
}
}

toc()

```

A3 Functions are used to calculate the coverage probability and the average width of confidence interval for the variance of delta-IG distribution.

The following R code is used to compute the confidence intervals, the coverage probability, and the average width of the confidence intervals for the variance of delta-IG distributions in Chapter 3.

```
#- This code was written by Wasurat Khumpasee -#

#####
###

library(VGAM)
library(MASS)
library(HDIInterval)
library(coda)
library(boot)
library(R2OpenBUGS)
library(statmod)
library(EnvStats)
library(beepr)

# M = number of simulation runs (iteration about 15,000 times)
# m = number of generalized computation (in GCI, AGCI about 5,000 times)
# n = number of sample size are (10,30,50,100, and 200)
# mu be 0.5 and 1
# lambda be 1,2,5, and 10
# delta be 0.1,0.3, and 0.5
# B = number of bootstrap sample

#Generating data
rZAIG <- function(x,mu,lambda,prob0){
  x_0 <- rbinom(x,size = 1,prob = 1-prob0)
  x0 <- subset(x_0,x_0 ==0)
  nzero <- length(x0)
  nNonzero <- x-nzero
  x1 <- rinvgauss(nNonzero,mu,lambda)
  x <- c(x0,x1)
  return(x)
}

#model from your distribution please change in phi[i] put in likelihood function
IGmodel <- function(){
```

```

for(i in 1:n){
  one[i] <- 1
  p[i]<-(sqrt(1/(2*3.14*pow(x[i],3))))*exp(-1*pow(x[i]-m,2)/2*pow(m,2)*x[i])/10000
  one[i] ~ dbern(p[i])
}
m ~ dunif(0.5,3) #define dunif(0,5) as m=0.5
l ~ dgamma(2,6); #define dgamma(5,16) as l=1
}

#define p as ratio
Binomialmodel <- function() {
  for (i in 1:n)
  {
    x[i] ~ dbern(p);
  }
  p ~ dbeta(8,6); #define dbeta(18,2) as p=0.5, dbeta(1,10) as p=0.3
}

CI.Var.Bayes <- function(M,n,m,l,p){
  #M = loop for montecarlo
  #n = number of sample size
  #m = mean parameter of ZAIG
  #l = shape parameter of ZAIG
  #p = probability of zero for ZAIG
  set.seed(999)
  CP.var.bayesian <- rep(0,M) # coverage probability of CI based on bayesianMCMC
  CP.BAYE2.var <- rep(0,M) # coverage probability of CI based on bayesianHPTD
  AW.var.bayesian <- rep(0,M)
  AW.BAYE2.var <- rep(0,M)
  mcoda <- rep(0,M)
  lcoda <- rep(0,M)
  pcoda <- rep(0,M)
  mean <- (1-p)*m
  var <- sqrt(m^2*(1-p)*((m/l)+p))
  #cv <- sqrt((m+(1*p))/((1-p)*l))
  ###stat iteration
  for(i in 1:M){
    x <- rZAIG(n,m,l,p)
    x0 <- subset(x,x==0)
    x1 <- subset(x,x>0)
    x0l <- ifelse(x>0,1,0)
    n0 <- length(x0)
    n1 <- length(x1)

```

```

#Bayesian
#for p
zerodata <- list(x = x0,n =length(x0))
inits1<-function(){ list( p=0) }
outputp <- bugs(data = zerodata, inits = inits1,parameters.to.save = c("p"),model.file
= Binomialmodel, n.chains = 1,n.iter = 20000,n.burnin =
5000,debug=FALSE,codaPkg=TRUE)
out.codap <- read.bugs(outputp)
codp <- as.vector(out.codap)
pcoda <- codp[[1]][,2]

#for m,1
ZAIGdata <- list(x = x1,n =length(x1))
inits2<-function(){ list(l=1, m=0.5) }
outputm <- bugs(data = ZAIGdata, inits = inits2, parameters.to.save =
c("l","m"),model.file = IGmodel, n.chains = 1,n.iter = 20000,n.burnin =
5000,debug=FALSE,codaPkg=TRUE)
out.codam <- read.bugs(outputm)
codm <- as.vector(out.codam)
mcoda <- codm[[1]][,3]
lcoda <- codm[[1]][,2]

baye.mean <- (1-pcoda)*mcoda
baye.var <- sqrt(mcodea^2*(1-pcoda)*((mcoda/lcoda)+pcoda))
#baye.cv <- sqrt((mcoda+(lcoda*pcoda))/((1-pcoda)*lcoda))
L.Baye.var <- quantile(baye.var,0.025,type = 8,na.rm = TRUE)
U.Baye.var <- quantile(baye.var,0.975,type = 8,na.rm = TRUE)
CP.var.bayesian[i] <- ifelse(L.Baye.var<var&&var<U.Baye.var,1,0)
AW.var.bayesian[i] <- U.Baye.var-L.Baye.var

#HPTD
HPD.baye.var =hdi(baye.var,0.95)
L.baye2.var =HPD.baye.var [1]
U.baye2.var =HPD.baye.var [2]
CP.BAYE2.var[i] = ifelse(L.baye2.var<var&&var<U.baye2.var,1,0) # compute
probability of CI based on BAYE of Baye
AW.BAYE2.var[i] = U.baye2.var- L.baye2.var # compute length of CI based on
BAYE of Baye

}

ACP.var.Baye <- mean(CP.var.bayesian)
AAW.var.Baye <- mean(AW.var.bayesian)
ACP.var.HPDBaye <- mean(CP.BAYE2.var)

```

```

AAW.var.HPDBaye <- mean(AW.BAYE2.var)
cat("CP var BayesianMCMC=",ACP.var.Baye,"\n")
cat("AW var BayesianMCMC=",AAW.var.Baye,"\n")
cat("CP var BayesianHPTD=",ACP.var.HPDBaye,"\n")
cat("AW var BayesianHPTD=",AAW.var.HPDBaye,"\n")
cat("Lower Bayes =",L.Baye.var,"Upper Bayes=",U.Baye.var,"\n")
cat("Lower HPTD =",L.baye2.var,"Upper HPTD=",U.baye2.var,"\n")

beep()
}

CI.dIG.Var=function(M,m,n,mu,lambda,delta)
{
  # Start

  starttime = proc.time()
  set.seed(999)
  B=1000      # number of bootstrap sample
  alpha=0.05

  CP.GCI = c()      # coverage probability of CI based on GCI
  Length.GCI = c()  # average length of CI based on GCI
  R_theta_GCI = c()

  CP.AGCI = c()      # coverage probability of CI based on AGCI
  Length.AGCI = c()  # average length of CI based on AGCI
  R_theta_AGCI = c()

  delta_GPQ <- c()
  Z.VST <- c()
  L.GCI <- c()
  U.GCI <- c()
  AV.GCI <- c()
  CP.GCI <- c()

  L.AGCI <- c()
  U.AGCI <- c()
  AV.AGCI <- c()
  CP.AGCI <- c()
  CP.BPCI = rep(0,M)      # coverage probability of CI based on BPCI
  Length.BPCI = rep(0,M)  # average length of CI based on BPCI
  theta.star = rep(0,B)

  CP.FCI = rep(0,M)      # coverage probability of CI based on Fiducial
  Length.FCI = rep(0,M)  # average length of CI based on Fiducial

```

```

CP.FHPD = rep(0,M)          # coverage probability of CI based on Fiducial HPD
Length.FHPD = rep(0,M)      # average length of CI based on Fiducial HPD

mu.t = c()
lambda.t = c()
mu.gibbs = c()
lambda.gibbs = c()

muhat.t = c()
lambdahat.t = c()

##### Estimation of parameter of interest #####
#delta <- n0/(n0+n1)
theta <- sqrt(mu^2*(1-delta)*((mu/lambda)+delta)) # var
#theta <- sqrt((mu+(lambda*delta))/((1-delta)*lambda)) # single CV
#####Start Iteration#####
for (i in 1:M)
{
  # _____ GENERATING X _____ #
  x_0 <- rbinom(n, size=1 , prob=delta) #Choose n_zero from bi(n,deltta) # generate
x0
  x0 <- x_0[which(x_0!=0)]
  n_zero <- length(x0)
  n_nonzero <- n-n_zero

  x1 <- rinvgauss(n_nonzero, mu, lambda) # generate x1
  x <- c(x0,x1) #combine x0,x1

  nonzero_value <- x[which(x!=0)]
  n1 <- n_nonzero
  n0 <- n_zero
  delta_hat <- n0/n

  muhat <- mean(x1) # compute mean of x (muhat)
  lambdahat_inverse <- (1/n1)*sum((1/x1)-(1/muhat))
  lambdahat <- 1/lambdahat_inverse # compute scale of x (lambdahat)

#####Compute CI based on GCI& AGCI#####
for (j in 1:m)
{
  # begin loop j
  xbar = rinvgauss(1, mu, n1*lambda)
  A=rchisq(1,n1-1)
  Z=rnorm(1,0,1)

```

```

V=A/(n1*lambda)
R_lambda = A/(n1*lambdahat_inverse) # compute GPQ for scale of x
R_mu_GCI = xbar/abs(1+Z*(sqrt(xbar/(n1*R_lambda)))) # compute GPQ for mean
of x
#generate Z from standard normal in Population
#Z.VST = rnorm(1,0,1)
Z.VST = 2*(sqrt(n))*(asin(sqrt(delta_hat))-asin(sqrt(delta)))

#Calculate GPQ for delta from VST
delta_VST = (sin(asin(sqrt(delta_hat))-(Z.VST/(2*sqrt(n))))^2

#Calculate GPQ for delta from Wilson
zw = rnorm(1,0,1)
q1 = (n0+((zw^2)/2))/(n+zw^2)
q2 = (zw)/(n+(zw^2))
q3 = (((n0*n1)/n)+(zw^2)/4)^(1/2)
delta_WS = q1-q2*q3

#Calculate GPQ for delta from Hannig
delta_HN = 0.5*(rbeta(1,n0,n1+1))+0.5*(rbeta(1,n0+1,n1)) #rbeta(1,n0,n1+1)

R_theta_GCI[j] = sqrt(R_mu_GCI^2*(1-
delta_WS)*((R_mu_GCI/R_lambda)+delta_WS)) # compute GPQ for single var

W = (sqrt(n1-1)*(xbar/mu-1))/sqrt(V*xbar)
R_mu_AGCI = xbar/max(0,1+(W*sqrt((V*xbar)/(n1-1)))) # compute GPQ
for mean of x
R_theta_AGCI[j] = sqrt(R_mu_AGCI^2*(1-
delta_WS)*((R_mu_AGCI/R_lambda)+delta_WS)) # compute GPQ for single CV
} # end loop j
L.GCI <- quantile(R_theta_GCI,0.025,type=8,na.rm = TRUE) # compute lower
based on GCI of theta
U.GCI <- quantile(R_theta_GCI,0.975,type=8,na.rm = TRUE) # compute upper
based on GCI of theta

L.AGCI <- quantile(R_theta_AGCI,0.025,type=8,na.rm = TRUE) # compute
lower based on AGCI of theta
U.AGCI <- quantile(R_theta_AGCI,0.975,type=8,na.rm = TRUE) # compute
upper based on AGCI of theta

CP.GCI[i] <- ifelse(L.GCI<theta&&theta<U.GCI,1,0) # compute probability of
CI based on GCI of theta

```

```

Length.GCI[i] <- U.GCI - L.GCI # compute length
of CI based on GCI of theta

CP.AGCI[i] <- ifelse(L.AGCI<theta&&theta<U.AGCI,1,0) # compute
probability of CI based on AGCI of theta
Length.AGCI[i] <- U.AGCI - L.AGCI # compute length of CI based on
AGCI of theta

#####Compute CI based on BPCI#####

for (k in 1:B)
{
  # begin loop k
  x.star= sample(x1,n1,replace=T) # generate bootstrap sample of x
  xbar.star= mean(x.star) # compute mean of bootstrap sample of x
  lambdahat_inv_bt = (1/n1) * sum((1/x.star)-(1/xbar.star))
  lambdahat_bt = 1/lambdahat_inv_bt # compute scale of bootstrap sample of x

  #generate Z from standard normal in Population
  Z.B = 2*(sqrt(n))*(asin(sqrt(delta_hat))-asin(sqrt(delta)))

  #Calculate for delta
  delta_B_VST = (sin(asin(sqrt(delta_hat))-(Z.B/(2*sqrt(n))))))^2

  #Calculate GPQ for delta from Wilson
  zw = rnorm(1,0,1)
  q1 = (n0+((zw^2)/2))/(n+zw^2)
  q2 = (zw)/(n+(zw^2))
  q3 = (((n0*n1)/n)+(zw^2)/4)^(1/2)
  delta_B_WS = q1-q2*q3

  #Calculate GPQ for delta from Hannig
  delta_B_HN = 0.5*(rbeta(1,n0,n1+1))+0.5*(rbeta(1,n0+1,n1)) #rbeta(1,n0,n1+1)

  theta.star[k]= sqrt(xbar.star^2*(1-
delta_B_WS)*((xbar.star/lambdahat_bt)+delta_B_WS)) # compute single
var of bootstrap sample
} # end loop k

L.BPCI <- quantile(theta.star,0.025,type=8,na.rm = TRUE) #compute lower
based on PBCI of theta
U.BPCI <- quantile(theta.star,0.975,type=8,na.rm = TRUE) #compute lower
based on PBCI of theta
CP.BPCI[i] <- ifelse( L.BPCI <theta&&theta< U.BPCI,1,0) #compute
probability of CI based on PBCI of theta

```

```
Length.BPCI[i] <- U.BPCI-L.BPCI      #compute length of CI based on PBCI of
theta
```

```
#####Compute CI based on FCI#####
```

```
muhat.t[1] = muhat      # MLE of mean x
```

```
lambdahat.t[1] = lambdahat      # MLE of scale x
```

```
### set t for Gibbs ###
```

```
for (t in 2:20000)
```

```
{      # begin loop t
```

```
  muhat.t[t]=muhat.t[t-1]
```

```
  lambdahat.t[t] =lambdahat.t[t-1]
```

```
  lambda.t[t] = (lambdahat.t[t]/n1)*rchisq(1,n1-1)
```

```
  mu.t[t] = rinvgauss(1, muhat.t[t], n1*lambdahat.t[t])
```

```
}      # end loop t
```

```
#generate Z from standard normal in Population
```

```
Z.F = 2*(sqrt(n))*(asin(sqrt(delta_hat))-asin(sqrt(delta)))
```

```
#Calculate for delta
```

```
delta_F_VST = (sin(asin(sqrt(delta_hat))-(Z.B/(2*sqrt(n)))))^2
```

```
#Calculate GPQ for delta from Wilson
```

```
zw =rnorm(1,0,1)
```

```
q1 = (n0+((zw^2)/2))/(n+zw^2)
```

```
q2 = (zw)/(n+(zw^2))
```

```
q3 = (((n0*n1)/n)+(zw^2)/4)^(1/2)
```

```
delta_F_WS = q1-q2*q3
```

```
#Calculate GPQ for delta from Hannig
```

```
delta_F_HN = 0.5*(rbeta(1,n0,n1+1))+0.5*(rbeta(1,n0+1,n1)) #rbeta(1,n0,n1+1)
```

```
mu.gibbs = mu.t
```

```
lambda.gibbs = lambda.t
```

```
## Gibbs sampling for fiducial distribution#
```

```
thetahat.FCI      =      sqrt(mu.gibbs^2*(1-
delta_F_WS)*((mu.gibbs/lambda.gibbs)+delta_F_WS))  # compute var base on
MCMC
```

```
theta.FCI = thetahat.FCI[-1:-1000]      # burn in 1000
```

```

L.CI.FCI <- quantile(theta.FCI,0.025,type=8, na.rm=TRUE)      # compute
lower based on Fiducial of theta
U.CI.FCI <- quantile(theta.FCI,0.975,type=8, na.rm=TRUE)      # compute
upper based on Fiducial of theta

CP.FCI[i] <- ifelse(L.CI.FCI<theta&&theta< U.CI.FCI,1,0) # compute probability
of CI based on Fiducial of theta
Length.FCI[i] <- U.CI.FCI-L.CI.FCI      # compute length of CI based on Fiducial
of theta

#####Compute CI based on F-HPDCI#####
FHPD = hdi(theta.FCI,0.95)
L.FHPD = FHPD[1]      # compute lower based on Fiducial HPD of theta
U.FHPD = FHPD[2]      # compute upper based on Fiducial HPD of theta
CP.FHPD[i] = ifelse(L.FHPD<theta&&theta<U.FHPD,1,0)      #compute
probability of CI based on Fiducial HPD of theta
Length.FHPD[i] <- U.FHPD-L.FHPD      #compute length of CI
based on Fiducial HPD of theta

} # end loop i

elapsedTime = proc.time()-starttime

ACP.GCI <- mean(CP.GCI)      # compute coverage probability based on
GCI of theta
Ex.length.GCI <- mean(Length.GCI)      # compute expected length based on
GCI of theta
cat("CP|GCI=", ACP.GCI,"Length|GCI=", Ex.length.GCI ,"\n")

ACP.AGCI <- mean(CP.AGCI)      #compute coverage probability based on AGCI
of theta
Ex.length.AGCI <- mean(Length.AGCI)      # compute expected length based on
AGCI of theta
cat("CP|AGCI=", ACP.AGCI,"Length|AGCI=", Ex.length.AGCI ,"\n")

ACP.BPCI <- mean(CP.BPCI)      # compute coverage probability based on BPCI
of theta
Ex.length.BPCI <- mean(Length.BPCI)      # compute expected length based
on BPCI of theta
cat("CP|BPCI=", ACP.BPCI,"Length|BPCI=", Ex.length.BPCI ,"\n")

ACP.FCI <- mean(CP.FCI)      # compute coverage probability based on Fiducial
of theta

```

```
Ex.length.FCI <- mean(Length.FCI) # compute expected length based on Fiducial
of theta
```

```
cat("CP|FCI=", ACP.FCI,"Length|FCI=", Ex.length.FCI,"\n")
```

```
ACP.FHPD <- mean(CP.FHPD) # compute coverage probability based on
Fiducial HPD of theta
```

```
Ex.length.FHPD <- mean(Length.FHPD) # compute expected length based on
Fiducial HPD of theta
```

```
cat("CP|FHPD=", ACP.FHPD,"Length|FHPD=", Ex.length.FHPD,"\n")
```

```
cat("Time =", elapsedTime["elapsed"],"\n")
```

```
} #end process.
```



A4 Functions are used to calculate the coverage probability and the average width of confidence interval for the difference and ratio between variances of delta-IG distribution.

The following R code is used to compute the confidence intervals, the coverage probability, and the average width of the confidence intervals for the difference and ratio between variances of delta-IG distributions in Chapter 3.

#- This code was written by Wasurat Khumpasee -#

```
library(VGAM)
library(MASS)
library(HDIInterval)
library(coda)
library(boot)
library(R2OpenBUGS)
library(statmod)
library(EnvStats)
library(beepr)

CI.DiffRatio.Var <- function(M, m, nx, ny, mu_x, mu_y, lambda_x, lambda_y, delta_x,
delta_y) {
  # --- Dependencies ---
  if (!requireNamespace("statmod", quietly = TRUE)) stop("Package 'statmod' is
required.")
  if (!requireNamespace("HDIInterval", quietly = TRUE)) stop("Package 'HDIInterval' is
required.")
  rinvgauss <- statmod::rinvgauss
  hdi <- HDIInterval::hdi

  starttime <- proc.time()
  set.seed(999)
  B <- 1000 # bootstrap replications

  # --- Storage ---
  # theta = CVx - CVy, eta = CVx / CVy
  CP.GCI_theta <- numeric(M); Length.GCI_theta <- numeric(M)
  CP.AGCI_theta <- numeric(M); Length.AGCI_theta <- numeric(M)
  CP.BPCI_theta <- numeric(M); Length.BPCI_theta <- numeric(M)
  CP.FCI_theta <- numeric(M); Length.FCI_theta <- numeric(M)
  CP.FHPD_theta <- numeric(M); Length.FHPD_theta <- numeric(M)

  CP.GCI_eta <- numeric(M); Length.GCI_eta <- numeric(M)
```

```

CP.AGCI_eta    <- numeric(M); Length.AGCI_eta  <- numeric(M)
CP.BPCI_eta    <- numeric(M); Length.BPCI_eta  <- numeric(M)
CP.FCI_eta     <- numeric(M); Length.FCI_eta   <- numeric(M)
CP.FHPD_eta    <- numeric(M); Length.FHPD_eta  <- numeric(M)

mu.t_x = c()
lambda.t_x = c()
mu.gibbs_x = c()
lambda.gibbs_x = c()

muhat.t_x = c()
lambdahat.t_x = c()

mu.t_y = c()
lambda.t_y = c()
mu.gibbs_y = c()
lambda.gibbs_y = c()

muhat.t_y = c()
lambdahat.t_y = c()

CP.MV_diff     <- numeric(M); AW.MV_diff      <- numeric(M)
CP.MV_ratio    <- numeric(M); AW.MV_ratio     <- numeric(M)

# true CVs and targets
var_x <- (mu_x^2)*(1-delta_x)*((mu_x/lambda_x)+delta_x)
var_y <- (mu_y^2)*(1-delta_y)*((mu_y/lambda_y)+delta_y)
theta <- var_x - var_y
eta   <- var_x / var_y

# --- Simulation loop ---
for (i in 1:M) {
  #----- Generate X -----#
  x0 <- rbinom(nx, 1, delta_x)    # 1 = zero, 0 = nonzero
  n0_x <- sum(x0 == 1)
  n1_x <- nx - n0_x
  if (n1_x <= 1) stop("n1_x <= 1 occurred; increase nx or reduce delta_x.")
  x1 <- rinvgauss(n1_x, mu_x, lambda_x)
  delta_hat_x <- n0_x / nx
  muhat_x <- mean(x1)
  lambdahat_inverse_x <- (1 / n1_x) * sum((1 / x1) - (1 / muhat_x))
  lambdahat_x <- 1 / lambdahat_inverse_x

  #----- Generate Y -----#

```

```

y0 <- rbinom(ny, 1, delta_y)
n0_y <- sum(y0 == 1)
n1_y <- ny - n0_y
if (n1_y <= 1) stop("n1_y <= 1 occurred; increase ny or reduce delta_y.")
y1 <- rinvgauss(n1_y, mu_y, lambda_y)
delta_hat_y <- n0_y / ny
muhat_y <- mean(y1)
lambdahat_inverse_y <- (1 / n1_y) * sum((1 / y1) - (1 / muhat_y))
lambdahat_y <- 1 / lambdahat_inverse_y

#----- GPQ Replications -----#
R_theta_GCI <- numeric(m)
R_eta_GCI <- numeric(m)
R_theta_AGCI <- numeric(m)
R_eta_AGCI <- numeric(m)

# also store per-rep CVs for AGCI to build CIMV bounds
R_Var_AGCI_x <- numeric(m)
R_Var_AGCI_y <- numeric(m)

for (j in 1:m) {
  # ===== X: GCI pieces =====
  xbar <- rinvgauss(1, mu_x, n1_x * lambda_x)
  A_x <- rchisq(1, n1_x - 1)
  V_x <- A_x / (n1_x * lambda_x)
  Z.VST_x <- 2 * sqrt(nx) * (asin(sqrt(delta_hat_x)) - asin(sqrt(delta_x)))
  delta_GPQ_x <- (sin(asin(sqrt(delta_hat_x)) - Z.VST_x / (2 * sqrt(nx))))^2
  R_lambda_x <- A_x / (n1_x * lambdahat_inverse_x)
  # GCI mean GPQ (Xu & Zhou style)
  Z_x <- rnorm(1)
  R_mu_GCI_x <- xbar / abs(1 + Z_x * sqrt(xbar / (n1_x * R_lambda_x)))
  R_Var_GCI_x <- (R_mu_GCI_x^2) * (1 - delta_GPQ_x) * ((R_mu_GCI_x / R_lambda_x) + delta_GPQ_x)

  # ===== Y: GCI pieces =====
  ybar <- rinvgauss(1, mu_y, n1_y * lambda_y)
  A_y <- rchisq(1, n1_y - 1)
  V_y <- A_y / (n1_y * lambda_y)
  Z.VST_y <- 2 * sqrt(ny) * (asin(sqrt(delta_hat_y)) - asin(sqrt(delta_y)))
  delta_GPQ_y <- (sin(asin(sqrt(delta_hat_y)) - Z.VST_y / (2 * sqrt(ny))))^2
  R_lambda_y <- A_y / (n1_y * lambdahat_inverse_y)
  Z_y <- rnorm(1)
  R_mu_GCI_y <- ybar / abs(1 + Z_y * sqrt(ybar / (n1_y * R_lambda_y)))

```

```

R_Var_GCI_y <- (R_mu_GCI_y^2)*(1-
delta_GPQ_y)*((R_mu_GCI_y/R_lambda_y)+delta_GPQ_y)

# GCI combos
R_theta_GCI[j] <- R_Var_GCI_x - R_Var_GCI_y
R_eta_GCI[j] <- R_Var_GCI_x / R_Var_GCI_y

# ===== AGCI pieces (adjusted) =====
W_x <- sqrt(n1_x - 1) * (xbar / mu_x - 1) / sqrt(V_x * xbar)
R_mu_AGCI_x <- xbar / max(0, 1 + W_x * sqrt((V_x * xbar) / (n1_x - 1)))
R_Var_AGCI_x[j] <- (R_mu_AGCI_x^2)*(1-
delta_GPQ_x)*((R_mu_AGCI_x/R_lambda_x)+delta_GPQ_x)

W_y <- sqrt(n1_y - 1) * (ybar / mu_y - 1) / sqrt(V_y * ybar)
R_mu_AGCI_y <- ybar / max(0, 1 + W_y * sqrt((V_y * ybar) / (n1_y - 1)))
R_Var_AGCI_y[j] <- (R_mu_AGCI_y^2)*(1-
delta_GPQ_y)*((R_mu_AGCI_y/R_lambda_y)+delta_GPQ_y)

R_theta_AGCI[j] <- R_Var_AGCI_x[j] - R_Var_AGCI_y[j]
R_eta_AGCI[j] <- R_Var_AGCI_x[j] / R_Var_AGCI_y[j]
}

# GCI bounds (theta, eta)
L.GCI_theta <- quantile(R_theta_GCI, 0.025, type = 8, na.rm = TRUE)
U.GCI_theta <- quantile(R_theta_GCI, 0.975, type = 8, na.rm = TRUE)
CP.GCI_theta[i] <- as.integer(L.GCI_theta < theta & theta < U.GCI_theta)
Length.GCI_theta[i] <- U.GCI_theta - L.GCI_theta

L.GCI_eta <- quantile(R_eta_GCI, 0.025, type = 8, na.rm = TRUE)
U.GCI_eta <- quantile(R_eta_GCI, 0.975, type = 8, na.rm = TRUE)
CP.GCI_eta[i] <- as.integer(L.GCI_eta < eta & eta < U.GCI_eta)
Length.GCI_eta[i] <- U.GCI_eta - L.GCI_eta

# AGCI bounds (theta, eta)
L.AGCI_theta <- quantile(R_theta_AGCI, 0.025, type = 8, na.rm = TRUE)
U.AGCI_theta <- quantile(R_theta_AGCI, 0.975, type = 8, na.rm = TRUE)
CP.AGCI_theta[i] <- as.integer(L.AGCI_theta < theta & theta < U.AGCI_theta)
Length.AGCI_theta[i] <- U.AGCI_theta - L.AGCI_theta

L.AGCI_eta <- quantile(R_eta_AGCI, 0.025, type = 8, na.rm = TRUE)
U.AGCI_eta <- quantile(R_eta_AGCI, 0.975, type = 8, na.rm = TRUE)
CP.AGCI_eta[i] <- as.integer(L.AGCI_eta < eta & eta < U.AGCI_eta)
Length.AGCI_eta[i] <- U.AGCI_eta - L.AGCI_eta

```

```

# ===== AGCI marginal bounds for each CV (needed for CIMV) =====
L.Var.AGCI_x <- quantile(R_Var_AGCI_x, 0.025, type = 8, na.rm = TRUE)
U.Var.AGCI_x <- quantile(R_Var_AGCI_x, 0.975, type = 8, na.rm = TRUE)
L.Var.AGCI_y <- quantile(R_Var_AGCI_y, 0.025, type = 8, na.rm = TRUE)
U.Var.AGCI_y <- quantile(R_Var_AGCI_y, 0.975, type = 8, na.rm = TRUE)

# ===== CIMV (Difference) =====
L.CIMV_diff.vec <- (R_CV_AGCI_x - R_CV_AGCI_y) -
  sqrt(((R_CV_AGCI_x - L.CV.AGCI_x)^2) + ((U.CV.AGCI_y -
R_CV_AGCI_y)^2))
U.CIMV_diff.vec <- (R_CV_AGCI_x - R_CV_AGCI_y) +
  sqrt(((U.CV.AGCI_x - R_CV_AGCI_x)^2) + ((R_CV_AGCI_y -
L.CV.AGCI_y)^2))
L.CIMV_diff <- quantile(L.CIMV_diff.vec, 0.025, na.rm = TRUE)
U.CIMV_diff <- quantile(U.CIMV_diff.vec, 0.975, na.rm = TRUE)
CP.MV_diff[i] <- as.integer(theta > L.CIMV_diff & theta < U.CIMV_diff)
AW.MV_diff[i] <- U.CIMV_diff - L.CIMV_diff

# ===== CIMV (Ratio) =====
L.CIMV_ratio.vec <- ((R_CV_AGCI_x * R_CV_AGCI_y) -
  sqrt(((R_CV_AGCI_x * R_CV_AGCI_y)^2) -
  (L.CV.AGCI_x * U.CV.AGCI_y) *
  (2 * R_CV_AGCI_x - L.CV.AGCI_x) *
  (2 * R_CV_AGCI_y - U.CV.AGCI_y)))) /
  (U.CV.AGCI_y * (2 * R_CV_AGCI_y - U.CV.AGCI_y))
U.CIMV_ratio.vec <- ((R_CV_AGCI_x * R_CV_AGCI_y) +
  sqrt(((R_CV_AGCI_x * R_CV_AGCI_y)^2) -
  (U.CV.AGCI_x * L.CV.AGCI_y) *
  (2 * R_CV_AGCI_x - U.CV.AGCI_x) *
  (2 * R_CV_AGCI_y - L.CV.AGCI_y)))) /
  (L.CV.AGCI_y * (2 * R_CV_AGCI_y - L.CV.AGCI_y))

L.CIMV_ratio <- quantile(L.CIMV_ratio.vec, 0.025, na.rm = TRUE)
U.CIMV_ratio <- quantile(U.CIMV_ratio.vec, 0.975, na.rm = TRUE)
CP.MV_ratio[i] <- as.integer(eta > L.CIMV_ratio & eta < U.CIMV_ratio)
AW.MV_ratio[i] <- U.CIMV_ratio - L.CIMV_ratio

# ===== BPCI (bootstrap percentile) =====
theta.star <- numeric(B)
eta.star <- numeric(B)
for (k in 1:B) {
  x.star <- sample(x1, n1_x, replace = TRUE)
  xbar.star <- mean(x.star)

```

```

lambdahat_inv_bt_x <- (1 / n1_x) * sum((1 / x.star) - (1 / xbar.star))
lambdahat_bt_x <- 1 / lambdahat_inv_bt_x

y.star <- sample(y1, n1_y, replace = TRUE)
ybar.star <- mean(y.star)
lambdahat_inv_bt_y <- (1 / n1_y) * sum((1 / y.star) - (1 / ybar.star))
lambdahat_bt_y <- 1 / lambdahat_inv_bt_y

# plug-in delta via variance-stabilizing approach
Z.B_x <- 2 * sqrt(nx) * (asin(sqrt(delta_hat_x)) - asin(sqrt(delta_x)))
delta_B_x <- (sin(asin(sqrt(delta_hat_x)) - Z.B_x / (2 * sqrt(nx))))^2

Z.B_y <- 2 * sqrt(ny) * (asin(sqrt(delta_hat_y)) - asin(sqrt(delta_y)))
delta_B_y <- (sin(asin(sqrt(delta_hat_y)) - Z.B_y / (2 * sqrt(ny))))^2

Var.star_x <- (xbar.star^2)*(1-delta_B_x)*((xbar.star/lambdahat_bt_x)+delta_B_x)
Var.star_y <- (ybar.star^2)*(1-delta_B_y)*((ybar.star/lambdahat_bt_y)+delta_B_y)

theta.star[k] <- Var.star_x - Var.star_y
eta.star[k] <- Var.star_x / Var.star_y
}
L.BPCI_theta <- quantile(theta.star, 0.025, type = 8, na.rm = TRUE)
U.BPCI_theta <- quantile(theta.star, 0.975, type = 8, na.rm = TRUE)
CP.BPCI_theta[i] <- as.integer(L.BPCI_theta < theta & theta < U.BPCI_theta)
Length.BPCI_theta[i] <- U.BPCI_theta - L.BPCI_theta

L.BPCI_eta <- quantile(eta.star, 0.025, type = 8, na.rm = TRUE)
U.BPCI_eta <- quantile(eta.star, 0.975, type = 8, na.rm = TRUE)
CP.BPCI_eta[i] <- as.integer(L.BPCI_eta < eta & eta < U.BPCI_eta)
Length.BPCI_eta[i] <- U.BPCI_eta - L.BPCI_eta

# ===== Fiducial (Gibbs) =====
T_iter <- 20000

muhat.t_x[1] = muhat_x # MLE of mean x
lambdahat.t_x[1] = lambdahat_x # MLE of scale x

muhat.t_y[1] = muhat_y # MLE of mean y
lambdahat.t_y[1] = lambdahat_y # MLE of scale y

#### set t for Gibbs ####
for (t in 2:20000)
{
# begin loop t
muhat.t_x[t]=muhat.t_x[t-1]

```

```

lambdahat.t_x[t] =lambdahat.t_x[t-1]
lambda.t_x[t] = (lambdahat.t_x[t]/n1_x)*rchisq(1,n1_x-1)
mu.t_x[t] = rinvgauss(1, muhat.t_x[t], n1_x*lambdahat.t_x[t])

muhat.t_y[t]=muhat.t_y[t-1]
lambdahat.t_y[t] =lambdahat.t_y[t-1]
lambda.t_y[t] = (lambdahat.t_y[t]/n1_y)*rchisq(1,n1_y-1)
mu.t_y[t] = rinvgauss(1, muhat.t_y[t], n1_y*lambdahat.t_y[t])

} # end loop t

#generate Z from standard normal in Population
Z.F_x = 2*(sqrt(nx))*(asin(sqrt(delta_hat_x))-asin(sqrt(delta_x)))

#Calculate for delta
delta_F_x = (sin(asin(sqrt(delta_hat_x))-(Z.F_x/(2*sqrt(nx))))))^2

#Calculate GPQ for delta from Wilson
zw_x=rnorm(1,0,1)
q1_x = (n0_x+((zw_x^2)/2))/(nx+zw_x^2)
q2_x = (zw_x)/(nx+(zw_x^2))
q3_x = (((n0_x*n1_x)/nx)+(zw_x^2)/4)^(1/2)
delta_F_WS_x = q1_x-q2_x*q3_x

#generate Z from standard normal in Population
Z.F_y = 2*(sqrt(ny))*(asin(sqrt(delta_hat_y))-asin(sqrt(delta_y)))

#Calculate for delta
delta_F_y = (sin(asin(sqrt(delta_hat_y))-(Z.F_y/(2*sqrt(ny))))))^2

#Calculate GPQ for delta from Wilson
zw_y=rnorm(1,0,1)
q1_y = (n0_y+((zw_y^2)/2))/(ny+zw_y^2)
q2_y = (zw_y)/(ny+(zw_y^2))
q3_y = (((n0_y*n1_y)/ny)+(zw_y^2)/4)^(1/2)
delta_F_WS_y = q1_y-q2_y*q3_y

mu.gibbs_x = mu.t_x
lambda.gibbs_x = lambda.t_x

mu.gibbs_y = mu.t_y
lambda.gibbs_y = lambda.t_y

## Gibbs sampling for fiducial distribution#

```

```

Var.FCI_x = (mu.gibbs_x^2)*(1-
delta_F_x)*((mu.gibbs_x/lambda.gibbs_x)+delta_F_x)
Var.FCI_y = (mu.gibbs_y^2)*(1-
delta_F_y)*((mu.gibbs_y/lambda.gibbs_y)+delta_F_y)
theta.FCI = Var.FCI_x - Var.FCI_y
eta.FCI = Var.FCI_x / Var.FCI_y
theta.FCI = theta.FCI[-1:-1000] # burn in 1000
eta.FCI = eta.FCI[-1:-1000]

L.CI.FCI_theta <- quantile(theta.FCI,0.025,type=8, na.rm=TRUE) #
compute lower based on Fiducial of theta
U.CI.FCI_theta <- quantile(theta.FCI,0.975,type=8, na.rm=TRUE) #
compute upper based on Fiducial of theta

CP.FCI_theta[i] <- ifelse(L.CI.FCI_theta<theta&&theta< U.CI.FCI_theta,1,0) #
compute probability of CI based on Fiducial of theta
Length.FCI_theta[i] <- U.CI.FCI_theta-L.CI.FCI_theta # compute
length of CI based on Fiducial of theta

L.CI.FCI_eta <- quantile(eta.FCI,0.025,type=8, na.rm=TRUE) #
compute lower based on Fiducial of theta
U.CI.FCI_eta <- quantile(eta.FCI,0.975,type=8, na.rm=TRUE) # compute
upper based on Fiducial of theta

CP.FCI_eta[i] <- ifelse(L.CI.FCI_eta<eta&&eta< U.CI.FCI_eta,1,0) # compute
probability of CI based on Fiducial of theta
Length.FCI_eta[i] <- U.CI.FCI_eta-L.CI.FCI_eta # compute
length of CI based on Fiducial of theta

#####Compute CI based on F-HPDCI#####
FHPD_theta = hdi(theta.FCI,0.95)
L.FHPD_theta = FHPD_theta[1] # compute lower
based on Fiducial HPD of theta
U.FHPD_theta = FHPD_theta[2] # compute upper
based on Fiducial HPD of theta
CP.FHPD_theta[i] = ifelse(L.FHPD_theta<theta&&theta<U.FHPD_theta,1,0) #
compute probability of CI based on Fiducial HPD of theta
Length.FHPD_theta[i] <- U.FHPD_theta-L.FHPD_theta # compute
length of CI based on Fiducial HPD of theta

FHPD_eta = hdi(eta.FCI,0.95)
L.FHPD_eta = FHPD_eta[1] # compute lower
based on Fiducial HPD of theta

```

```

    U.FHPD_eta = FHPD_eta[2]                                # compute upper
    based on Fiducial HPD of theta
    CP.FHPD_eta[i] = ifelse(L.FHPD_eta<eta&&eta<U.FHPD_eta,1,0) # compute
    probability of CI based on Fiducial HPD of theta
    Length.FHPD_eta[i] <- U.FHPD_eta-L.FHPD_eta              # compute
    length of CI based on Fiducial HPD of theta
  }

# --- Summaries ---
elapsedTime <- (proc.time() - starttime)[["elapsed"]]

  ACP.GCI_theta <- mean(CP.GCI_theta);      Ex.length.GCI_theta <-
  mean(Length.GCI_theta)
  ACP.AGCI_theta <- mean(CP.AGCI_theta);    Ex.length.AGCI_theta <-
  mean(Length.AGCI_theta)
  ACP.BPCI_theta <- mean(CP.BPCI_theta);    Ex.length.BPCI_theta <-
  mean(Length.BPCI_theta)
  ACP.FCI_theta <- mean(CP.FCI_theta);      Ex.length.FCI_theta <-
  mean(Length.FCI_theta)
  ACP.FHPD_theta <- mean(CP.FHPD_theta);    Ex.length.FHPD_theta <-
  mean(Length.FHPD_theta)

  ACP.GCI_eta <- mean(CP.GCI_eta);          Ex.length.GCI_eta <-
  mean(Length.GCI_eta)
  ACP.AGCI_eta <- mean(CP.AGCI_eta);        Ex.length.AGCI_eta <-
  mean(Length.AGCI_eta)
  ACP.BPCI_eta <- mean(CP.BPCI_eta);        Ex.length.BPCI_eta <-
  mean(Length.BPCI_eta)
  ACP.FCI_eta <- mean(CP.FCI_eta);          Ex.length.FCI_eta <-
  mean(Length.FCI_eta)
  ACP.FHPD_eta <- mean(CP.FHPD_eta);        Ex.length.FHPD_eta <-
  mean(Length.FHPD_eta)

  ACP.MV_diff <- mean(CP.MV_diff, na.rm = TRUE)
  AWbar.MV_diff <- mean(AW.MV_diff, na.rm = TRUE)
  ACP.MV_ratio <- mean(CP.MV_ratio, na.rm = TRUE)
  AWbar.MV_ratio <- mean(AW.MV_ratio, na.rm = TRUE)

# --- Print ---
cat("CP|GCI_theta=", ACP.GCI_theta, "AW|GCI_theta=", Ex.length.GCI_theta,
"\n")
cat("CP|AGCI_theta=", ACP.AGCI_theta, "AW|AGCI_theta=",
Ex.length.AGCI_theta, "\n")

```

```

cat("CP|BPCI_theta=", ACP.BPCI_theta, "AW|BPCI_theta=", Ex.length.BPCI_theta,
"\n")
cat("CP|FCI_theta=", ACP.FCI_theta, "AW|FCI_theta=", Ex.length.FCI_theta, "\n")
cat("CP|FHPD_theta=",          ACP.FHPD_theta,          "AW|FHPD_theta=",
Ex.length.FHPD_theta, "\n")

```

```

cat("CP|GCI_eta=", ACP.GCI_eta, "AW|GCI_eta=", Ex.length.GCI_eta, "\n")
cat("CP|AGCI_eta=", ACP.AGCI_eta, "AW|AGCI_eta=", Ex.length.AGCI_eta,
"\n")
cat("CP|BPCI_eta=", ACP.BPCI_eta, "AW|BPCI_eta=", Ex.length.BPCI_eta,
"\n")
cat("CP|FCI_eta=", ACP.FCI_eta, "AW|FCI_eta=", Ex.length.FCI_eta, "\n")
cat("CP|FHPD_eta=", ACP.FHPD_eta, "AW|FHPD_eta=", Ex.length.FHPD_eta,
"\n")

```

```

cat("CP|CIMV_diff=", ACP.MV_diff, "AW|CIMV_diff=", AWbar.MV_diff,
"\n")
cat("CP|CIMV_ratio=", ACP.MV_ratio, "AW|CIMV_ratio=", AWbar.MV_ratio,
"\n")
cat("Time =", elapsedTime, "\n")

```

```

# --- Return all summaries as a list ---
invisible(list(
  theta = theta, eta = eta,
  GCI_theta_CP = ACP.GCI_theta, GCI_theta_AW = Ex.length.GCI_theta,
  AGCI_theta_CP = ACP.AGCI_theta, AGCI_theta_AW = Ex.length.AGCI_theta,
  BPCI_theta_CP = ACP.BPCI_theta, BPCI_theta_AW = Ex.length.BPCI_theta,
  FCI_theta_CP = ACP.FCI_theta, FCI_theta_AW = Ex.length.FCI_theta,
  FHPD_theta_CP = ACP.FHPD_theta, FHPD_theta_AW = Ex.length.FHPD_theta,
  GCI_eta_CP = ACP.GCI_eta, GCI_eta_AW = Ex.length.GCI_eta,
  AGCI_eta_CP = ACP.AGCI_eta, AGCI_eta_AW = Ex.length.AGCI_eta,
  BPCI_eta_CP = ACP.BPCI_eta, BPCI_eta_AW = Ex.length.BPCI_eta,
  FCI_eta_CP = ACP.FCI_eta, FCI_eta_AW = Ex.length.FCI_eta,
  FHPD_eta_CP = ACP.FHPD_eta, FHPD_eta_AW = Ex.length.FHPD_eta,
  CIMV_diff_CP = ACP.MV_diff, CIMV_diff_AW = AWbar.MV_diff,
  CIMV_ratio_CP = ACP.MV_ratio, CIMV_ratio_AW = AWbar.MV_ratio,
  elapsed_time = elapsedTime
))
}

```

```
#####BAYES#####
```

```

#Generating data
rZAIG <- function(x,mu,lambda,prob0){

```

```

x_0 <- rbinom(x,size = 1,prob = 1-prob0)
x0 <- subset(x_0,x_0 ==0)
nzero <- length(x0)
nNonzero <- x-nzero
x1 <- rinvgauss(nNonzero,mu,lambda)
x <- c(x0,x1)
return(x)
}

#model from your distribution please change in phi[i] put in likelihood function
IGmodel <- function(){
  for(i in 1:n){
    one[i] <- 1
    p[i] <- (sqrt(1/(2*3.14*pow(x[i],3)))*exp(-1*pow(x[i]-
m,2)/2*pow(m,2)*x[i]))/10000
    one[i] ~ dbern(p[i])
  }
  m ~ dunif(1,3) #define uniform for parameter mu
  l ~ dgamma(25,3); #define gamma for shape parameter lambda
}

#define p as ratio
Binomialmodel <- function() {
  for (i in 1:n)
  {
    x[i] ~ dbern(p);
  }
  p ~ dbeta(33,4); #define beta for probability parameter
}

CI.CV.Bayes <- function(M,n1,n2,m1,m2,l1,l2,p1,p2){
  #M = loop for montecarlo
  #n = number of sample size
  #m = mean parameter of ZAIG
  #l = shape parameter of ZAIG
  #p = probability of zero for ZAIG
  set.seed(999)
  CP.theta.bayesian <- rep(0,M) # coverage probability of CI based on bayesianMCMC
  CP.eta.bayesian <- rep(0,M)
  CP.BAYE2.theta <- rep(0,M) # coverage probability of CI based on bayesianHPTD
  CP.BAYE2.eta <- rep(0,M)
  AW.theta.bayesian <- rep(0,M)
  AW.eta.bayesian <- rep(0,M)
  AW.BAYE2.theta <- rep(0,M)

```

```

AW.BAYE2.eta <- rep(0,M)
mcoda1 <- rep(0,M)
lcoda1 <- rep(0,M)
pcoda1 <- rep(0,M)
mcoda2 <- rep(0,M)
lcoda2 <- rep(0,M)
pcoda2 <- rep(0,M)
mean1 <- (1-p1)*m1
mean2 <- (1-p2)*m2
var1 <- m1^2*(1-p1)*((m1/l1)+p1)
var2 <- m2^2*(1-p2)*((m2/l2)+p2)
cv1 <- sqrt((m1+(l1*p1))/((1-p1)*l1))
cv2 <- sqrt((m2+(l2*p2))/((1-p2)*l2))
theta1 <- var1 - var2
eta1 <- var1 / var2
###stat iteration
for(i in 1:M){
  x <- rZAIG(n1,m1,l1,p1)
  x0 <- subset(x,x==0)
  x1 <- subset(x,x>0)
  x01 <- ifelse(x>0,1,0)
  n0_x <- length(x0)
  n1_x <- length(x1)

  y <- rZAIG(n2,m2,l2,p2)
  y0 <- subset(y,y==0)
  y1 <- subset(y,y>0)
  y01 <- ifelse(y>0,1,0)
  n0_y <- length(y0)
  n1_y <- length(y1)

  #Bayesian
  #for p1
  zerodata <- list(x = x0,n =length(x0))
  inits1<-function(){ list( p=0) }
  outputp <- bugs(data = zerodata, inits = inits1,parameters.to.save = c("p"),model.file
= Binomialmodel, n.chains = 1,n.iter = 20000,n.burnin =
5000,debug=FALSE,codaPkg=TRUE)
  out.codap <- read.bugs(outputp)
  codp <- as.vector(out.codap)
  pcoda1 <- codp[[1]][,2]

  #for m1,l1
  ZAIGdata <- list(x = x1,n =length(x1))

```

```

inits2<-function(){ list(l=2, m=3) }
outputm <- bugs(data = ZAIGdata, inits = inits2, parameters.to.save =
c("l","m"),model.file = IGmodel, n.chains = 1,n.iter = 20000,n.burnin =
5000,debug=FALSE,codaPkg=TRUE)
out.codam <- read.bugs(outputm)
codm <- as.vector(out.codam)
mcoda1 <- codm[[1]][,3]
lcoda1 <- codm[[1]][,2]

#for p2
zerodata <- list(x = y0,n =length(y0))
inits1<-function(){ list( p=0) }
outputp <- bugs(data = zerodata, inits = inits1,parameters.to.save = c("p"),model.file
= Binomialmodel, n.chains = 1,n.iter = 20000,n.burnin =
5000,debug=FALSE,codaPkg=TRUE)
out.codap <- read.bugs(outputp)
codp <- as.vector(out.codap)
pcoda2 <- codp[[1]][,2]

#for m2,l2
ZAIGdata <- list(x = y1,n =length(y1))
inits2<-function(){ list(l=2, m=3) }
outputm <- bugs(data = ZAIGdata, inits = inits2, parameters.to.save =
c("l","m"),model.file = IGmodel, n.chains = 1,n.iter = 20000,n.burnin =
5000,debug=FALSE,codaPkg=TRUE)
out.codam <- read.bugs(outputm)
codm <- as.vector(out.codam)
mcoda2 <- codm[[1]][,3]
lcoda2 <- codm[[1]][,2]

baye.mean1 <- (1-pcoda1)*mcoda1
baye.var1 <- mcoda1^2*(1-pcoda1)*((mcoda1/lcoda1)+pcoda1)
baye.cv1 <- sqrt((mcoda1+(lcoda1*pcoda1))/((1-pcoda1)*lcoda1))
baye.mean2 <- (1-pcoda2)*mcoda2
baye.var2 <- mcoda2^2*(1-pcoda2)*((mcoda2/lcoda2)+pcoda2)
baye.cv2 <- sqrt((mcoda2+(lcoda2*pcoda2))/((1-pcoda2)*lcoda2))
baye.theta <- baye.var1 - baye.var2
baye.eta <- baye.var1 / baye.var2
L.Baye.theta <- quantile(baye.theta,0.025,type = 8,na.rm = TRUE)
U.Baye.theta <- quantile(baye.theta,0.975,type = 8,na.rm = TRUE)
L.Baye.eta <- quantile(baye.eta,0.025,type = 8,na.rm = TRUE)
U.Baye.eta <- quantile(baye.eta,0.975,type = 8,na.rm = TRUE)
CP.theta.bayesian[i] <- ifelse(L.Baye.theta<theta1&&theta1<U.Baye.theta,1,0)
AW.theta.bayesian[i] <- U.Baye.theta-L.Baye.theta

```

```

CP.eta.bayesian[i] <- ifelse(L.Baye.eta<eta1&&eta1<U.Baye.eta,1,0)
AW.eta.bayesian[i] <- U.Baye.eta-L.Baye.eta

#HPTD
HPD.baye.theta =hdi(baye.theta,0.95)
L.baye2.theta =HPD.baye.theta [1]
U.baye2.theta =HPD.baye.theta [2]
CP.BAYE2.theta[i] = ifelse(L.baye2.theta<theta1&&theta1<U.baye2.theta,1,0) #
compute probability of CI based on BAYE of Baye
AW.BAYE2.theta[i] = U.baye2.theta- L.baye2.theta # compute length of CI
based on BAYE of Baye

HPD.baye.eta =hdi(baye.eta,0.95)
L.baye2.eta =HPD.baye.eta [1]
U.baye2.eta =HPD.baye.eta [2]
CP.BAYE2.eta[i] = ifelse(L.baye2.eta<eta1&&eta1<U.baye2.eta,1,0) # compute
probability of CI based on BAYE of Baye
AW.BAYE2.eta[i] = U.baye2.eta- L.baye2.eta # compute length of CI based on
BAYE of Baye

}

ACP.theta.Baye <- mean(CP.theta.bayesian)
AAW.theta.Baye <- mean(AW.theta.bayesian)
ACP.theta.HPDBaye <- mean(CP.BAYE2.theta)
AAW.theta.HPDBaye <- mean(AW.BAYE2.theta)
ACP.eta.Baye <- mean(CP.eta.bayesian)
AAW.eta.Baye <- mean(AW.eta.bayesian)
ACP.eta.HPDBaye <- mean(CP.BAYE2.eta)
AAW.eta.HPDBaye <- mean(AW.BAYE2.eta)
cat("CP theta BayesianMCMC=",ACP.theta.Baye,"\n")
cat("AW theta BayesianMCMC=",AAW.theta.Baye,"\n")
cat("CP theta BayesianHPTD=",ACP.theta.HPDBaye,"\n")
cat("AW theta BayesianHPTD=",AAW.theta.HPDBaye,"\n")
cat("CP eta BayesianMCMC=",ACP.eta.Baye,"\n")
cat("AW eta BayesianMCMC=",AAW.eta.Baye,"\n")
cat("CP eta BayesianHPTD=",ACP.eta.HPDBaye,"\n")
cat("AW eta BayesianHPTD=",AAW.eta.HPDBaye,"\n")

beep()
}

```

```
#####NA#####
tic()

set.seed(95)

# Single

# begin process
starttime <- proc.time()

# x = data
# M = number of simulation runs
# n0 = size of zero in population
# n1 = size of non-zero in population
# tau = mean of delta-IG dist
# alpha_sig = level of significance

M <- 5000

n_x=100 #,30,50,100,200
n_y=200 #,30,50,100,200
delta_x=0.3
delta_y=0.3
mu_x=0.5
mu_y=0.5
lambda_x=1
lambda_y=1

for (mu_x in c(0.5,1)) #0.3,0.5
{
  for (mu_y in c(0.5,1))
  {
    for (lambda_x in c(1,5,10)) #0.5,1
    {
      for (lambda_y in c(1,5,10)) #0.5,1
      {
        for(delta_x in c(0.3,0.5)) # <- 1,2,5,10
        {
          for(delta_y in c(0.3,0.5)) # <- 1,2,5,10
          {
            alpha_sig <-0.05
```

```

deltaprime_x <- 1-delta_x
deltaprime_y <- 1-delta_y

#estimator Variance
var_x <- mu_x^2*(1-delta_x)*((mu_x/lambda_x)+delta_x)
var_y <- mu_y^2*(1-delta_y)*((mu_y/lambda_y)+delta_y)
theta <- var_x - var_y
eta <- var_x/var_y

#-----NA-----#
L.NA_theta=c()
U.NA_theta=c()
L.NA_eta=c()
U.NA_eta=c()
AV.NA_theta=c()
CP.NA_theta=c()
AverLengthNAVar_theta=c()
CoveragePNAVar_theta=c()
AV.NA_eta=c()
CP.NA_eta=c()
AverLengthNAVar_eta=c()
CoveragePNAVar_eta=c()

mle_x=c()
mle_mu_x=c()
mle_lambda_x=c()
mle_delta_x=c()
mle_y=c()
mle_mu_y=c()
mle_lambda_y=c()
mle_delta_y=c()
theta_NA=c()
se_theta_NA=c()
eta_NA=c()
se_eta_NA=c()

options(digits = 4)

#####
###      Start Iteration      ###
#####

NAfunc=function(x1_1,x1_2){

```

```

mle_x=fitdist(x1_1,      "invgauss",      lower=c(0,0),      start=list(mean=1,
shape=1))$estimate
mle_y=fitdist(x1_2,      "invgauss",      lower=c(0,0),      start=list(mean=1,
shape=1))$estimate
mle_mu_x=mle_x[1]
mle_mu_y=mle_y[1]
mle_lambda_x=mle_x[2]
mle_lambda_y=mle_y[2]
mle_delta_x=n_zero_1/n_x
mle_delta_y=n_zero_2/n_y
z.alpha=qnorm(1-alpha_sig/2)

varhat_x <- mle_mu_x^2*(1-
mle_delta_x)*((mle_mu_x/mle_lambda_x)+mle_delta_x)
varhat_y <- mle_mu_y^2*(1-
mle_delta_y)*((mle_mu_y/mle_lambda_y)+mle_delta_y)

thetahat <- varhat_x - varhat_y
etahat <- varhat_x / varhat_y

var_varhat_x <- (((1-
mle_delta_x)*((mle_mu_x^2)*(1/mle_lambda_x))+((mle_mu_x/mle_lambda_x)+mle
_delta_x)*2*mle_mu_x^2)*(mle_mu_x^3/(mle_lambda_x*n1_1))+
(((mle_mu_x^2)*(1-mle_delta_x)*(-
mle_mu_x/mle_lambda_x^2))^2)*((2*mle_lambda_x^2)/n1_1)+
(((mle_mu_x^2)*(1-
mle_delta_x+((mle_mu_x/mle_lambda_x)+mle_delta_x)*(-1)))^2)*((mle_delta_x*(1-
mle_delta_x))/n_x)

var_varhat_y <- (((1-
mle_delta_y)*((mle_mu_y^2)*(1/mle_lambda_y))+((mle_mu_y/mle_lambda_y)+mle
_delta_y)*2*mle_mu_y^2)*(mle_mu_y^3/(mle_lambda_y*n1_2))+
(((mle_mu_y^2)*(1-mle_delta_y)*(-
mle_mu_y/mle_lambda_y^2))^2)*((2*mle_lambda_y^2)/n1_2)+
(((mle_mu_y^2)*(1-
mle_delta_y+((mle_mu_y/mle_lambda_y)+mle_delta_y)*(-1)))^2)*((mle_delta_y*(1-
mle_delta_y))/n_y)

var_var_diff <- var_varhat_x + var_varhat_y

se_theta_NA <- sqrt(var_var_diff)
se_eta_NA <- sqrt(var_var_diff)

```

```
L.NA_theta=thetahat-z.alpha*se_theta_NA
U.NA_theta=thetahat+z.alpha*se_theta_NA
```

```
L.NA_eta=etahat-z.alpha*se_eta_NA
U.NA_eta=etahat+z.alpha*se_eta_NA
return(data.frame(L.NA_theta,U.NA_theta,L.NA_eta,U.NA_eta))
}
```

```
for (i in 1:M)
{ #begin loop i
  ###      Generate x      ###
  x_0_1 <- rbinom(n_x, size=1 , prob=delta_x) #Choose n_zero from bi(n,deltta)
  x0_1 <- x_0_1[which(x_0_1!=0)]
  n_zero_1 <- length(x0_1)
  n_nonzero_1 <- n_x-n_zero_1

  x_0_2 <- rbinom(n_y, size=1 , prob=delta_y) #Choose n_zero from bi(n,deltta)
  x0_2 <- x_0_2[which(x_0_2!=0)]
  n_zero_2 <- length(x0_2)
  n_nonzero_2 <- n_y-n_zero_2

  x1_1 <- rinvgauss(n_nonzero_1, mu_x, lambda_x)
  x1_2 <- rinvgauss(n_nonzero_2, mu_y, lambda_y)
  x1 <- c(x0_1-1,x1_1)
  x2 <- c(x0_2-1,x1_2)

  n0_1 <- n_zero_1
  n1_1 <- n_nonzero_1
  nonzero_value_1 <- x1[which(x1!=0)]
  delta_hat_1 <- n_zero_1/n_x

  n0_2 <- n_zero_2
  n1_2 <- n_nonzero_2
  nonzero_value_2 <- x2[which(x2!=0)]
  delta_hat_2 <- n_zero_2/n_y

  #-----Begin NA-----#
  #MME of alpha and beta from the original sample
  #Call NAFunc
  NAVar<-NAfunc(x1_1,x1_2)
  L.NA_theta[i]=NAVar[,1]
  U.NA_theta[i]=NAVar[,2]
```

```

L.NA_eta[i]=NAVar[,3]
U.NA_eta[i]=NAVar[,4]

#-----END NA-----#

#calculate lower, upper bound and length for NA
AV.NA_theta[i]=U.NA_theta[i]-L.NA_theta[i]
AV.NA_eta[i]=U.NA_eta[i]-L.NA_eta[i]

if ( L.NA_theta[i]<=theta && theta<=U.NA_theta[i] ) { #for NA
  CP.NA_theta[i]=1 } else {CP.NA_theta[i]=0}

if ( L.NA_eta[i]<=eta && eta<=U.NA_eta[i] ) { #for NA
  CP.NA_eta[i]=1 } else {CP.NA_eta[i]=0}
}

AverLengthNAVar_theta=mean(AV.NA_theta) #calculate the average length for
NA
CoveragePNAVar_theta=mean(CP.NA_theta)

AverLengthNAVar_eta=mean(AV.NA_eta) #calculate the average length for NA
CoveragePNAVar_eta=mean(CP.NA_eta)

theta
eta

alpbe=c(n_x,n_y,mu_x,mu_y,lambda_x,lambda_y,delta_x,delta_y)

print(alpbe)
print(CoveragePNAVar_theta)
print(AverLengthNAVar_theta)
print(CoveragePNAVar_eta)
print(AverLengthNAVar_eta)
print("-----")
}
}
}
}
}
}

toc()

```

A5 Functions are used to calculate the coverage probability and the average width of confidence interval for the common coefficient of variation for multiple delta-IG distributions.

The following R code is used to compute the confidence intervals, the coverage probability, and the average width of the confidence intervals for the common coefficient of variation for multiple delta-IG distributions in Chapter 4.

```
#- This code was written by Wasurat Khumpasee -#

# for k = 3
CommonCVZIIG <- function(M, m, n1, n2, n3,
  delta1, delta2, delta3,
  mu1, mu2, mu3,
  lambda1, lambda2, lambda3) {

  library(statmod)
  library(HDIInterval)
  library(tictoc)

  tic()
  set.seed(3)

  G <- 3
  n_all <- c(n1, n2, n3)
  delta <- c(delta1, delta2, delta3)
  mu <- c(mu1, mu2, mu3)
  lambda <- c(lambda1, lambda2, lambda3)

  B <- 5000
  n_iter <- 20000
  burn_in <- 5000

  proposal_sd_mu <- 0.1
  proposal_sd_lambda <- 0.1

  # --- True CV ---
  cv_true <- sqrt((mu + lambda * delta) / ((1 - delta) * lambda))

  # --- Variance of CV (true) ---
  var_cv <- (((1/2)*(((1-delta)*lambda)/(mu+(lambda*delta)))^0.5 *
    (1/(((1-delta)*lambda)))^2) * (mu^3/(lambda*(n_all*delta))) +
    (((1/2)*(((1-delta)*lambda)/(mu+(lambda*delta)))^0.5 *
```

```

(((1-delta)*lambda*delta - (mu+lambda*delta)*(1-delta))/
  ((1-delta)*lambda^2)^2) * ((2*lambda^2)/(n_all*delta))) +
(((1/2)*(((1-delta)*lambda)/(mu+(lambda*delta)))^0.5 *
  (((1-delta)*lambda^2 - (mu+lambda*delta)*(-lambda))/
  ((1-delta)*lambda^2)^2) * ((delta*(1-delta))/n_all))

w <- 1/var_cv
theta_true <- sum(cv_true * w) / sum(w)

# --- ZIIG generator ---
rZIIG <- function(n, delta, mu, lambda) {
  ind <- rbinom(n, 1, 1 - delta)
  ig <- rinvgauss(n, mean = mu, shape = lambda)
  ifelse(ind == 1, ig, 0)
}

# --- storage ---
CP <- AV <- matrix(0, M, 3)
colnames(CP) <- colnames(AV) <- c("PBPCI", "BayeCI", "HPDCI")

for (i in 1:M) {

  # --- simulate ---
  x <- list(
    rZIIG(n1, delta1, mu1, lambda1),
    rZIIG(n2, delta2, mu2, lambda2),
    rZIIG(n3, delta3, mu3, lambda3)
  )

  nonzero <- lapply(x, function(z) z[z != 0])
  zero <- lapply(x, function(z) z[z == 0])

  n1_i <- sapply(nonzero, length)
  n0_i <- sapply(zero, length)

  if (any(n1_i == 0)) next

  mean_i <- sapply(nonzero, mean)
  deltahat <- n0_i / n_all

  # --- MLE ---
  inv_lambda <- mapply(function(x, m) sum(1/x - 1/m), nonzero, mean_i)
  lambdahat <- n1_i / inv_lambda
  muhat <- (1 - deltahat) * mean_i

```

```

cvhat <- sqrt((muhat + lambdahat * deltahat) / ((1 - deltahat) * lambdahat))

# ===== PBPCI =====
theta_boot <- numeric(B)

for (b in 1:B) {

  xb <- lapply(x, function(z) sample(z, replace=TRUE))
  nz <- lapply(xb, function(z) z[z!=0])
  n1_b <- sapply(nz, length)

  if (any(n1_b == 0)) next

  mean_b <- sapply(nz, mean)
  delta_b <- n0_i / n_all
  V <- mapply(function(x,m) sum(1/x - 1/m), nz, mean_b)
  lambda_b <- n1_b / V
  cv_b <- sqrt((mean_b + lambda_b*delta_b)/((1-delta_b)*lambda_b))

  # weighted common CV
  var_b <- (((1/2)*(((1-delta_b)*lambda_b)/(mean_b+(lambda_b*delta_b)))^0.5 *
    (1/((1-delta_b)*lambda_b)))^2) * (mean_b^3/(lambda_b*n1_b)) +
    (((1/2)*(((1-delta_b)*lambda_b)/(mean_b+(lambda_b*delta_b)))^0.5 *
    (((1-delta_b)*lambda_b*delta_b-(mean_b+lambda_b*delta_b)*(1-delta_b))/
    ((1-delta_b)*lambda_b)^2)^2) * ((2*lambda_b^2)/n1_b)) +
    (((1/2)*(((1-delta_b)*lambda_b)/(mean_b+(lambda_b*delta_b)))^0.5 *
    (((1-delta_b)*lambda_b^2-(mean_b+lambda_b*delta_b)*(-lambda_b))/
    ((1-delta_b)*lambda_b)^2)^2) * ((delta_b*(1-delta_b))/n_all))

  w_b <- 1 / pmax(var_b, 1e-10)
  theta_boot[b] <- sum(cv_b * w_b) / sum(w_b)
}

L <- quantile(theta_boot, 0.025, na.rm=TRUE)
U <- quantile(theta_boot, 0.975, na.rm=TRUE)
CP[i,"PBPCI"] <- theta_true >= L & theta_true <= U
AV[i,"PBPCI"] <- U - L

# ===== Bayesian =====

# --- Non-informative priors ---
# mu_i ~ Uniform(0, M_upper) : M_upper = 3 * max of nonzero
# lam_i ~ Gamma(0.001, 0.001) : vague prior

```

```

# delta ~ Beta(1, 1)      : conjugate → posterior Beta(1+n0, 1+n1)

all_nonzero <- unlist(nonzero)
M_upper    <- 3 * max(all_nonzero) # upper bound of Uniform prior for mu
gamma_shape <- 0.01                # hyperparameter of Gamma prior for lambda
gamma_rate  <- 0.01

# --- initialize at MLE ---
mu_curr    <- mean_i
lambda_curr <- lambdahat
delta_curr  <- deltahat

theta_post <- numeric(n_iter)

# --- tracking acceptance (optional: acceptance rate check) ---
accept_mu <- numeric(G)
accept_lam <- numeric(G)

for (it in 1:n_iter) {
  for (g in 1:G) {
    if (n1_i[g] > 0) {
      # ----- update mu_g -----
      # Prior: Uniform(0, M_upper)
      # Posterior ∝ likelihood × Uniform(0, M_upper)
      # log prior ratio = 0 if proposal in (0, M_upper)

      mu_prop <- rnorm(1, mu_curr[g], proposal_sd_mu)

      if (mu_prop > 0 && mu_prop < M_upper) {

        log_lik_mu <-
          sum(dinvgauss(nonzero[[g]], mu_prop, lambda_curr[g], log=TRUE)) -
          sum(dinvgauss(nonzero[[g]], mu_curr[g], lambda_curr[g], log=TRUE))

        # log prior ratio = log(1/M_upper) - log(1/M_upper) = 0

        if (log(runif(1)) < log_lik_mu) {
          mu_curr[g] <- mu_prop
          accept_mu[g] <- accept_mu[g] + 1
        }
      }
    }
  }
}

```

```

# if proposal out (0, M_upper) → reject

# ----- update lambda_g -----
# Prior: Gamma(0.001, 0.001)
# log posterior ratio = log_lik_ratio + log_prior_ratio

lam_prop <- rnorm(1, lambda_curr[g], proposal_sd_lambda)

if (lam_prop > 0) {

  log_lik_lam <-
    sum(dinvgauss(nonzero[[g]], mu_curr[g], lam_prop, log=TRUE)) -
    sum(dinvgauss(nonzero[[g]], mu_curr[g], lambda_curr[g], log=TRUE))

  # log prior ratio: log[Gamma(lam_prop)] - log[Gamma(lambda_curr)]
  log_prior_lam <-
    dgamma(lam_prop, shape=gamma_shape, rate=gamma_rate, log=TRUE)
    -
    dgamma(lambda_curr[g], shape=gamma_shape, rate=gamma_rate,
log=TRUE)

  log_ratio_lam <- log_lik_lam + log_prior_lam

  if (log(runif(1)) < log_ratio_lam) {
    lambda_curr[g] <- lam_prop
    accept_lam[g] <- accept_lam[g] + 1
  }
}
# if lam_prop <= 0 → reject
}
}

# ----- update delta_g -----
# Prior: Beta(1,1) = Uniform(0,1)
# Likelihood: Binomial(n_all, delta)
# Posterior conjugate: Beta(1 + n0_i, 1 + n1_i)
delta_curr <- rbeta(G, 1 + n0_i, 1 + n1_i)

# ----- posterior CV -----
cv_post <- sqrt((mu_curr + lambda_curr * delta_curr) /
  ((1 - delta_curr) * lambda_curr))

# weighted common CV posterior
var_post <- (((1/2)*((1-delta_curr)*lambda_curr)/

```

```

      (mu_curr+(lambda_curr*delta_curr)))^0.5 *
      (1/((1-delta_curr)*lambda_curr))^2) *
(mu_curr^3/(lambda_curr*n1_i)) +
(((1/2)*(((1-delta_curr)*lambda_curr)/
      (mu_curr+(lambda_curr*delta_curr)))^0.5 *
      (((1-delta_curr)*lambda_curr*delta_curr -
      (mu_curr+lambda_curr*delta_curr)*(1-delta_curr)) /
      ((1-delta_curr)*lambda_curr)^2)^2) *
      ((2*lambda_curr^2)/n1_i)) +
(((1/2)*(((1-delta_curr)*lambda_curr)/
      (mu_curr+(lambda_curr*delta_curr)))^0.5 *
      (((1-delta_curr)*lambda_curr^2 -
      (mu_curr+lambda_curr*delta_curr)*(-lambda_curr)) /
      ((1-delta_curr)*lambda_curr)^2)^2) *
      ((delta_curr*(1-delta_curr))/n_all))

w_post      <- 1 / pmax(var_post, 1e-10)
theta_post[it] <- sum(cv_post * w_post) / sum(w_post)
}

# ----- burn-in -----
post <- theta_post[(burn_in + 1):n_iter]

# ----- BayeCI: equal-tailed -----
L <- quantile(post, 0.025)
U <- quantile(post, 0.975)
CP[i,"BayeCI"] <- theta_true >= L & theta_true <= U
AV[i,"BayeCI"] <- U - L

# ----- HPDCI: highest posterior density -----
hpd <- hdi(post)
CP[i,"HPDCI"] <- theta_true >= hpd[1] & theta_true <= hpd[2]
AV[i,"HPDCI"] <- diff(hpd)
}

toc()

return(list(
  mean_CP = colMeans(CP, na.rm=TRUE),
  mean_AV = colMeans(AV, na.rm=TRUE)
))
}

```

```

# for k = 6
CommonCVZIIG <- function(M, m, n1, n2, n3, n4, n5, n6,
  delta1, delta2, delta3, delta4, delta5, delta6,
  mu1, mu2, mu3, mu4, mu5, mu6,
  lambda1, lambda2, lambda3, lambda4, lambda5, lambda6) {

  # Check and install packages if not already installed
  if (!requireNamespace("statmod", quietly = TRUE)) {
    install.packages("statmod")
  }
  if (!requireNamespace("HDInterval", quietly = TRUE)) {
    install.packages("HDInterval")
  }
  if (!requireNamespace("tictoc", quietly = TRUE)) {
    install.packages("tictoc")
  }

  library(statmod)
  library(HDInterval)
  library(tictoc)

  tic()
  set.seed(3)

  G <- 6
  n_all <- c(n1, n2, n3, n4, n5, n6)
  delta <- c(delta1, delta2, delta3, delta4, delta5, delta6)
  mu <- c(mu1, mu2, mu3, mu4, mu5, mu6)
  lambda <- c(lambda1, lambda2, lambda3, lambda4, lambda5, lambda6)

  B <- 5000
  n_iter <- 20000
  burn_in <- 5000

  proposal_sd_mu <- 0.1
  proposal_sd_lambda <- 0.1

  # --- True CV ---
  cv_true <- sqrt((mu + lambda * delta) / ((1 - delta) * lambda))

  # --- Variance of CV (true) ---
  var_cv <- (((1/2)*(((1-delta)*lambda)/(mu+(lambda*delta))))^0.5 *
    (1/(((1-delta)*lambda)))^2) * (mu^3/(lambda*(n_all*delta))) +
    (((1/2)*(((1-delta)*lambda)/(mu+(lambda*delta))))^0.5 *

```

```

(((1-delta)*lambda*delta - (mu+lambda*delta)*(1-delta))/
 ((1-delta)*lambda^2)^2) * ((2*lambda^2)/(n_all*delta))) +
(((1/2)*(((1-delta)*lambda)/(mu+(lambda*delta)))^0.5 *
 (((1-delta)*lambda^2 - (mu+lambda*delta)*(-lambda))/
 ((1-delta)*lambda^2)^2) * ((delta*(1-delta))/n_all))

w <- 1/var_cv
theta_true <- sum(cv_true * w) / sum(w)

# --- ZIIG generator ---
rZIIG <- function(n, delta, mu, lambda) {
  ind <- rbinom(n, 1, 1 - delta)
  ig <- rinvgauss(n, mean = mu, shape = lambda)
  ifelse(ind == 1, ig, 0)
}

# --- storage ---
CP <- AV <- matrix(0, M, 3)
colnames(CP) <- colnames(AV) <- c("PBPCI", "BayeCI", "HPDCI")

for (i in 1:M) {

  # --- simulate ---
  x <- list(
    rZIIG(n1, delta1, mu1, lambda1),
    rZIIG(n2, delta2, mu2, lambda2),
    rZIIG(n3, delta3, mu3, lambda3),
    rZIIG(n4, delta4, mu4, lambda4),
    rZIIG(n5, delta5, mu5, lambda5),
    rZIIG(n6, delta6, mu6, lambda6)
  )

  nonzero <- lapply(x, function(z) z[z != 0])
  zero <- lapply(x, function(z) z[z == 0])

  n1_i <- sapply(nonzero, length)
  n0_i <- sapply(zero, length)

  if (any(n1_i == 0)) next

  mean_i <- sapply(nonzero, mean)
  deltahat <- n0_i / n_all

  # --- MLE ---

```

```

inv_lambda <- mapply(function(x, m) sum(1/x - 1/m), nonzero, mean_i)
lambdahat <- n1_i / inv_lambda
muhat <- (1 - deltahat) * mean_i

cvhat <- sqrt((muhat + lambdahat * deltahat) / ((1 - deltahat) * lambdahat))

# ===== PBPCI =====
theta_boot <- numeric(B)

for (b in 1:B) {

  xb <- lapply(x, function(z) sample(z, replace=TRUE))
  nz <- lapply(xb, function(z) z[z!=0])
  n1_b <- sapply(nz, length)

  if (any(n1_b == 0)) next

  mean_b <- sapply(nz, mean)
  delta_b <- n0_i / n_all
  V <- mapply(function(x,m) sum(1/x - 1/m), nz, mean_b)
  lambda_b <- n1_b / V
  cv_b <- sqrt((mean_b + lambda_b*delta_b)/((1-delta_b)*lambda_b))

  # weighted common CV
  var_b <- (((1/2)*(((1-delta_b)*lambda_b)/(mean_b+(lambda_b*delta_b)))^0.5 *
    (1/(((1-delta_b)*lambda_b))^2) * (mean_b^3/(lambda_b*n1_b)) +
    (((1/2)*(((1-delta_b)*lambda_b)/(mean_b+(lambda_b*delta_b)))^0.5 *
    (((1-delta_b)*lambda_b*delta_b-(mean_b+lambda_b*delta_b)*(1-delta_b))/
    ((1-delta_b)*lambda_b)^2)^2) * ((2*lambda_b^2)/n1_b)) +
    (((1/2)*(((1-delta_b)*lambda_b)/(mean_b+(lambda_b*delta_b)))^0.5 *
    (((1-delta_b)*lambda_b^2-(mean_b+lambda_b*delta_b)*(-lambda_b))/
    ((1-delta_b)*lambda_b)^2)^2) * ((delta_b*(1-delta_b))/n_all))

  w_b <- 1 / pmax(var_b, 1e-10)
  theta_boot[b] <- sum(cv_b * w_b) / sum(w_b)
}

L <- quantile(theta_boot, 0.025, na.rm=TRUE)
U <- quantile(theta_boot, 0.975, na.rm=TRUE)
CP[i,"PBPCI"] <- theta_true >= L & theta_true <= U
AV[i,"PBPCI"] <- U - L

# ===== Bayesian =====

```

```

# --- Non-informative priors ---
# mu_i ~ Uniform(0, M_upper)
# lam_i ~ Gamma(0.001, 0.001) : vague prior
# delta ~ Beta(1, 1) : conjugate → posterior Beta(1+n0, 1+n1)

all_nonzero <- unlist(nonzero)
M_upper <- 3 * max(all_nonzero) # upper bound of Uniform prior for mu
gamma_shape <- 0.001 # hyperparameter of Gamma prior for lambda
gamma_rate <- 0.001

# --- initialize at MLE ---
mu_curr <- mean_i
lambda_curr <- lambdahat
delta_curr <- deltahat

theta_post <- numeric(n_iter)

# --- tracking acceptance (optional: acceptance rate check) ---
accept_mu <- numeric(G)
accept_lam <- numeric(G)

for (it in 1:n_iter) {
  for (g in 1:G) {
    if (n1_i[g] > 0) {
      # ----- update mu_g -----
      # Prior: Uniform(0, M_upper)
      # Posterior ∝ likelihood × Uniform(0, M_upper)
      # log prior ratio = 0 if proposal in (0, M_upper)

      mu_prop <- rnorm(1, mu_curr[g], proposal_sd_mu)

      if (mu_prop > 0 && mu_prop < M_upper) {

        log_lik_mu <-
          sum(dinvgauss(nonzero[[g]], mu_prop, lambda_curr[g], log=TRUE)) -
          sum(dinvgauss(nonzero[[g]], mu_curr[g], lambda_curr[g], log=TRUE))

        # log prior ratio = log(1/M_upper) - log(1/M_upper) = 0

        if (log(runif(1)) < log_lik_mu) {

```

```

    mu_curr[g] <- mu_prop
    accept_mu[g] <- accept_mu[g] + 1
  }
}
# if proposal out (0, M_upper) → reject

# ----- update lambda_g -----
# Prior: Gamma(0.001, 0.001)
# log posterior ratio = log_lik_ratio + log_prior_ratio

lam_prop <- rnorm(1, lambda_curr[g], proposal_sd_lambda)
if (lam_prop > 0) {
  log_lik_lam <-
    sum(dinvgauss(nonzero[[g]], mu_curr[g], lam_prop, log=TRUE)) -
    sum(dinvgauss(nonzero[[g]], mu_curr[g], lambda_curr[g], log=TRUE))

  # log prior ratio: log[Gamma(lam_prop)] - log[Gamma(lambda_curr)]
  log_prior_lam <-
    dgamma(lam_prop, shape=gamma_shape, rate=gamma_rate, log=TRUE)
  -
    dgamma(lambda_curr[g], shape=gamma_shape, rate=gamma_rate,
log=TRUE)

  log_ratio_lam <- log_lik_lam + log_prior_lam

  if (log(runif(1)) < log_ratio_lam) {
    lambda_curr[g] <- lam_prop
    accept_lam[g] <- accept_lam[g] + 1
  }
}
# if lam_prop <= 0 → reject
}
}

# ----- update delta_g -----
# Prior: Beta(1,1) = Uniform(0,1)
# Likelihood: Binomial(n_all, delta)
# Posterior conjugate: Beta(1 + n0_i, 1 + n1_i)
delta_curr <- rbeta(G, 1 + n0_i, 1 + n1_i)

# ----- posterior CV -----
cv_post <- sqrt((mu_curr + lambda_curr * delta_curr) /

```

```

      ((1 - delta_curr) * lambda_curr))

# weighted common CV posterior
var_post <- (((1/2)*(((1-delta_curr)*lambda_curr)/
      (mu_curr+(lambda_curr*delta_curr)))^0.5 *
      (1/(((1-delta_curr)*lambda_curr)))^2) *
      (mu_curr^3/(lambda_curr*n1_i)) +
      (((1/2)*(((1-delta_curr)*lambda_curr)/
      (mu_curr+(lambda_curr*delta_curr)))^0.5 *
      (((1-delta_curr)*lambda_curr*delta_curr -
      (mu_curr+lambda_curr*delta_curr)*(1-delta_curr)) /
      ((1-delta_curr)*lambda_curr)^2)^2) *
      ((2*lambda_curr^2)/n1_i)) +
      (((1/2)*(((1-delta_curr)*lambda_curr)/
      (mu_curr+(lambda_curr*delta_curr)))^0.5 *
      (((1-delta_curr)*lambda_curr^2 -
      (mu_curr+lambda_curr*delta_curr)*(-lambda_curr)) /
      ((1-delta_curr)*lambda_curr)^2)^2) *
      ((delta_curr*(1-delta_curr))/n_all))

w_post <- 1 / pmax(var_post, 1e-10)
theta_post[it] <- sum(cv_post * w_post) / sum(w_post)
}

# ----- burn-in -----
post <- theta_post[(burn_in + 1):n_iter]

# ----- BayeCI: equal-tailed -----
L <- quantile(post, 0.025)
U <- quantile(post, 0.975)
CP[i,"BayeCI"] <- theta_true >= L & theta_true <= U
AV[i,"BayeCI"] <- U - L

# ----- HPDCI: highest posterior density -----
hpd <- hdi(post)
CP[i,"HPDCI"] <- theta_true >= hpd[1] & theta_true <= hpd[2]
AV[i,"HPDCI"] <- diff(hpd)
}

toc()

return(list(
  mean_CP = colMeans(CP, na.rm=TRUE),
  mean_AV = colMeans(AV, na.rm=TRUE)

```

))
{



A6 Functions are used to calculate the coverage probability and the average width of simultaneous confidence intervals for all pairwise differences between coefficients of variation for multiple delta-IG distributions.

The following R code is used to compute the simultaneous confidence intervals, the coverage probability, and the average width of the simultaneous confidence intervals for all pairwise differences between coefficients of variation for multiple delta-IG distributions in Chapter 5.

```
#- This code was written by Wasurat Khumpasee -#

library(EnvStats)
library(HDIInterval)
library(VGAM)
library(MASS)
library(coda)
library(boot)
library(R2OpenBUGS)
library(statmod)
library(beepr)
library(plotrix)

CI.SCIdiffCVs_k <- function(
  M, m,
  n, mu, lambda, delta,
  alpha = 0.05,
  methods = c("GCI", "AGCI", "FCI", "CIMV", "N_A"),
  seed = 999
){
  stopifnot(length(n)==length(mu),
            length(mu)==length(lambda),
            length(lambda)==length(delta))

  library(statmod)
  set.seed(seed)
  options(digits=6)

  k <- length(n)
  pairs <- combn(k, 2)
  P <- ncol(pairs)

  # ===== True parameters =====
  CV.true <- sqrt((mu + lambda * delta) / ((1 - delta) * lambda))
  theta.true <- CV.true[pairs[1, ]] - CV.true[pairs[2, ]]
```

```

# ===== Storage =====
CP <- Length <- lapply(methods, function(x) numeric(M))
names(CP) <- names(Length) <- methods

starttime <- proc.time()

# =====
for (r in 1:M) {

  # ===== Data generation =====
  x <- vector("list", k)
  n0 <- n1 <- n2 <- muhat <- lambdahat <- deltahat <- cvhat <- numeric(k)

  for (i in 1:k) {
    z <- rbinom(n[i], 1, delta[i])
    n0[i] <- sum(z)
    n1[i] <- n[i] - n0[i]
    n2[i] <- n0[i]+n2[i]
    if (n1[i] <= 1) stop("Degenerate sample.")

    x[[i]] <- rinvgauss(n1[i], mu[i], lambda[i])

    deltahat[i] <- n0[i] / n[i]
    muhat[i] <- mean(x[[i]])
    lambdahat[i] <- 1 / mean(1/x[[i]] - 1/muhat[i])

    cvhat[i] <- sqrt((muhat[i] + lambdahat[i]*deltahat[i]) /
      ((1-deltahat[i])*lambdahat[i]))
  }

  # ===== GCI =====
  if ("GCI" %in% methods) {
    R_CV_GCI <- matrix(NA, m, k)
    for (j in 1:m) {
      for (i in 1:k) {
        A <- rchisq(1, n1[i]-1)
        R_lambda <- A / (n1[i] * (1/lambdahat[i]))
        Z <- rnorm(1)
        R_mu <- muhat[i] /
          abs(1 + Z * sqrt(muhat[i] / (n1[i]*R_lambda)))
        R_delta <- deltahat[i]

        R_CV_GCI[j,i] <- sqrt((R_mu + R_lambda*R_delta) /

```

```

        ((1-R_delta)*R_lambda))
    }
}

R_theta <- combn(k, 2, FUN = function(id)
  R_CV_GCI[,id[1]] - R_CV_GCI[,id[2]])

L <- apply(R_theta, 2, quantile, alpha/2)
U <- apply(R_theta, 2, quantile, 1-alpha/2)

CP$GCI[r] <- mean(L < theta.true & theta.true < U)
Length$GCI[r] <- mean(U - L)
}

# ===== AGCI =====
if ("AGCI" %in% methods | "CIMV" %in% methods | "N_A" %in% methods) {
  R_CV_AGCI <- matrix(NA, m, k)
  for (j in 1:m) {
    for (i in 1:k) {
      A <- rchisq(1, n1[i]-1)
      R_lambda <- A / (n1[i] * (1/lambdahat[i]))
      Z <- rnorm(1)
      R_mu <- muhat[i] /
        abs(1 + Z * sqrt(muhat[i] / (n1[i]*R_lambda)))
      ZD <- rnorm(1)
      R_delta <- (sin(asin(sqrt(deltahat[i])) -
        ZD/(2*sqrt(n[i]))))^2
      R_CV_AGCI[j,i] <- sqrt((R_mu + R_lambda*R_delta) /
        ((1-R_delta)*R_lambda))
    }
  }
}

if ("AGCI" %in% methods) {
  R_theta <- combn(k, 2, FUN = function(id)
    R_CV_AGCI[,id[1]] - R_CV_AGCI[,id[2]])

  L <- apply(R_theta, 2, quantile, alpha/2)
  U <- apply(R_theta, 2, quantile, 1-alpha/2)

  CP$AGCI[r] <- mean(L < theta.true & theta.true < U)
  Length$AGCI[r] <- mean(U - L)
}

```

```

# ===== CIMV =====
if ("CIMV" %in% methods) {
  Lcv <- apply(R_CV_AGCI, 2, quantile, alpha/2)
  Ucv <- apply(R_CV_AGCI, 2, quantile, 1-alpha/2)

  theta.hat <- combn(cvhat, 2, FUN = function(z) z[1]-z[2])
  L.mv <- U.mv <- numeric(P)

  for (p in 1:P) {
    i <- pairs[1,p]; j <- pairs[2,p]
    L.mv[p] <- theta.hat[p] -
      sqrt((cvhat[i]-Lcv[i])^2 + (Ucv[j]-cvhat[j])^2)
    U.mv[p] <- theta.hat[p] +
      sqrt((Ucv[i]-cvhat[i])^2 + (cvhat[j]-Lcv[j])^2)
  }

  CP$CIMV[r] <- mean(L.mv < theta.true & theta.true < U.mv)
  Length$CIMV[r] <- mean(U.mv - L.mv)
}

# ===== FCI =====
if ("FCI" %in% methods) {
  R_CV_FCI <- matrix(NA, m, k)

  trim <- function(x, lower = 1e-6, upper = 1e6)
    pmin(pmax(x, lower), upper)

  for (i in 1:k) {

    # ---- initialize fiducial sequences ----
    mu.t <- numeric(m)
    lambda.t <- numeric(m)

    mu.t[1] <- muhat[i]
    lambda.t[1] <- lambdahat[i]

    # ---- iterative fiducial generation ----
    for (t in 2:m) {
      lambda.t[t] <- trim((lambda.t[t-1] / n1[i]) *
        rchisq(1, n1[i] - 1))
      mu.t[t] <- rinvgauss(1, mu.t[t-1], n1[i] * lambda.t[t])
    }
  }
}

```

```

}

# ---- fiducial delta (arcsin-sqrt) ----
Z.F <- 2 * sqrt(n1[i]) * (asin(sqrt(deltahat[i])) - asin(sqrt(delta[i])))
R_delta <- (sin(asin(sqrt(deltahat[i])) - Z.F / (2 * sqrt(n1[i]))))^2

# ---- fiducial CV ----
R_CV_FCI[, i] <- sqrt(
  (mu.t + lambda.t * R_delta) /
  ((1 - R_delta) * lambda.t)
)
}

# ---- pairwise fiducial differences ----
R_theta <- combn(k, 2, FUN = function(id)
  R_CV_FCI[, id[1]] - R_CV_FCI[, id[2]])

# ---- confidence intervals ----
L <- apply(R_theta, 2, quantile, alpha / 2)
U <- apply(R_theta, 2, quantile, 1 - alpha / 2)

CP$FCI[r] <- mean(L < theta.true & theta.true < U)
Length$FCI[r] <- mean(U - L)
}

# ===== NA =====
if("N_A" %in% methods) {
  var_CV_asym <- function(mu, lambda, delta, n1, n) {

    A <- ((1 - delta) * lambda) / (mu + lambda * delta)
    sqrtA <- sqrt(A)

    d_mu <- (1/2) * sqrtA * (1 / ((1 - delta) * lambda))

    d_lambda <- (1/2) * sqrtA *
      (((1 - delta) * lambda * delta -
        (mu + lambda * delta) * (1 - delta)) /
        ((1 - delta) * lambda)^2)

    d_delta <- (1/2) * sqrtA *
      (((1 - delta) * lambda^2 -
        (mu + lambda * delta) * (-lambda)) /
        ((1 - delta) * lambda)^2)
  }
}

```

```

var_mu <- mu^3 / (lambda * n1)
var_lambda <- 2 * lambda^2 / n1
var_delta <- delta * (1 - delta) / n

max(
  d_mu^2 * var_mu +
  d_lambda^2 * var_lambda +
  d_delta^2 * var_delta,
  0
)
}

var.cv <- sapply(1:k, function(i)
  var_CV_asym(
    mu = muhat[i],
    lambda = lambdahat[i],
    delta = deltahat[i],
    n1 = n1[i],
    n = n[i]
  )
)

theta.hat <- combn(cvhat, 2, FUN = function(z) z[1]-z[2])

se.na <- sapply(1:P, function(p){
  i <- pairs[1,p]; j <- pairs[2,p]
  sqrt(var.cv[i] + var.cv[j])
})

z <- qnorm(1 - alpha/2)
L.na <- theta.hat - z * se.na
U.na <- theta.hat + z * se.na

CP$N_A[r] <- mean(L.na < theta.true & theta.true < U.na)
Length$N_A[r] <- mean(U.na - L.na)
}
}
# =====

elapsed <- (proc.time() - starttime)["elapsed"]

data.frame(

```

```

Method = methods,
Coverage = sapply(methods, function(m) mean(CP[[m]])),
AvgLength = sapply(methods, function(m) mean(Length[[m]])),
Time = elapsed
)
}

```





APPENDIX B PUBLICATIONS

Article

Confidence Intervals for the Coefficient of Variation in Delta Inverse Gaussian Distributions

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Abstract: The inverse Gaussian distribution is characterized by its asymmetry and right-skewed shape, indicating a longer tail on the right side. This distribution represents extreme values in one direction, such as waiting times, stochastic processes, and accident counts. Moreover, depending on if the accident counts data can occur or not and may have zero value, the Delta Inverse Gaussian (Delta-IG) distribution is more suitable. The confidence interval (CI) for the coefficient of variation (CV) of the Delta-IG distribution in accident counts is essential for risk assessment, resource allocation, and the creation of transportation safety policies. Our objective is to establish CIs of CV for the Delta-IG population using various methods. We considered seven CI construction methods, namely Generalized Confidence Interval (GCI), Adjusted Generalized Confidence Interval (AGCI), Parametric Bootstrap Percentile Confidence Interval (PBPCI), Fiducial Confidence Interval (FCI), Fiducial Highest Posterior Density Confidence Interval (F-HPDCI), Bayesian Credible Interval (BCI), and Bayesian Highest Posterior Density Credible Interval (B-HPDCI). We utilized Monte Carlo simulations to assess the proposed CI technique for average widths (AWs) and coverage probability (CP). Our findings revealed that F-HPDCI and AGCI exhibited the most effective coverage probability and average widths. We applied these methods to generate CIs of CV for accident counts in India.

Keywords: bootstrap; generalized confidence interval; Bayesian credible interval; fiducial confidence interval; the highest posterior density interval



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1. Introduction

Observations that are non-negative with an excess of zeros are frequently seen in areas such as economics, biomedical sciences, life sciences, and meteorology. These data often exhibit asymmetry, with a distribution tilted to the right (positively skewed) and heteroscedastic, meaning that the spread of the data varies. In such cases, many observations cluster at zero, while the positive values follow a right-skewed pattern. For example, in a clinical experiment, the duration of episodes such as low blood pressure, excessive perspiration, and rapid heartbeat is measured minutes after administering a particular medicine. Some patients may show a measurement value of zero, while those with non-zero values tend to follow an asymmetric, right-skewed distribution.

The data model that was appropriate for incorporating zero-inflated non-negative observations into a continuous distribution was examined by Liu et al. [1]. The data that were utilized encompassed information derived from the microbiota, medical bills, and the amount of alcohol consumed. Later, several researchers proposed utilizing the Inverse Gaussian (IG) distribution to analyze these kinds of data. To illustrate this, Heller et al. [2] investigated insurance data with zero-inflated information and discovered that the Inverse Gaussian distribution was better for analysis than the Gamma distribution. Ma and Yan [3] found that the Inverse Gaussian distribution was suitable for analyzing data above zero values through examining the frequency of traffic accidents in Colorado, USA. Regression models were employed to predict accident frequencies and estimate parameters using



Research article

Confidence Intervals for the Difference and Ratio of Two Coefficients of Variation of Delta-Inverse Gaussian Distributions

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Abstract: The inverse Gaussian (IG) distribution demonstrates asymmetry and right skewness. This distribution exhibits values uniformly, including waiting duration, stochastic processes, and accident occurrence rates. The delta inverse Gaussian (delta-IG) distribution is suitable for modeling traffic accident data as a count of fatalities, particularly in instances where incidents may not occur. The confidence interval (CI) for the difference and ratio of two coefficients of variation of delta-IG distributions in accident frequencies is essential for assessing risk, distributing resources, and developing improvement strategies for transportation safety. Our objective is to establish confidence intervals for the difference and ratio of the coefficients of variation of two delta-IG populations through various methods: Adjusted GCI (AGCI), Fiducial CI (FCI), Bayesian Credible Interval (BCI), Method of Variance Estimates Recovery (MOVER), and Normal Approximation (NA). Monte Carlo simulations were assessed, and the proposed confidence interval approach was implemented for average width (AW) and coverage probability (CP). The results indicated that the AGCI and MOVER methods employed efficient procedures, as evidenced by their CP and AW. We utilized these approaches to generate confidence intervals for the difference and ratio of two coefficient of variations of delta-IG distributions in the mortality count data from the central region of Thailand.

Keywords: Adjusted generalized confidence interval; Bayesian credible interval; Coverage probability; Fiducial confidence interval; Method of variance estimates recovery; Normal approximation.

Mathematics Subject Classification: 62F25.

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Article

Confidence Intervals for the Variance and Standard Deviation of Delta-Inverse Gaussian Distributions with Application to Traffic Mortality Count

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Abstract: The inverse Gaussian (IG) distribution exhibits asymmetry and right skewness. This distribution presents values uniformly, encompassing wait length, stochastic processes, and rates of accident occurrences. The delta-inverse Gaussian (delta-IG) distribution is suitable for modeling traffic accident data as a mortality count, especially in cases when accidents may not occur. The confidence interval (CI) for the variance and standard deviation of the delta-IG distribution for the accident count is crucial for evaluating risk, allocating resources, and formulating enhancement protocols for transportation safety. We aim to construct confidence intervals for variance and standard deviation in the delta-IG population using several approaches: Adjusted GCI (AGCI), Parametric Bootstrap Percentile CI (PBPCI), fiducial CI (FCI), and Bayesian credible interval (BCI). The AGCI, PBPCI, and FCI will be utilized with estimation methods for proportions which are VST, Wilson's score, and Hannig approaches. Monte Carlo simulations were evaluated, and the suggested confidence interval approach was employed for the average width (AW) and coverage probability (CP). The findings demonstrated that the AGCI based on the VST method employed successful approaches, as seen in their CP and AW. We employed these approaches to produce CIs for the variance and S.D. of the mortality count in Bangkok.

Keywords: parametric bootstrap confidence interval; adjusted generalized confidence interval; Bayesian credible interval; fiducial confidence interval



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1. Introduction

Parameter estimation is a fundamental component of statistical inference, widely utilized in theoretical and practical statistics. Population parameters may be expressed as individual values or ranges depending on the purpose of the analytical study. A point estimate derives a singular value for an unknown parameter based on sampled data. Conversely, interval estimation is utilized to establish a range of plausible values for the parameter. Interval estimation entails determining the upper and lower limits that define this range, sometimes using confidence measures. Unlike point estimation, interval estimation requires a comprehensive grasp of the probability distribution and statistical characteristics of the unknown parameter, offering a significant advantage in inferential depth. This study uses interval estimation to compute the confidence interval (CI).

In several fields, including economics, meteorology, biology, and medicine, data characterized by non-negativity and a significant number of zeros are commonly seen. These sorts of data frequently exhibit asymmetry with right-skewed distributions and heteroscedasticity, characterized by a significant presence of both zero and positive values.

Article

Confidence Intervals for the Difference and Ratio of Two Variances of Delta-Inverse Gaussian Distributions

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Abstract

Accurate statistical inference for zero-inflated and highly skewed data requires confidence interval procedures with a strong finite-sample performance. The delta-inverse Gaussian distribution provides a flexible framework for modeling such data by combining a point mass at zero with an inverse Gaussian distribution for positive observations, making it suitable for application in various fields such as traffic mortality, insurance, and environmental studies. This paper develops and compares several confidence interval estimation methods for the difference and the ratio of two variances from independent delta-IG distributions. The proposed approaches include adjusted generalized confidence intervals, fiducial confidence intervals, Bayesian credible intervals, the method of variance estimates recovery, and normal approximation methods used as benchmarks. The finite-sample performance of these methods is evaluated through Monte Carlo simulations under various parameter configurations and both balanced and unbalanced sample sizes, with an emphasis on coverage probability and interval width. The simulation results show that AGCI and MOVER generally achieve coverage probabilities close to the nominal level while producing relatively narrow intervals. The MOVER performs particularly well when zero-inflation probabilities are equal, whereas AGCI is more effective when they differ. Illustrative real-data examples are provided to demonstrate practical implementations.

Keywords: adjusted generalized confidence interval; Bayesian credible interval; fiducial confidence interval; method of variance estimates recovery; normal approximation

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1. Introduction

Estimating population parameters is a fundamental task in statistical inference, where point estimation and interval estimation play complementary roles. While point estimates provide single-value summaries, confidence intervals (CIs) quantify sampling uncertainty and are therefore essential for reliable statistical conclusions. In many practical situations, the performance of classical CI procedures based on normal approximations or large-sample theory can be unsatisfactory, particularly when the underlying data are highly skewed, heavy-tailed, or exhibit a substantial proportion of exact zeros. Developing CI methods with strong finite-sample properties for such data structures remains an important challenge in modern statistical inference.

The zero-inflated and semi-continuous data arise naturally in a wide range of applied fields, including insurance claim modeling, environmental and hydrological studies, reliability and survival analysis, epidemiology, and transportation-related research. In these

Article

Simultaneous Confidence Intervals for All Pairwise Differences of Coefficients of Variation of Delta-Inverse Gaussian Distributions

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Abstract

This study develops and evaluates simultaneous confidence interval procedures for all pairwise differences of coefficients of variation under delta-inverse Gaussian distributions. The objective is to provide reliable comparative inference for relative variability in zero-inflated and highly skewed data, where standard normal-based methods may be unreliable. Five approaches were studied and compared in terms of coverage probabilities and average widths: generalized confidence interval, adjusted generalized confidence interval, fiducial confidence interval, method of variance estimates recovery, and normal approximation. A Monte Carlo simulation study was conducted under varying shape parameters, zero-inflation probabilities, sample sizes, and numbers of populations ($k = 3, 6$, and 10). Although most methods produced CPs near the nominal 0.95 level, meaningful differences emerged when both coverage accuracy and interval efficiency were considered. The AGCI method consistently delivered stable coverage across parameter settings and remained robust as dimensionality increased. The MOVER approach achieved competitive coverage while frequently yielding narrower intervals. In contrast, GCI occasionally showed mild undercoverage, and FCI tended to produce overly wide intervals. An empirical application to zero-inflated mortality data supports the simulation findings. Overall, AGCI and MOVER provide reliable and practical tools for simultaneous inference on differences in CVs across delta-IG populations.

Keywords: adjusted generalized confidence interval; fiducial confidence interval; generalized confidence interval; method of variance estimates recovery; normal approximation



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1. Introduction

Quantifying variability is a fundamental objective in statistical analysis across numerous scientific disciplines. Although variance and standard deviation are widely used measures of dispersion, their dependence on measurement scale limits their effectiveness when comparisons among heterogeneous populations are required. The coefficient of variation (CV), defined as the ratio of the standard deviation to the mean, provides a scale-independent measure of relative variability and is therefore particularly useful for comparing stochastic behavior across populations with different magnitudes or units of measurement. Consequently, statistical inference for the CV has been widely investigated in environmental studies, reliability engineering, finance, and biomedical applications [1–4].

In many real-world situations, interest lies not only in estimating a single CV but also in comparing relative variability across multiple populations. Over the past decades, various



APPENDIX C
TRACE PLOTS FOR CONVERGENCE DIAGNOSTICS OF MARKOV
CHAIN MONTE CARLO GIBBS SAMPLING

Trace plots for convergence diagnostics of MCMC Gibbs Sampling.

The trace plots presented in this appendix were generated to assess the convergence of the Markov Chain Monte Carlo (MCMC) Gibbs sampling algorithm used in the Fiducial confidence interval (FCI) and Bayesian credible interval (BCI) estimation. For instance, the results shown are representative of the simulation scenario with $n = 50$, $\mu = 0.5$, $\lambda = 10$, and $\delta = 0.3$, based on a total of 20,000 iterations with a burn-in period of 5,000 iterations, yielding 15,000 post-burn-in samples used for posterior inference.

Figure C-1 presents the trace plots and corresponding posterior density plots for parameters μ and λ of the inverse Gaussian component, based on 15,000 post-burn-in samples. The first row displays the deviance monitored automatically by OpenBUGS, where the trace oscillates stably around approximately 570 with no systematic drift, and the posterior density is unimodal and approximately symmetric, confirming that the model has reached a stationary state. The second row presents the results for parameter λ , where the trace plot exhibits consistent random fluctuation within the range of approximately 6 to 16 throughout the sampling period, and the posterior density is unimodal and approximately bell-shaped and symmetric with a peak near 10, indicating good mixing and stable convergence. The third row corresponds to parameter μ , where the trace similarly demonstrates stationary behavior within the range of approximately 0.5 to 0.9, and the posterior density is unimodal with a right-skewed shape, with a peak near 0.6, reflecting the asymmetric nature of the posterior arising from the positivity constraint on μ .

Figure C-2 presents the trace plots and posterior density plots for the zero-inflation parameter δ . The first row shows the deviance for the binomial component model, where the trace fluctuates randomly within the range of approximately 10 to 40 and the density is unimodal with a right-skewed shape centered near 15, confirming model stability. The second row displays the results for parameter δ , where the trace plot shows stationary oscillation within the range of approximately 0.1 to 0.7 with no apparent trend throughout the sampling period, and the posterior density is unimodal and right-skewed with a peak near 0.4. The slight upward shift from the true parameter value of $\delta = 0.3$ reflects the influence of the beta prior specification used in this study.

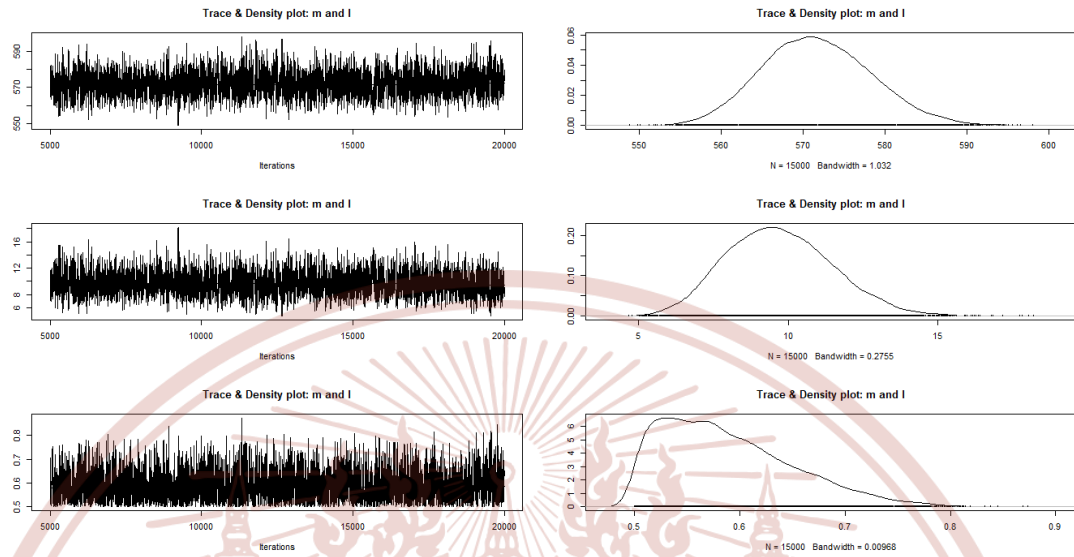


FIGURE C-1 Trace plots and posterior density plots for parameters μ and λ obtained from the MCMC Gibbs sampling.

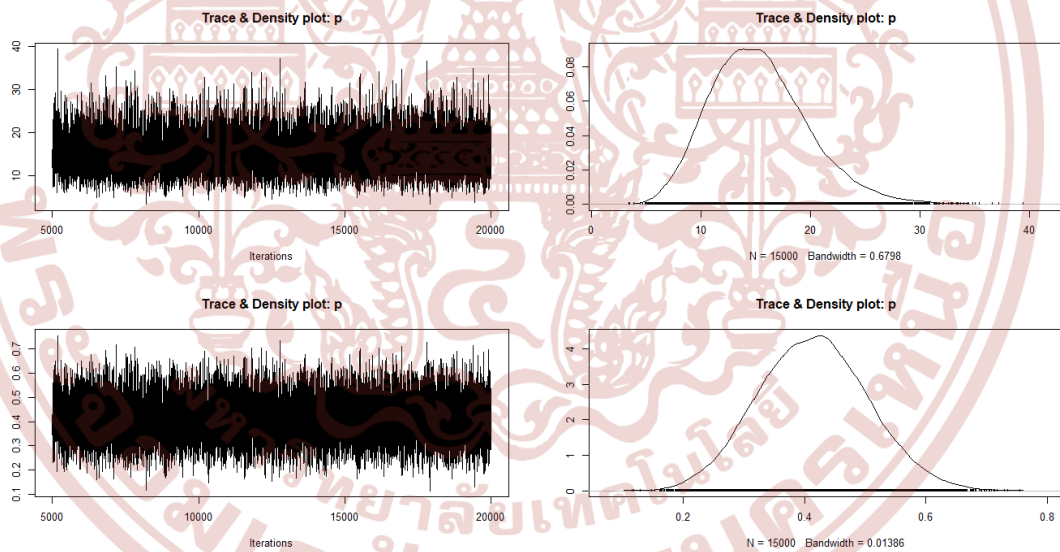


FIGURE C-2 Trace plots and posterior density plots for parameter δ obtained from the MCMC Gibbs sampling.

Across all parameters, the trace plots confirm that the MCMC chains had reached their stationary distributions well within the 5,000 burn-in iterations. The absence of systematic trends, drift, or periods of slow mixing in all trace plots supports the adequacy of the chosen burn-in length and the reliability of the posterior samples used for constructing the FCI and BCI reported in this study.

VITA

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Biography

Education

I received a Bachelor's degree in Mathematics with second-class honors from King Mongkut's University of Technology Thonburi in 2017 and a Master's degree in Applied Statistics from Silpakorn University 2021.

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